

UNIT IV

LIMIT STATE DESIGN OF COLUMNS

DESIGN OF AXIALLY LOADED SHORT COLUMNS

INTRODUCTION

- Members in compression are called columns and struts.
- The term 'column' is reserved for members which transfer loads to the ground and the term 'strut' is applied to a compression member in any direction, as those on a truss.
- Column members whose height is not more than three times its lateral dimension are called pedestals while the term 'wall' is used to compression members whose breadth is more than four times the thickness of the wall.
- It is well known from the theory of structures that the modes of failure of a column depend on its slenderness ratio.
- This ratio is expressed in IS and BS practice for reinforced concrete rectangular columns as the ratio of the effective length L_e to its least lateral dimension (d), (L_e/d) ratio.
- In steel columns the slenderness ratio is generally expressed as the effective length to its least radius of gyration (L) ratio.
- This practice is continued for R.C.C. columns in ACI code. Effective length L_e of a column is different from its unsupported length L .
- Columns, when centrally loaded, fail in one of the three following modes, depending on the slenderness ratio (see Fig. 13.1).

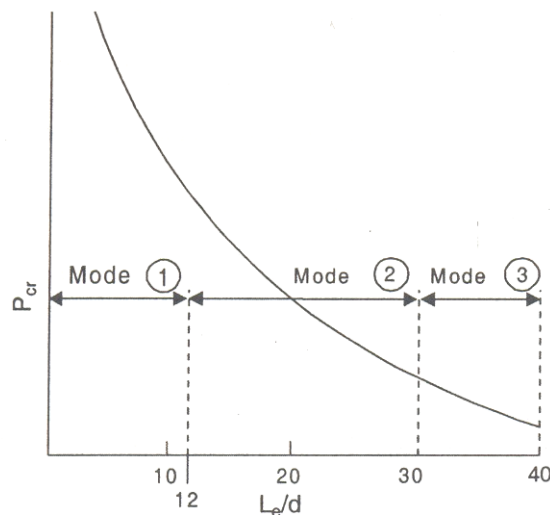


Fig. 13.1 Modes of failure of columns.

- **Mode 1: Pure compression failure**
 - The column fails under axial loads without undergoing any lateral deformation. Steel and concrete reach the yield stress values at failure.
 - The collapse of the column is due to material failure.
- **Mode 2: Combined compression and bending failure**
 - Short columns can be subjected to direct load (F) and moment (M). Slender columns even when loaded axially undergo deflection along their length as beam columns, and these deflections produce additional moments in the columns.
 - When material failure is reached under the combined action of these direct loads and bending moment, it is called combined compression and bending failure.
- **Mode 3: Failure by elastic instability**
 - Very long columns can become unstable even under small loads well before the material reaches yield stresses. Under such cases the member fails by lateral 'elastic buckling'
 - Failure by the third mode is unacceptable in practical construction (see Fig. 13.1).
 - R.C.C. members that may fail by this type of failure is prevented by the code provision that columns beyond a specified slenderness (30 for unbraced columns) should not be allowed in structures.

SHORT COLUMNS

- IS 456 classifies rectangular columns as short when the ratio of the effective length (L_i) to the least dimension is less than 12.
- This ratio is called the slenderness ratio of the column. If the column is of dimension $b \times D$, then there are two slenderness ratios namely,
 - **Slenderness ratio about major axis** $= L_{ex}/D$
 - **Slenderness ratio about minor axis** $= L_{ey}/b$

- If any of these two ratios is equal to or more than 12, it is called a slender column.
- If both ratios are less than 12, it is a short column.
- In BS the dividing ratio is taken as 15 for braced column and 10 for unbraced columns.
- In ACI the D_r ratio is used instead of the l/b ratio. The dividing line is taken as $D_r = 34$ for braced columns and $D_r = 22$ for unbraced columns.

BRACED AND UNBRACED COLUMNS

- columns can be planned in a structure so that they do not have to withstand any horizontal load like wind and earthquake loads.
- Thus, for example, when the column of water-tower are braced, the wind load is taken by the interaction of column bracings.
- In tall buildings, lateral supports like shear walls can be provided so that the lateral loads are taken by them.
- Such columns are called braced columns.
- Other columns, where the lateral loads have to be resisted in addition to vertical loads by the strength of the columns themselves, are considered as unbraced columns.
- Bracings can be in one direction or in more than one direction, depending on the likelihood of the direction

Design of Short Columns with Moments

- Columns, such as the external columns of framed buildings, or columns carrying crane loads through corbels of a column, are subjected not only to direct loads (F), but also to moments (M) due to the eccentricity in application of the load (Fig. 14.1).
- In the above columns, the eccentricity is with respect to one axis only and these columns are said to be under uniaxial bending.

- On the other hand, a corner column of a building is subjected to eccentric load along both the X and Y axes.
- Such columns are said to be under biaxial bending. In Chapter 13 it was pointed out that the short column formula takes into account accidental eccentricity to a certain extent.
- (whichever is more) is greater than $0.05D$ already provided in the formula for short column, then the short column formula cannot be used for design of such columns.
- The theory of short columns subjected to axial load and moments should be used for their strength calculations.
- Slender columns, even when they are subjected to central load, bending moments are produced as they undergo deflection along its length as a beam column.
- Such columns also have to be designed as eccentrically loaded columns.

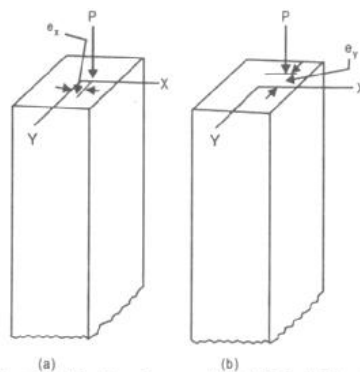


Fig. 14.1 Columns under direct load and moments: (a) Uniaxial bending; (b) Biaxial bending.

METHODS OF DESIGN

- The three methods that are commonly used to design these columns are:
 - Use of design charts (interaction diagrams)
 - Use of equilibrium equation to determine the minimum steel required
 - A simplified approximate method considering the section as a doubly reinforced beam.
- Of these, the interaction diagram is extensively used for design of rectangular or circular columns with symmetric arrangement of steel.

- The equilibrium method is based on fundamental concepts and is applicable to any cross-section and any arrangement of steel.
- The simplified method is found useful for columns with large eccentricities where the column acts more like a beam.

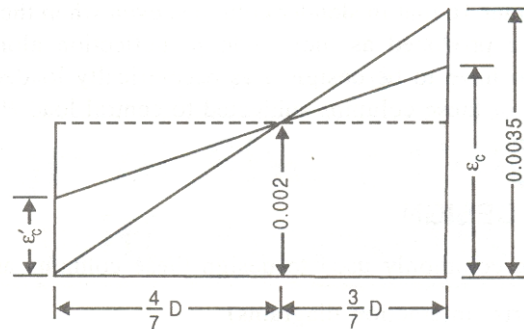
UNIAXIAL BENDING (DESIGN ASSUMPTIONS)

- In eccentrically loaded columns the strain distribution across the section will not be uniform as in the case of centrally loaded columns.
- As there is bending in addition to direct load, the strain distribution will vary linearly across the section as in the case of beams.
- The following assumptions as given in IS 456, Clause 39.1(b) are used to calculate the value of P and M of a given section:
- Plane sections remain plane even after bending.
- The strain at different points in the section will be different. The maximum compressive strain in concrete at failure is the governing criterion for ultimate failure. The magnitude of this failure strain is given by the expression (see also Fig. 14.2)

$$\epsilon_c = 0.0035 - 0.75\epsilon'_c$$

- where ϵ is the 'maximum' strain in compression in the section at failure and ϵ' is the 'minimum' strain in compression in the section at failure.
- The minimum compression strain in a beam ϵ' is equal to zero so that $\epsilon = 0$.
- In an axially loaded column, $\epsilon' = \epsilon$ so that ϵ at failure = 0.002.
- It is interesting to note that the above expression is also equivalent to assuming that the strain distribution diagram rotates around a fulcrum at a distance $3D/7$ from the highly compressed edge.
- This fulcrum point is the same point where the concrete strain will be 0.002 at failure with the neutral axis at the far edge of the section, as shown in Fig. 14.2.
- The design compression stress block under the varying strain is the same as assumed for beams. It is rectangular parabolic with the maximum stress value σ_c (approximately equal to f_{ck}) at failure.
- The design stress-strain curve for steel in compression is the same as in tension. The same design curve as for beams is assumed for columns also.

- The tensile strength of concrete is ignored.



Failure strain in concrete under direct load and moments.

STRESS-STRAIN CURVE FOR STEEL

- One should have a good idea of the stress-strain curves recommended by IS for the different types of steels to correctly assess the stresses corresponding to the strain in the steel.
- According to IS 456, mild steel (Fe 250) has a bilinear stress-strain curve, and failure strain on the design stress- strain curve is given by the expression

$$\epsilon_s = \frac{0.87 \times 250}{200 \times 10^3} = 0.00109$$

- However, the design stress-strain curves of cold drawn bars, like Fe 415 steel, is not bilinear.
- There is a linear and a non-linear strain, for the stress levels beyond 0.801, so that the strains on the design stress-strain curve corresponding to the various design stress levels .

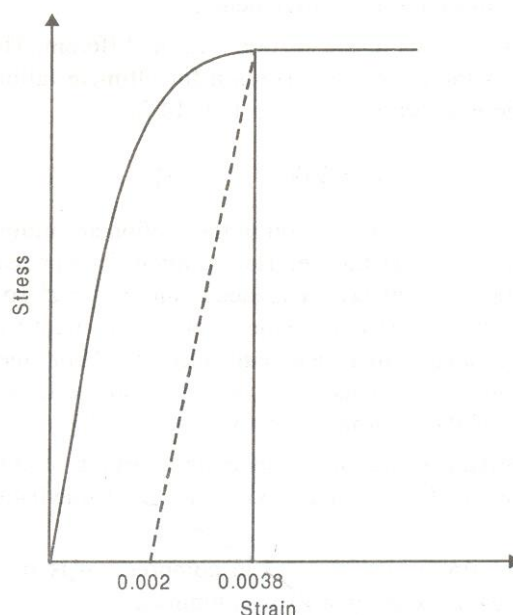


Fig. 14.3 Stress-strain curve for Fe 415 steel.

Points on the X-ordinate will be given by rearranging the terms as

$$\frac{M_u}{f_{ck}bD^2} = 0.36k(0.5 - 0.42k) \quad (14.4b)$$

Putting different values of k in Eqs. (14.4a) and (14.4b), the Y and X ordinates are obtained as

1. For $k = 1$, $Y = 0.36$ and $X = 0.029$
2. For $k = 0.5$, $Y = 0.18$ and $X = 0.12$
3. For $k = 0$, $Y = 0$ and $X = 0$

These are the values plotted in the interaction diagrams given in SP 16.

14.10 SHAPE OF INTERACTION CURVES

The shape of the interaction diagram will vary with the shape of the stress-strain diagram of steel, with kinks corresponding to the kinks in the stress-strain for steel. This can be noted from the differences in the shapes of the curves given in SP 16 for Fe 250, 415 and 500 steels.

14.11 ACCIDENTAL ECCENTRICITY IN COLUMNS

It was shown in Chapter 13 that the accidental eccentricity that should be assumed in a R.C. column according to IS 456 is

$$e_{\min} = \frac{L}{500} + \frac{D}{30}, \text{ but not less than 20 mm (see Section 13.7)}$$

With small values of M , the consequent eccentricity $e = M/P$ may be small. In such cases one should note that the design moment—should be always larger than both of the following:

$$M = M \text{ (given)}$$

$$M = Pe_{\min} \text{ (due to accidental eccentricity)}$$

14.12 USE OF INTERACTION DIAGRAMS FOR DESIGN AND ANALYSIS (METHOD 1)

Case 1: Use of interaction curves to determine the area of steel required for a given column size for specified P and M

Step 1: Check whether the column is short or long. If it is long, proceed as in Chapter 15. If short, proceed as follows:

- Step 3: Determination of areas of steel from interaction curve is as follows:
Choosing proper curve for grades of steel and d' find P/f_{ck} .
- Calculate $A = p/(bD)$. Distribute this total area A as distributed in the sketch given in SP 16 for the interaction diagram.
- Case 2: Use of interaction curve to analyzed the safety of the given column for given P and M , with symmetrical distribution of steel
- **Step 1:** Find the following parameters:

$$\frac{a}{D}, \quad \frac{P}{f_{ck}}, \quad \frac{P_u}{f_{ck}bD}, \quad \frac{M_u}{f_{ck}bD^2}$$

Step 2: Determine safety of column as follows: Determine the point corresponding to the given

$$\frac{P_u}{f_{ck}bD}, \quad \frac{M_u}{f_{ck}bD^2}$$

on the corresponding interaction curve for P/f_{ck} . If the point is on or inside the interaction curve, the column is safe. If the point is outside the curve, it is unsafe.

DESIGN OF ECCENTRIC COLUMNS BY EQUILIBRIUM EQUATION (METHOD 2)

- Another method that can be used for design of eccentrically loaded column is to work from fundamentals by using equilibrium equation and to arrive at the necessary steel for a given section of breadth b , depth D , with given P and M , as follows:
- Step 1: Assume the arbitrary depth of neutral axis. Let the extreme fibre in concrete reach failure strain as explained in the assumptions in Section 14.3 above.
- Step 2: Determine the strains in the steels.
- Step 3: Determine the compression force in concrete by using Table 14.1 or by other means. Find also the stress f_s in compression steel that will be provided near the compression face from the strain at the level of steel.
- Step 4: Determine the area of steel to be provided at the compression face (A_{s1}) by taking moments of all the forces about the position of the steel at the tension face. The moment equilibrium equation will be

$$P[e + D/2 - d'] = \left(\begin{array}{l} \text{Moment of compression} \\ \text{in concrete about tension} \\ \text{steel} + f_{sc} A_{s1}(d - d') \end{array} \right)$$

Step 5: Determine A_{s2} , the steel required on the tension face, from the second equation of equilibrium of forces

$$P = C_c + f_{s1}A_{s1} \pm f_{s2}A_{s2}$$

Step 6: From these, determine the total area of steel

$$A_s = A_{s1} + A_{s2}$$

Plot the value of A_s against the value of the depth of the neutral axis assumed (Fig. 14.9).

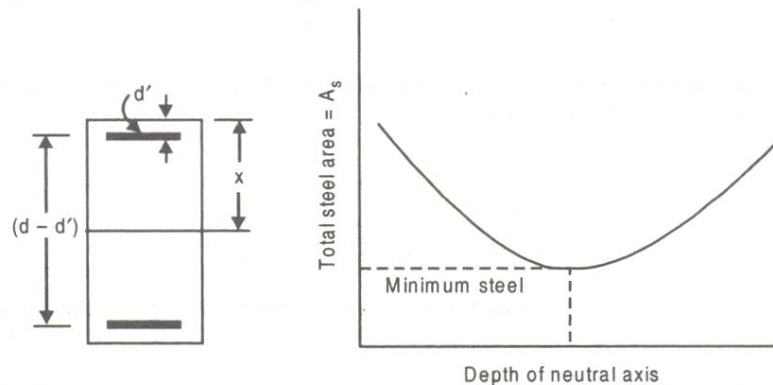


Fig. 14.9 Plot to determine minimum steel in section under direct load and moment.

- Step 7: Assume other suitable values of the depth of neutral axis and plot the values of A (total steel) needed, in the above diagram. The values of the neutral axis corresponding to minimum value of A can be taken as the optimum solution of the problem.
- Step 8: Provide the steel A and A_s as obtained in step 7 for optimum value of x . The procedure is similar.

SIMPLIFIED METHOD (METHOD 3)

- Columns under large eccentricity of load may be solved by considering them as equivalent to a doubly reinforced beam with a concentrated load acting on it (for equilibrium of forces) as shown in Fig. 14.10.
- The effect of P and M may be regarded as equivalent to a modified moment ($M + M_a$) and a force P applied along the steel on the tension side, where

$$M + M_a = M + P \left(\frac{D}{2} - d_2 \right)$$

- In addition, P acts through the tension steel.
- As tables for doubly reinforced beams for these large eccentricities will not be available, calculation has to be made from the basic equations.
- The simplified stress block as uniform compression can be used for satisfying the equilibrium of forces.

- Taking moments about the tension steel, we get

$$M_a = (0.4f_{ck}b)(0.5d)(0.75d) + 0.72f_y A_{sc}(d - d')$$

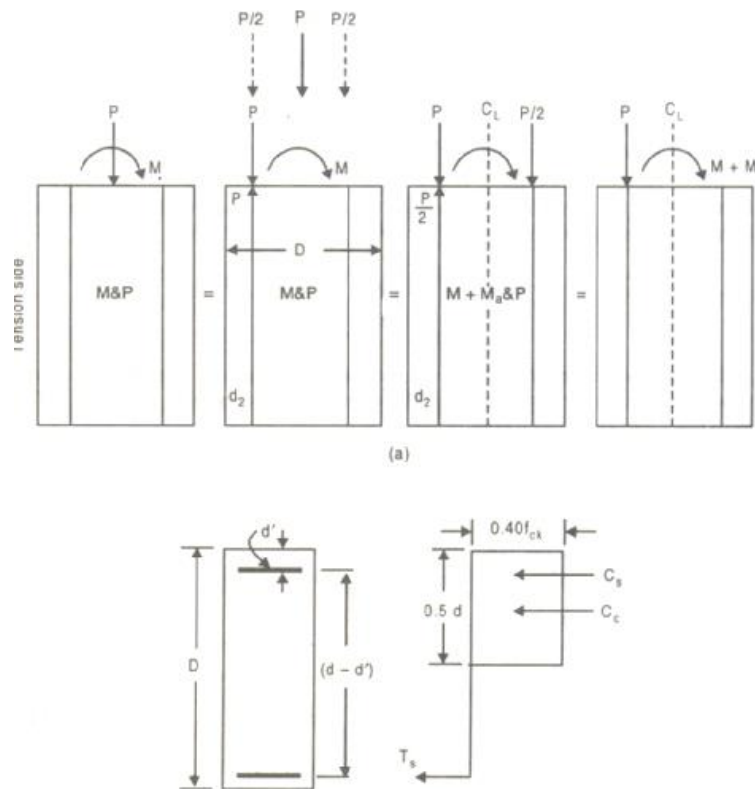
$$= 0.15f_{ck}bd^2 + 0.72f_y A_{sc}(d - d')$$

from which the value of A_{sc} , the area of compression steel, can be calculated.

For equilibrium of forces, the area of tension steel A_{st} is given by the equation

$$0.87f_y A_{st} = (0.4f_{ck}b)(0.5d) + 0.72f_y A_{sc}$$

$$= 0.2f_{ck}bd + 0.72f_y A_{sc}$$



- However, a correction has to be made for the value of P acting as compression P on tension steel.
- This reduces the tension steel required and the area of the reduced tension steel is given $A_{st1} = A_{st} - \left(\frac{P}{0.87f_y} \right)$
- It is evident from Eq. (14.3) that the equations are valid only when the beam theory as is valid for the situation, i.e. the eccentricity is larger than $(D/2 - 2)$.

MEMBER SUBJECTED TO BIAXIAL BENDING

- IS 456, Clause 38.6 deals with biaxial bending. The analysis may be used by one of the following two methods:
 - By choosing the neutral axis which is in the XY-plane. Calculations are made from fundamentals to satisfy the equilibrium of load and moments about both the axes. (This method is quite tedious and is not generally recommended for routine design).
 - By the use of the formula recommended for use of IS 456. The above code recommends the use of the following relation:

$$\left(\frac{M_x}{M_{x1}}\right)^{\alpha_n} + \left(\frac{M_y}{M_{y1}}\right)^{\alpha_n} \leq 1.0 \quad (14.9)$$

where M_x and M_y are the applied moments about the X and Y axes and M_{x1} and M_{y1} are the maximum uniaxial moments the column can take under the actual load P by bending along the XX - and YY -axis, respectively.

α_n is related to the value of P/P_z

where

P = design load on the column

$P_z = 0.45f_{ck} + 0.75f_y A_{sc}$ (i.e. value of P_u when $M = 0$)

α_n = exponent whose value is to be taken as follows:

P/P_z	α_n
0.2	1.0
0.8	2.0

The intermediate values are to be obtained by interpolation, as shown in Fig. 14.11. The value of α_n can be also determined by the equation

$$\alpha_n = \frac{2}{3} \left(1 + \frac{5}{2} \frac{P}{P_z} \right)$$

which should be within 1.0 and 2.0 as above.

- The IS code formula follows the “Bresler Load Contour Method”. It is based on the concept of a failure surface which is the envelope of a number of interaction curves for different axes of bending of a column, as shown in Fig.14.12.
- Any point of the failure surface corresponds to failure condition in a column about a neutral axis and any point inside the surface can be considered safe.

- A horizontal section at each level defines a load contour corresponding to a particular PIPE value. The general form of the contour is defined by relation (14.9).
- It may be noted that the limiting value $\alpha = 1.0$ for $P/P_t = 0.2$ represents the equation of a straight line and the value of $\alpha = 2$ for $P/P_t = 0.8$ represents a circle.
- These values are meant to represent the shape of the interaction diagrams at these points. Application of this formula is greatly facilitated by Chart 64 given in SP 16 (Chart 14.3 of the text).

SIMPLIFIED BS 8110 METHOD FOR BIAXIAL BENDING

- For the design of symmetrically reinforced rectangular columns under biaxial bending, results comparable with those obtained by the Bresler method can be obtained by the simplified design procedure recommended in BS 8110, Clause 3.8.4.5.
- The principle of the method is to transform the biaxial bending case to a uniaxial bending case which should withstand an increased moment about that axis according to the two conditions which are now given.
- Let the column be subjected to $(P, M_x \text{ and } M_y)$. Then it can be designed for uniaxial bending of (P, M') or (P, M'') , depending on the following conditions:

Condition 1: When $M_x/d \geq M_y/b'$, M_x controls the design and the column is to be designed for P and M'_x , where

$$M'_x = M_x + \alpha \frac{d}{b'} M_y$$

Condition 2: When $M_x/d < M_y/b'$, M_y controls the design and the column is to be designed for P and M'_y , where

$$M'_y = M_y + \alpha \frac{b'}{d} M_x$$

In the above expression,

d = effective depth with respect to major axis and total depth D

b' = effective depth with respect to minor axis and total depth b

$$\alpha = \text{coefficient} = \left(1 - \frac{7}{6} \frac{P}{f_{ck} b D^2} \right)$$

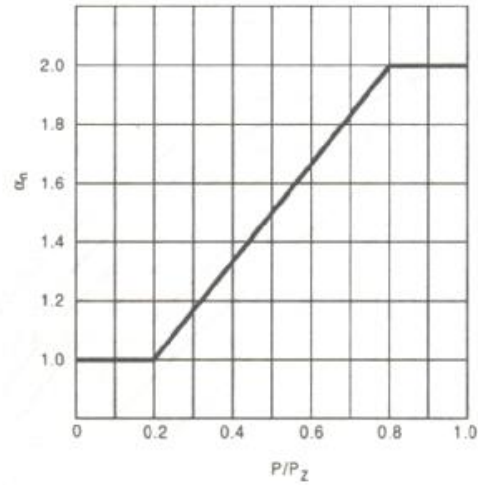


Fig. 14.11 Illustration of coefficient α_n for biaxial bending of columns.

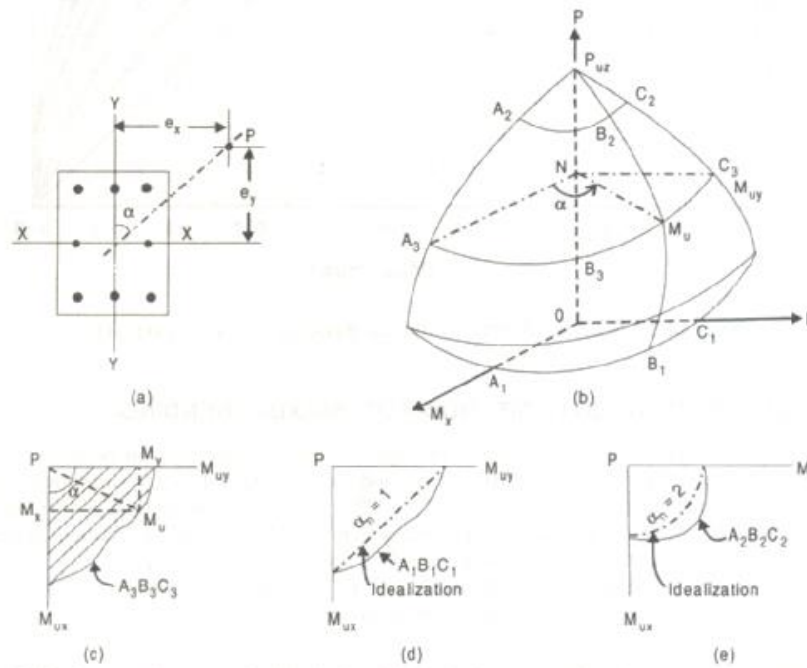


Fig. 14.12 Interaction curve for biaxial bending of columns: (a) Section; (b) Interaction surface; (c) Section at A_3C_3 ; (d) Section at A_1C_1 ; (e) Section at A_2C_2 .

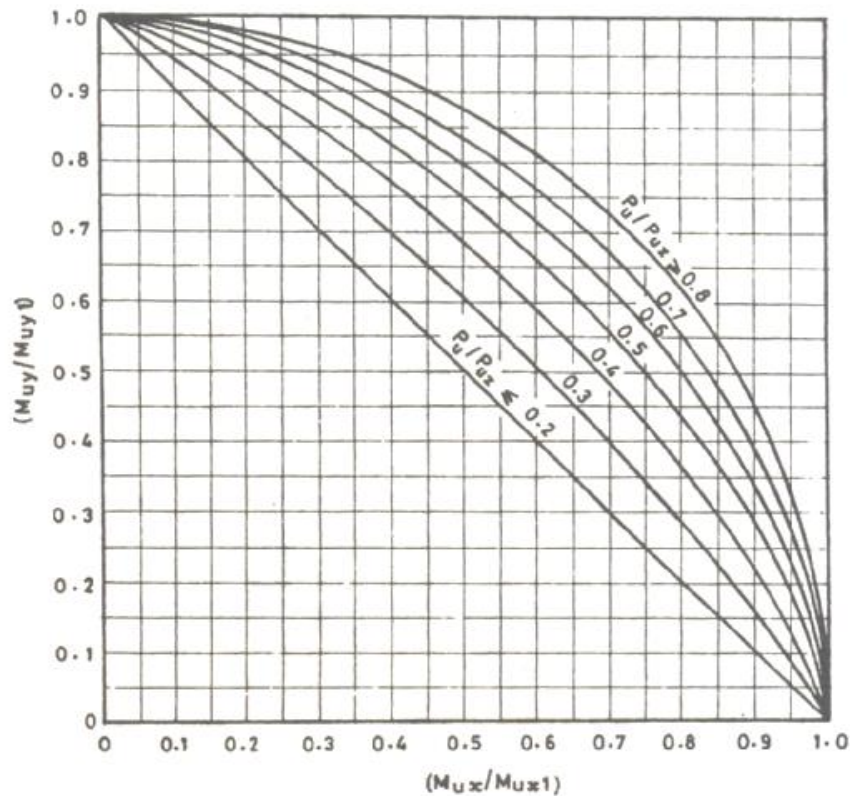
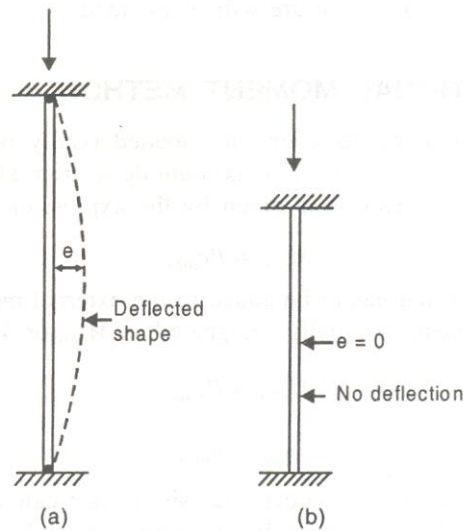


Chart 14.3 Check for Safety in Biaxial Bending [SP 16 Chart 64].

Design of R.C. Slender Columns

INTRODUCTION

- Rectangular columns when the ratio of the effective length of the columns to its lateral dimension exceeds 12, it is called a slender or long column according to IS code.
- In BS 8110, the limits for a short column are put as 15 for braced and 10 for unbraced columns.
- The difference between the behavior of short and slender columns is that, when slender columns are loaded even with axial loads, the lateral deflection (measured from their original centre lines along its length) becomes appreciable, whereas in short columns this lateral deflection is very small and can be neglected as shown in Fig.



1 Behaviour of long and short columns: (a) Long column; (b) Short column.

- In slender columns the moment produced by this deflection is large and has to be taken into account in design. Hence slender columns, even if centrally loaded, have to be designed for not secondary moment produced by the lateral deflection.
- There are three major methods that are used to take into account the slenderness effect of these columns. They are:
 - The reduction coefficient method
 - The additional moment method
 - The moment magnification method.
- The reduction coefficient method as given in IS 4 generally recommended for working stress design where designs are made for service loads (not factored loads) using allowable stresses in steel and concrete.
- The procedure is to reduce the allowable load by a reduction factor that depends on the slenderness of the column as given in IS 456, Clause 8.3.3 of Annex. B.
- In the limit state method, however, where one works with factored loads and ultimate strengths of steel and concrete, one of the other two methods is generally recommended.
- The British and the Indian codes specify the use of the additional moment method as given in IS 456, Clause 38.7.1, whereas ACT recommends the use of moment-magnification method.

MAXIMUM PERMITTED LENGTH OF COLUMNS

- In order to avoid buckling failures, IS 456, Clause 25.3.1 limits the unsupported length between restraints to 60 times the least lateral dimension. In practice, the unsupported length to breadth ratio of column is restricted to 60 in braced columns and 30 in unbraced columns.
- This restriction will ensure that the final failure will be due to material failure only and the classical buckling failure will be avoided.

BASIS OF ADDITIONAL MOMENT METHOD

- Slender columns, even when loaded axially, produce moments along their length due to lateral deflection.
- If δ is the maximum deflection, along the axis of the column, the moment produced by this deflection is given by the expression

$$M_{\max} = Pe_{\max}$$

This is the additional moment that has to be added to the external moments to obtain the design moment. This additional moment is usually designated as M_{add} or M_a so that

$$M_{\text{add}} = Pe_{\text{add}}$$

where

$$e_{\text{add}} = e_{\max}$$

- The analysis for add, which is a second order analysis, is also called **P-J analysis**. In un braced frames such an analysis should also include the effect of sway deflections. However, this aspect is not dealt with in this text and is found in literature on structural analysis of frames.

EXPRESSION FOR LATERAL DEFLECTION

- In theory of structures the elastic deflection of a bending member is represented by the $\frac{d^2y}{dx^2} = \frac{M}{EI} = \frac{1}{R}$

- $\frac{1}{R}$ is called the curvature, which may be defined as the change in slope over unit length of the member, assuming it to be constant along this length.
- Integrating the above expression twice, we get

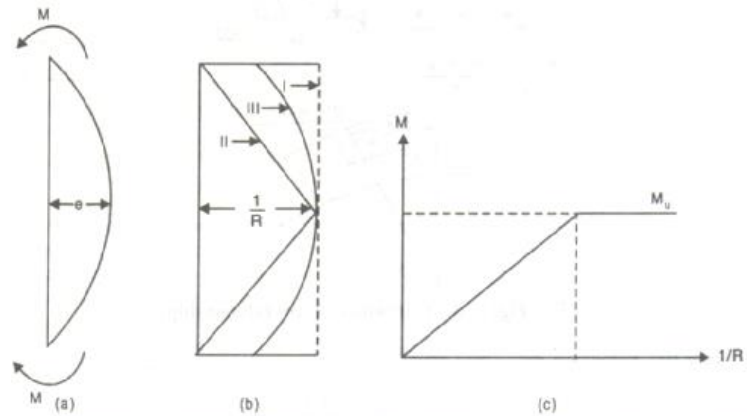
$$y = \iint \left(\frac{1}{R} dx \right) dx = \iint \left(\frac{M}{EI} dx \right) dx$$

- This means that the deflection depends on the distribution of curvature of M/EI diagram, along the length of the member.
- Assuming that the moment curvature relationship is linear and is not influenced by axial load, one can express the maximum deflection in terms of $1/r$.
- Thus for a UDL on a simply supported beam with a parabolic distribution of bending moment, the maximum deflection can be expressed as

$$e_{\max} = \frac{5}{384} \left(\frac{wL^4}{EI} \right) = \frac{(5)8}{384} \left(\frac{wL^2}{8} \right) \frac{L^2}{EI}$$

$$= \frac{(M_{\max})}{EI} \frac{L^2}{9.6}$$

- Taking a very conservative estimate of a uniform rectangular distribution of the bending moment .
- Taking a very unconservative estimate of a triangular distribution of the bending moment diagram, we obtain



2 Curvature of bent columns: (a) Deflected shape at ultimate load; (b) Curvature (M/EI) diagrams showing rectangular, triangular, and assumed probable distribution; (c) Linear moment-curvature relationship.

$$e_{\max} = \frac{L^2}{12} (1/R)_{\max}$$

Hence, one may assume that a reasonable estimate of $e_{\max}$ can be obtained from the expression

$$e_{\max} = \frac{L^2}{11} (1/R)_{\max}$$

Thus the eccentricity to be taken into account for calculation of the additional moment is equal to

$$e_{\text{add}} = \frac{L_e^2}{11} (1/R)_{\max}$$

where L_e is the effective length of the column.

Assuming that the bending strains at failure (ultimate conditions) are as given in Fig. 16. (balanced failure), we have

$$(1/R)_{\max} = \frac{\epsilon_c + \epsilon_s}{D} = \frac{0.0035 + 0.002}{D} = \frac{1}{182D}$$

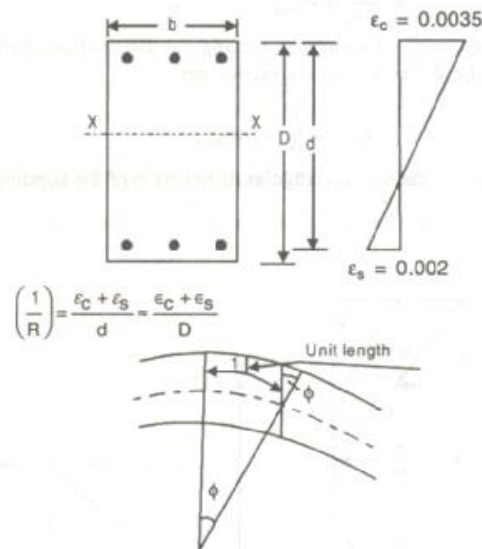


Fig. 16.3 Curvature-strain relationship.

$$e_{add} = \frac{D}{2000} \left(\frac{L_e}{D} \right)^2$$

Both IS 456 and BS 8110 use the reduced form of this equation, viz.

$$\frac{e_{add}}{D} = \frac{1}{2000} \left(\frac{L_e}{D} \right)^2$$

Putting the right-hand side expression as β , we have

$$e_{add} = \beta D$$

Therefore,

$$M_{add} = M_a = P e_{add}$$

The value of β for (L_e/D) ratios can be read off from Table I of SP 16 or calculated from the above formula. If the value of additional moment on the XX -axis is designated by M_{ax} and that on YY -axis as M_{ay} , they can be calculated by the formulae

$$M_{ax} = \frac{PD}{2000} \left(\frac{L_{ex}}{D} \right)^2$$

$$M_{ay} = \frac{Pb}{2000} \left(\frac{L_{ey}}{b} \right)^2 \quad (16.1)$$

where

P = axial load on the member

L_{ex} = effective length in respect to major axis

L_{ey} = effective length in respect to minor axis

D = depth at right angles to the X -axis

b = breadth at right angles to the Y -axis

REDUCTION FACTOR FOR ADDITIONAL MOMENT

- It is quite obvious from physical considerations that the lateral deflection of a column must be less when a large portion of the column section is in compression.
- The expression for δ derived above was on the assumption that the curvature is the one corresponding to the balanced failure where the maximum strain in tension steel $\epsilon_s = 0.002$ and the maximum compression strain in concrete $\epsilon_c = 0.0035$ at failure.
- For any value of P for which the strain ϵ is less than that of balanced failure, the deflection and hence the additional moment should also be less.
- Even though IS has made this modification optional, this reduction in many cases can be substantial, and should be taken into account from economy point of view.

$$k = \frac{P_z - P}{P_z - P_b} \leq 1 \quad \text{or} \quad k = \frac{1 - P/P_z}{1 - P_b/P_z} \leq 1$$

- The reduced deflection can be expressed by the reduction factor k given by IS where

$$\frac{d_c}{d} = \frac{0.0035}{0.0035 + 0.002}$$

$$d_c = 0.636d$$

Assuming tension steel and compression steels are equal and are equally stressed so that $f_{s2} A_{s2} = f_{s1} A_{s1}$ for a section where equal steel is put on the two opposite sides, we have

$$P_b = (0.4f_{ck}b)(0.636d) = 0.254f_{ck}bd$$

The value of P_{fr} as obtained above is slightly different from that obtained by

16.5.2 Use of SP 16 for Determination of P_b

Table 60 on page 171 of SP 16 (Table 16.2 of the text) gives a more accurate method to calculate P_{bal} , especially when steel is distributed on all faces. It uses an expression in the following forms

1. For rectangular section,

$$\frac{P_b}{f_{ck} b D} = \left(k_1 + k_2 \frac{p}{f_{ck}} \right) \quad (16.4)$$

2. For circular section,

$$\frac{P_b}{f_{ck} D^2} = \left(k_1 + k_2 \frac{p}{f_{ck}} \right) \quad (16.4a)$$

k_1 and k_2 can be read off from the Table 16.1, where p is the percentage of the total steel in the column.

TABLE 16.1 VALUES OF k_1 AND k_2 FOR VALUES OF P_b
(SP 16, Table 60)

		d/D			
		0.05	0.10	0.15	0.20
Value of k_1	Rectangular	0.219	0.207	0.196	0.184
	Circular	0.172	0.160	0.149	0.138
Value of k_2 for Fe 415 steel	Rectangular				
	Equal steel on two sides	0.096	0.082	0.046	- 0.022
	Equal steel on four sides	0.424	0.328	0.203	0.028
	Circular	0.410	0.323	0.201	0.036

- It may however be noted that the value of k cannot be estimated until the area of the steel in the section is known. Hence, one has to initially assume a percentage of steel and determine the value of k and check for safety by successive approximation.
- Having obtained b as above, Chart 65 on page 150 of SP 16 (Chart 16.2 here) gives a quick method of obtaining k , the reduction factor from $PIPE$ and P values.

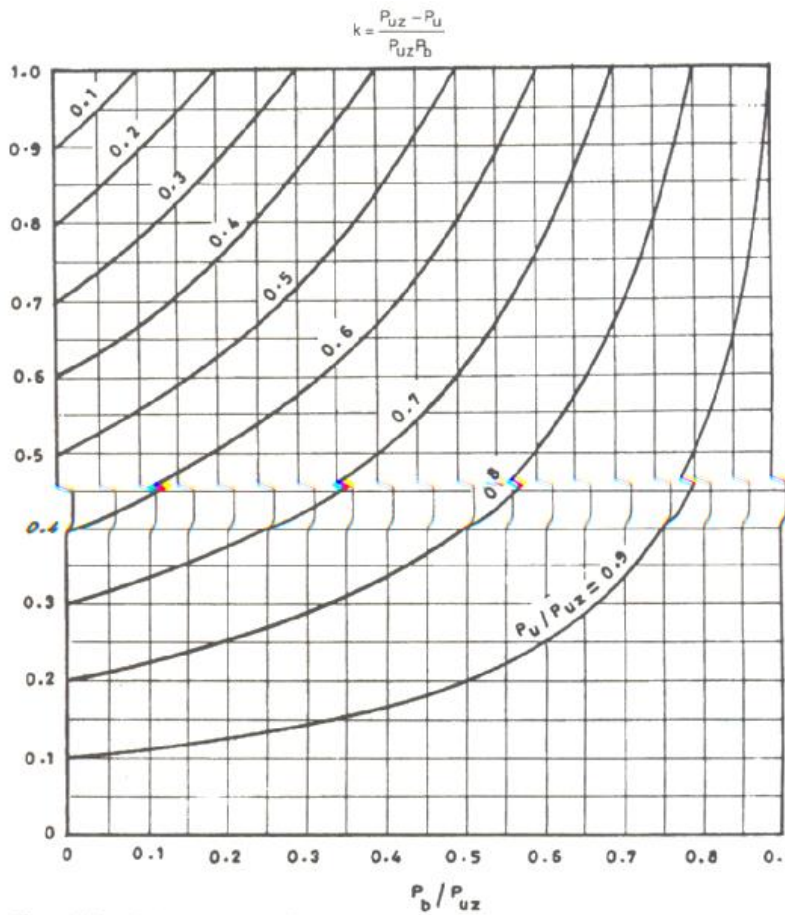


Chart 16.2 Reduction factor k for additional moment in columns (Chart 65 of SP 16)

FACTORS AFFECTING BEHAVIOUR OF SLENDER COLUMNS

- Columns are usually classified as one of the following types:
 - **Pin-ended columns**
 - **Braced columns**
 - **Unbraced columns.**
- They can also bend in single curvature or double curvature.
- The various types of bending that can occur in column is shown in Figs.
- Analysis of structures like building frames gives the value of the axial load and the moments at top and bottom of the columns.

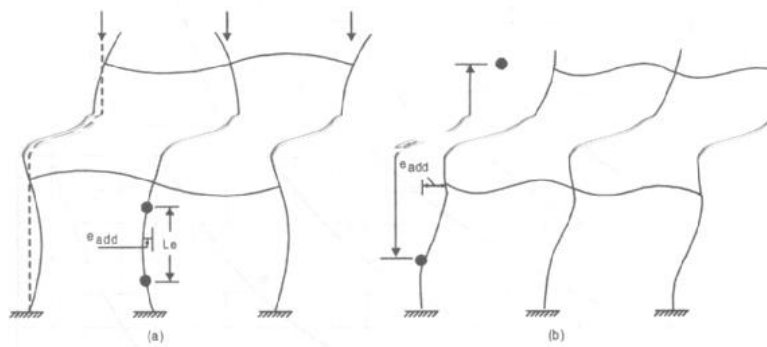


Fig. 16.5 Bending of columns in frames: (a) Braced; (b) Unbraced.

- The resulting bending moment diagram will indicate whether the column is bent in single or double curvature.
- If the moments on the two ends are opposite in sign, the column bends in single curvature.
- In designing slender columns to get the total moment M , for final design. one has to determine the combined effects of all the following three factors:
- The initial moment M caused by the end moments, M and M (The larger value is taken as positive and designated as M If the bending is in double curvature, M is taken as negative.)
- Moment due to accidental eccentricity usually designated by the term M or M_m .
- The magnitude of M will depend on whether the columns are in single or double curvature and whether they are braced or unbraced as explained in notes 1 and 2 of IS 456, Clause 39.7.1.
- The Explanatory Handbook on IS 456, SP 24, Section 38.7 may also be referred to for more details.