# 4.8 TORQUE EQUATION OF PERMANENT MAGNET SYNCHRONOUS MOTOR

### 5.7.1 Torque equation of ideal PMSM

When a balanced three phase voltage is applied to the armature, a three phase current flows through the conductors. This current produces armature flux for deriving the torque equation, the concept of armature ampere conductor density is used. A sinusoidally distributed ampere conductor density is assumed as shown in figure 5.9

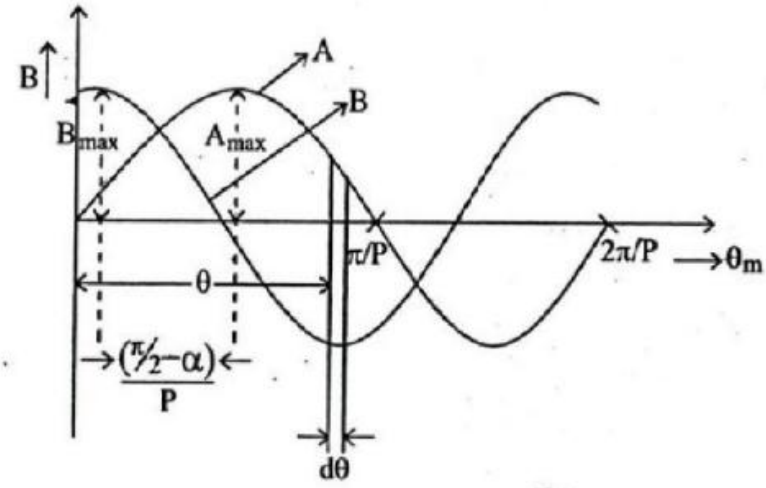


*Figure 5.9 Ampere conductor and flux density distribution*

Let the operation point of PMSM is such that the ampere conductor density and the flux density are as shown in figure5.9 In figure 5.9 the angle between the axes of ampere conductor and flux density is $\left(\frac{\pi}{2}-\alpha\right)$. A strip of width $d\theta$ is considered at $\theta$

From figure 5.9,

$$B = B_{max}\sin\left(P\theta+\left(\frac{\pi}{2}-\alpha\right)\right)$$

$$= B_{max}\sin\left(\frac{\pi}{2}+(P\theta-\alpha)\right)$$

$$\therefore B = B_{max}\cos(P\theta-\alpha) \qquad \text{... (5.21)}$$

$$A = A_{max}\sin P\theta$$

Force experinced by the armature conductors in $d\theta$ is

$$dF = BlA\,d\theta$$

$$= A_{max}\,B_{max}\,l\sin P\theta\cos(P\theta-\alpha)\,d\theta$$

Torque experienced by the armature conductors in $d\theta$ is

$$dT = A_{max}\,B_{max}\,rl\sin P\theta\cos(P\theta-\alpha)\,d\theta$$

Torque experienced by armature conductors/pole $= \int_{\theta=0}^{\theta=\pi/P} dT$

$$= A_{max}\,B_{max}\,rl\int_{o}^{\pi/P}\sin P\theta\cos(P\theta-\alpha)\,d\theta$$

$$= \frac{A_{max}\,B_{max}}{2}rl\int_{o}^{\pi/P}[\sin(P\theta+P\theta-\alpha)+\sin\alpha]\,d\theta$$

$$= \frac{A_{max}\,B_{max}}{2}rl\left[\frac{-\cos(2P\theta-\alpha)}{2P}+\theta\sin\alpha\right]_{o}^{\pi/P}$$

$$= \frac{A_{max} B_{max} rl}{2}\left[\frac{-\cos\left(2P \times \frac{\pi}{P} - \alpha\right)}{2P} + \frac{\pi}{P}\sin\alpha + \frac{\cos(-\alpha)}{2P}\right]$$

$$= \frac{A_{max} B_{max} rl}{2}\left[\frac{-\cos\alpha}{2P} + \frac{\cos\alpha}{2P} + \frac{\pi}{P}\sin\alpha\right]$$

$$= \frac{A_{max} B_{max} rl}{2} \frac{\pi}{P}\sin\alpha \qquad \text{... (5.22)}$$

Total electromagnetic torque developed by all the armature conductors = 2P × Torque per pole

$$= 2P \frac{\pi}{P} \frac{A_{max} B_{max} rl \sin\alpha}{2}$$

$$= \pi A_{max} B_{max} rl \sin\alpha \qquad \text{... (5.23)}$$

As armature is stationary, this torque is experienced by the rotor and rotor rotates

$$T = -\pi A_{max} B_{max} rl \sin\alpha \qquad \text{... (5.24)}$$

Since $\beta = -\alpha$

$$\Rightarrow T = \pi A_{max} B_{max} rl \sin\beta \qquad \text{... (5.25)}$$

Where $\beta$ is the torque angle or power angle

### 5.7.2 Ampere conductor density distribution

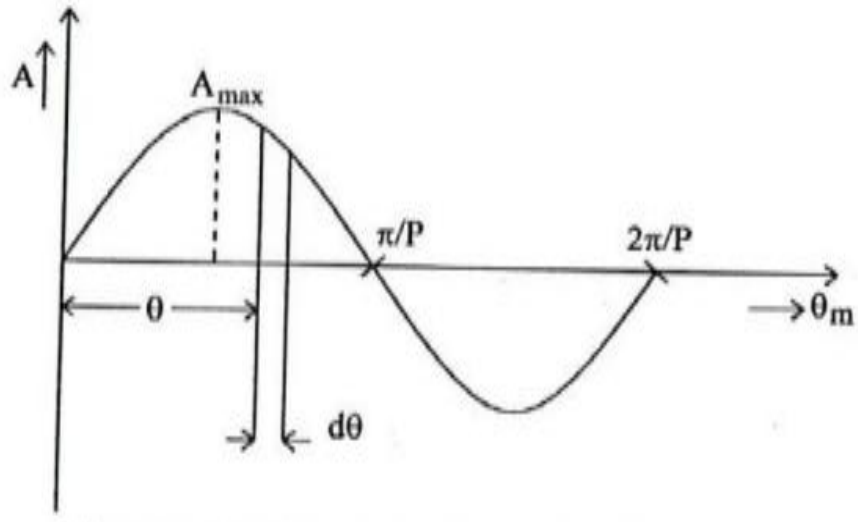


*Figure: 5.10 Ampere conductor density*

The above figure shown the ampere conductor density distribution in the air gap due to current carrying armature winding

$$A = A_{max} \sin P\theta$$

Where A $\rightarrow$ ampere conductor density

consider a strip of width $d\theta$ at angle $\theta$ from the reference axis.

Ampere conductors in the strip $d\theta$ is $A\,d\theta$.

$$A d\theta = A_{max} \sin P\theta \, d\theta \qquad \text{... (5.26)}$$

Ampere conductors per pole $= \int_{o}^{\pi/P} A d\theta$

$$= \int_{o}^{\pi/P} A_{max} \sin P\theta \, d\theta$$

$$= A_{max} \left[ \frac{\cos P\theta}{P} \right]_{o}^{\pi/P}$$

$$= \frac{A_{max}}{P} (-1 - 1)$$

$$= \frac{2A_{max}}{P} \qquad \text{... (5.27)}$$

Let $T_{ph}$ be the number of full pitched turns per phase

i be the current

Total ampere conductiors $= 2iT_{ph}$

Sinusoidally distributed ampere conductors /pole $= \frac{2iT_{ph}}{2P}$

$$= \frac{iT_{ph}}{P} \qquad \text{... (5.28)}$$

equating eqations (5.27) & (5.28)

$$= \frac{2A_{max}}{P} = \frac{iT_{ph}}{P}$$

$$A_{max} = \frac{iT_{ph}}{2} \qquad \text{...(5.29)}$$

For a PMSM supplied by balanced three phase sinusoidal voltage, the phase currents are given by

$$i_R = I_{max} \cos\omega t$$

$$i_y = I_{max} \cos\left(\omega t - \frac{2\pi}{3}\right)$$

$$i_B = I_{max} \cos\left(\omega t - \frac{4\pi}{3}\right)$$

The turns are given by

$$T_{phR} = T_{ph} \cos\theta$$

$$T_{phy} = T_{ph} \cos\left(\theta - \frac{2\pi}{3}\right)$$

$$T_{phB} = T_{ph} \cos\left(\theta - \frac{4\pi}{3}\right)$$

Ampere turns at any instant is given by

$$iT_{ph} = i_R T_{phR} + i_y T_{phy} + i_B T_{phB}$$

$$= I_{max} T_{ph} \cos\omega t \cos\theta + I_{max} T_{ph} \cos\left(\omega t - \frac{2\pi}{3}\right)$$

$$\cos\left(\theta - \frac{2\pi}{3}\right) + I_{max} T_{ph} \cos\left(\omega t - \frac{4\pi}{3}\right) \cos\left(\theta - \frac{4\pi}{3}\right)$$

By simplifying the above equation

$$iT_{ph} = \frac{3}{2} I_{max} T_{ph} \cos(\omega t - \theta)$$

$$= \frac{3}{2} \sqrt{2}\, I_{ph} T_{ph} \cos(\omega t - \theta)$$

$$A_{max} = \frac{iT_{ph}}{2}$$

$$A_{max} = \frac{3\sqrt{2}}{2} I_{ph} T_{ph} \qquad \text{... (5.30)}$$

In practical motor, the armature turns are short pitched and distributed further they may and be accommodated in skewed slots in such case for getting slots fundamental component of ampere turn distribution, the turns per phase is modified as $K_{\omega 1} T_{ph}$

Where $k_{\omega 1} = K_{s1} K_{c1} K_{d1}$

$K_{s1} \rightarrow$ skew factor

$K_{s1} = \dfrac{\sin \sigma / 2}{\sigma / 2}$ where $\sigma \rightarrow$ skew angle

$K_{c1} = \dfrac{\cos \delta}{2}$; $K_{c1} \rightarrow$ chording factor

$K_{d1} \rightarrow$ distribution factor

$$K_{d1} = \frac{\sin q V/2}{q \sin V/2}$$

Fundamental component of ampere turns per phase of a practical motor

$$= \frac{4}{\pi} i T_{ph} K_{\omega 1} \quad ...(5.31)$$

When a balanced sinusoidally varying three phase ac current pass through a balanced three phase winding, it can be shown that total sinussoidally distributed ampere turns is equal to

$$= \frac{3}{2} \frac{4}{\pi} I_{max} K_{\omega 1} T_{ph}$$

$$= \frac{3 \cdot 2\sqrt{2}}{\pi} I_{ph} K_{\omega 1} T_{ph} \quad ...(5.32)$$

The amplitude of ampere conductor density distribution is equal to the total sinusoidally distributed ampere turns divided by 2

$\therefore A_{max}$ in practical $3\phi$ motor

$$= \frac{3 \cdot 2\sqrt{2}}{\pi} \frac{I_{ph} K_{\omega 1}}{2} T_{ph}$$

$$= \frac{3\sqrt{2}}{\pi} I_{ph} K_{\omega 1} T_{ph} \qquad \text{... (5.33)}$$

electromagnetic torque developed in practical PMSM is

$$= \pi A_{max} B_{max} rl \sin\beta$$

$$= \pi \left[\frac{3\sqrt{2}}{\pi} I_{ph} K_{\omega 1} T_{ph}\right] B_{max} rl \sin\beta$$

$$= 3\sqrt{2} K_{\omega 1} I_{ph} T_{ph} B_{max} rl \sin\beta$$

$$= (3\sqrt{2} K_{\omega 1} T_{ph} B_{max} rl) I_{ph} \sin\beta$$

$$T = 3 \frac{E_{ph}}{\omega_m} I_{ph} \sin\beta \qquad \text{... (5.34)}$$