Ampere's Circuital Law

Ampere's circuital law states that the line integral of the magnetic field around a closed path is the net current enclosed by this path. Mathematically,

$$\oint \vec{H}.d\vec{l} = I_{enc} \quad ....(3.8)$$

The total current I_{enc} can be written as,

$$I_{enc} = \int_S \vec{J}.d\vec{s} \quad ....(3.9)$$

By applying Stoke's theorem, we can write

$$\oint \vec{H}.d\vec{l} = \int_S \nabla \times \vec{H}.d\vec{s}$$

$$\therefore \int_S \nabla \times \vec{H}.d\vec{s} = \int_S \vec{J}.d\vec{s}$$

$$\therefore \nabla \times \vec{H} = \vec{J} \quad ....(3.10)$$

which is the Ampere's law in the point form.

Estimation of Magnetic field intensity for straight and circular conductors:

We illustrate the application of Ampere's Law with some examples.

We compute magnetic field due to an infinitely long thin current carrying conductor as shown in Fig. 2.1. Using Ampere's Law, we consider the close path to be a circle of radius ρ

If we consider a small current element $Id\vec{l}(= Idz\hat{a}_z)$, $d\vec{H}$ is perpendicular to the plane

containing both $d\vec{l}$ and $\vec{R}(= \rho\hat{a}_\rho)$. Therefore only that component of $\vec{H}$

will be present i.e., $\vec{H} = H_\phi \hat{a}_\phi$

,i.e., H_ϕ .

By applying Ampere's law we can write,

$$\vec{H} = \frac{I}{2\pi\rho}\hat{a}_\phi \int_0^{2\pi} H_\phi \rho \, d\phi = H_\phi \rho 2\pi = I$$

.................(4.11)

Therefore, $\vec{H} = \frac{I}{2\pi\rho}\hat{a}_\phi$ which is same as equation (3.7)

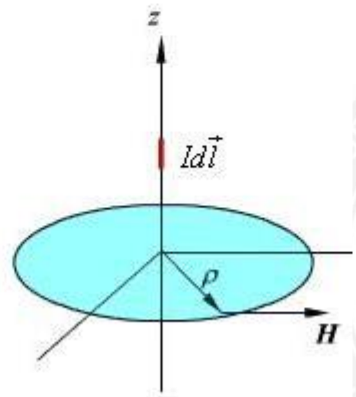

Fig. 2.1: Magnetic field due to an infinite thin current carrying conductor

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We consider the cross section of an infinitely long coaxial conductor, the inner conductor carrying a current I and outer conductor carrying current - I as shown in figure 2.2. We compute the magnetic field as a function of ρ as follows:

In the region $0 \le \rho \le R_1$

$$I_{enc} = I\frac{\rho^2}{R_1^2}$$

......................................(3.12)

$$H_\phi = \frac{I_{enc}}{2\pi\rho} = \frac{I\rho}{2\pi a^2}$$

............................(3.13)

In the region $R_1 \le \rho \le R_2$

$I_{enc} = I$

$$H_\phi = \frac{I}{2\pi\rho} \quad ....(3.14)$$

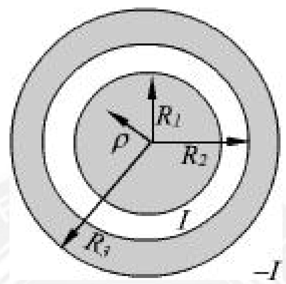

Fig. 2.2: Coaxial conductor carrying equal and opposite

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$\rho \le R_3$

$I_{enc} = I$

$$- I \frac{\rho^2 - R_2^2}{R_3^2 - R_2^2} \quad ....(3.15)$$

$$H_\phi = \frac{I}{2\pi\rho} \frac{R_3^2 - \rho^2}{R_3^2 - R_2^2} \quad ....(3.16)$$

$> R_3$ $\quad I_{enc} = 0 \quad H_\phi = 0$ ....(3.17)