## **3.5 RANDOM TELEGRAPH SIGNAL**

Define semi random telegraph signal process and random telegraph signal process and Prove also that the former is evolutionary and later is WSS.

Sol :

If {N(t) } is a poisson process and  $X(t) = (-1)^{N(t)}$ , then {X(t)} is called a semi random telegraph signal process.

$$\therefore X(t) = \begin{cases} -1; & N(t) \text{ is odd} \\ +1 & N(t) \text{ is even} \end{cases}$$

Since N(t) is a poisson process, its probability law is

P [ N(t) = n ] = 
$$\frac{e^{-\lambda t} (\lambda t)^n}{n!}$$
,  $n = 0, 1, 2, 3, ..., \infty$   
P[X(t) = -1] = P[N(t) is odd]  
= P [ N(t) = 1] + P[ N(T) = 3 ] + P [N(t) = 5 ] +.....  
=  $\frac{e^{-\lambda t} (\lambda t)^1}{1!} + \frac{e^{-\lambda t} (\lambda t)^3}{3!} + \frac{e^{-\lambda t} (\lambda t)^5}{5!} + ....$ 

$$= e^{-\lambda t} \left[ \frac{\lambda t}{1!} + \frac{(\lambda t)^3}{3!} + \frac{(\lambda t)^5}{5!} + \dots \right] \text{ (since } e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \text{, Sinhx} = \frac{x}{1!} + \frac{x^3}{3!} + \dots,$$
$$Coshx = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots \text{ )}$$

$$P[X(t) = -1] = e^{-\lambda t} \sinh \lambda t$$

$$P[X(t) = 1] = P[N(t) \text{ is even}]$$

$$=e^{-\lambda t}+\frac{e^{-\lambda t}(\lambda t)^2}{2!}+\frac{e^{-\lambda t}(\lambda t)^4}{4!}+\ldots\ldots$$

$$= e^{-\lambda t} \left[ 1 + \frac{(\lambda t)^2}{2!} + \frac{(\lambda t)^4}{4!} + \dots \right] \text{ (since } e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \text{, Sinhx} = \frac{x}{1!} + \frac{x^3}{3!} + \dots,$$
$$Coshx = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots \text{ )}$$
$$P[X(t) = 1] = e^{-\lambda t} \operatorname{Cosh} \lambda t$$

The mean of the process is given by

 $E[X(t)] = \sum nP_n(t)$   $= (-1)e^{-\lambda t} \sinh \lambda t + e^{-\lambda t} \cosh \lambda t$   $= e^{-\lambda t} [\cosh \lambda t - \sinh \lambda t]$   $= e^{-\lambda t} \left[ \left( \frac{e^{\lambda t} + e^{-\lambda t}}{2} \right) - \left( \frac{e^{\lambda t} - e^{-\lambda t}}{2} \right) \right]$   $= e^{-\lambda t} \left[ \frac{e^{\lambda t}}{2} + \frac{e^{-\lambda t}}{2} - \frac{e^{\lambda t}}{2} + \frac{e^{-\lambda t}}{2} \right]$   $= e^{-\lambda t} e^{-\lambda t}$ 

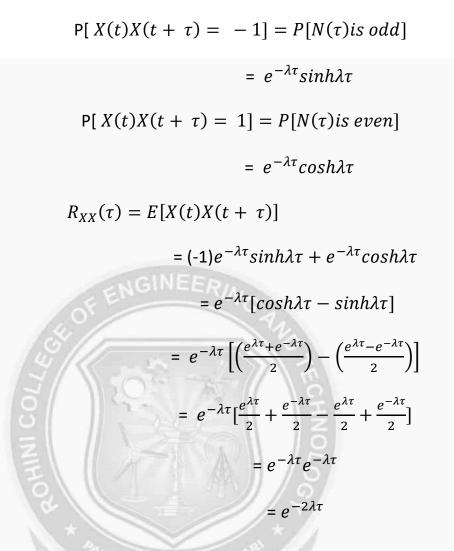
 $E[X(t)] = e^{-2\lambda t}$  which is not a constant.

 $\therefore$  {*X*(*t*)} is not stationary.

Hence semi random telegraph signal is an evolutionary process.

The autocorrelation function is given by

$$R_{XX}(\tau) = E[X(t)X(t + \tau)]$$
$$X(t)X(t + \tau) = \begin{cases} -1; & N(\tau)is \text{ odd} \\ +1 & N(\tau) \text{ is even} \end{cases}$$
$$P[X(t)X(t + \tau) = -1] = P[N(\tau)is \text{ odd}]$$



## RANDOM TELEGRAPH SIGNAL

Let {X(t)} be semi random telegraph signal process and Y(t) =  $\alpha X(t)$  where  $\alpha$  is a R.V which takes value -1 and +1 with probability  $\frac{1}{2}$  and  $\frac{1}{2}$  respectively, which is independent of { X(t)}, then {Y(t)} is called a random telegraph signal process.

## Probability distribution for $\alpha$

α	-1	1
$P(\alpha)$	1/2	1/2

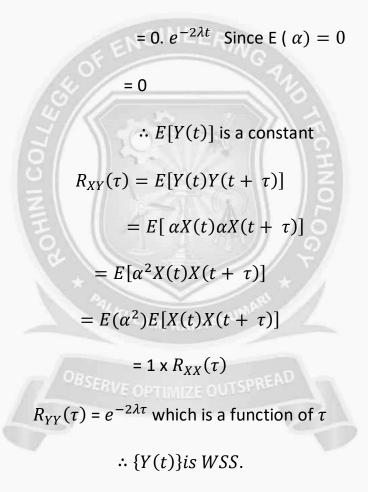
$$E(\alpha) = \sum \alpha P(\alpha) = -\frac{1}{2} + \frac{1}{2} = 0$$

$$E(\alpha^2) = \sum \alpha^2 P(\alpha) = \frac{1}{2} + \frac{1}{2} = 1$$

To prove {X(t)} is a WSS process

$$E[Y(t)] = E[\alpha X(t)]$$

=  $E(\alpha)E[X(t)]$  Since  $\alpha$  is independent of X(t)



Hence the random telegraph signal process is WSS.