

4.1. FLEXIBILITY METHOD

4.1.1.INTRODUCTION

These are the two basic methods by which an indeterminate skeletal structure is analyzed. In these methods flexibility and stiffness properties of members are employed. These methods have been developed in conventional and matrix forms. Here conventional methods are discussed.

The number of releases required is equal to statically indeterminacy. Introduction of releases results in displacement discontinuities at these releases under the externally applied loads. Pairs of unknown by actions (forces and moments) are applied at these releases in order to restore the continuity or compatibility of structure.

The computation of these unknown by actions involves solution of linear simultaneous equations. The number of these equations is equal to statically indeterminacy. After the unknown by actions are computed all the internal forces can be computed in the entire structure using equations of equilibrium and free bodies of members. The required displacements can also be computed using methods of displacement computation.

In flexibility methods the unknowns are forces at the releases the method is also called force method. Since computation of displacement is also required at releases for imposing conditions of compatibility the method is also called compatibility method. In computation of displacements use is made of flexibility properties, hence, the method is also called flexibility method.

4.1.2.EQUILIBRIUM AND COMPATABILITY CONDITIONS

The three conditions of equilibrium are the sum of horizontal forces, vertical forces and moments at any joint should be equal to zero.

i.e., ($H=0$; $V=0$; $M=0$)

Forces should be in equilibrium

i.e., ($F_X=0$; $F_Y=0$; $F_Z=0$) i.e., ($M_X=0$; $M_Y=0$; $M_Z=0$)

Displacement of a structure should be compactable

The compatibility conditions for the supports

can be given as 1. Roller Support ($V=0$)

2. Hinged Support ($V=0$, $H=0$)

3. Fixed Support ($V=0$, $H=0$, $M=0$)

4.1.3.DETERMINATE AND INDETERMINATE STRUCTURAL SYSTEMS

If skeletal structure is subjected to gradually increasing loads, without distorting the initial geometry of structure, that is, causing small displacements, the structure is said to be stable. Dynamic loads and buckling or instability of structural system are not considered here. If for a stable structure it is possible to find the internal forces in all the members constituting the structure and supporting reactions at all the supports provided from statically equations of equilibrium only, the structure is said to be determinate.

If it is possible to determine all the support reactions from equations of equilibrium alone the structure is said to be externally determinate externally indeterminate. If structure is externally determinate but it is not possible to determine all internal forces then structure is said to be internally indeterminate. There are four structural system may be:

- Externally indeterminate but internally determinate
- Externally determinate but internally indeterminate
- Externally and Internally indeterminate
- Externally and Internally determinate

4.1.4.DETERMINATE Vs INDETERMINATE STRUCTURES.

Determinate structures can be solved using conditions of equilibrium alone ($H=0$; $V=0$; $M=0$). No other conditions are required.

Indeterminate structures cannot be solved using conditions of equilibrium because ($H=0$; $V=0$; $M=0$). Additional conditions are required for solving such structures. Usually matrix methods are adopted.

4.1.5.INDETERMINACY OF STRUCTURAL SYSTEM

The indeterminacy of a structure is measured as statically (s) or kinematical (k) Indeterminacy.

$S = P (M - N + 1) - r = PM - r$; $K = P (N - 1) + r - s$; $K = PM - CP = 6$ for space frames subjected to general loading

$P = 3$ for plane frames subjected to in plane or normal to plane loading. N = Number of nodes in structural system.

M =Number of members of completely stiff structure which includes foundation as a singly connected system of members.

Incompletely stiff structure there is no release present. In singly connected system of rigid foundation members there is only one route between any two points in which tracks are not retraced. The system is considered comprising of closed rings or loops.

R = Number of loops or rings in completely stiff structure. r = Number of releases in the system.

C = Number of constraints in the system. $R = (M - N + 1)$ For plane and space trusses reduces $S = M - (NDOF) N + P$

M = Number of members in completely stiffness.

$P = 6$ and 3 for space and plane truss respectively
 N = Number of nodes in truss.

$NDOF$ = Degrees of freedom at node which is 2 for plane truss and 3 for space truss

For space truss = $M - 3N + 6$

For plane truss = $M - 2N + 3$

Test for statically indeterminacy of structures system;

If $S \geq 0$ structure is statically indeterminate

If $S = 0$ structure is statically determinate and

If $S \leq 0$ structure is mechanism

It may be noted that structure may be mechanism even if $S > 0$ if the releases are present in such away so as to cause collapse as mechanism. The situation of mechanism is unacceptable.

4.1.6.STATIC AND KINEMATIC INDETERMINACY

Statically Indeterminacy

It is difference of the unknown forces (internal forces plus external reactions) and the equations of equilibrium.

Kinematic Indeterminacy

It is the number of possible relative displacements of the nodes in the directions of stress resultants.

4.1.7.PRIMARY STRUCTURE

A structure formed by the removing the excess or redundant restraints from an indeterminate structure making it statically determinate is called primary structure. This is required for solving indeterminate structures by flexibility matrix method.

Indeterminate structure and Primary Structure



4.1.8. ANALYSIS OF INDETERMINATE STRUCTURES: (CONTINUOUS BEAMS)

Introduction

Solve statically indeterminate beams of degree more than one.

- To solve the problem in matrix notation.
- To compute reactions at all the supports.
- To compute internal resisting bending moment at any section of the continuous beam.

Beams which are statically indeterminate to first degree, were considered. If the structure is statically indeterminate to a degree more than one, then the approach presented in the force method is suitable.

Example Problems;

Problem 1.1

Calculate the support reactions in the continuous beam ABC due to loading as shown in Fig.1.1 Assume EI to be constant throughout.

Select two reactions wise, at B(R1) and C(R2) as redundant, since the given beam is statically indeterminate to second degree. In this case the primary structure is a cantilever beam AC. The primary structure with a given loading is shown in Fig. 1.2

In the present case, the deflections (L)1 and (L)2 of the released structure at B and C can be readily calculated by moment-area method. Thus

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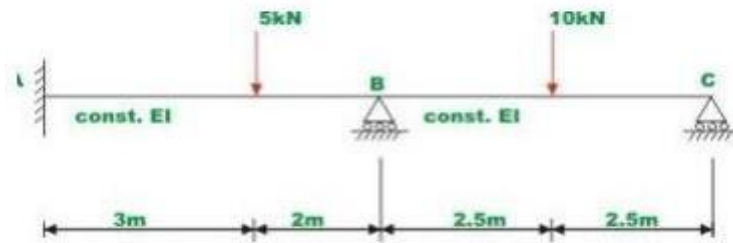


Fig 1.1

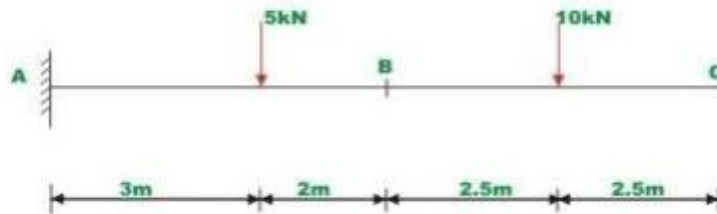


Fig 1.2

$$\begin{aligned} (L)_1 &= 819.16 / EI \\ (L)_2 &= 2311.875 / EI \quad \text{--- (1)} \end{aligned}$$

For the present problem the flexibility matrix is,

$$\begin{aligned} a_{11} &= 125/3EI, a_{21} = 625/6EI \\ a_{12} &= 625/6EI, a_{22} = 1000/3EI \quad \text{--- (2)} \end{aligned}$$

In the actual problem the displacements at B and C are zero. Thus the compatibility conditions for the problem may be written as,

$$\begin{aligned} a_{11} R_1 + a_{12} R_2 + (L)_1 &= 0 \\ a_{21} R_1 + a_{22} R_2 + (L)_2 &= 0 \quad \text{--- (3)} \end{aligned}$$

Substituting the value of E and I in the above equation,

$$R_1 = 10.609 \text{ KN and } R_2 = 3.620 \text{ KN}$$

Using equations of static equilibrium,

$$R_3 = 0.771 \text{ KN and } R_4 = 0.755 \text{ KN}$$

Problem 1.2

A Fixed beam AB of constant flexural rigidity is shown in Fig.1.3 The beam is subjected to a uniform distributed load of w moment $M=wL^2$ kN.m. Draw Shear force and bending moment diagrams by force method.

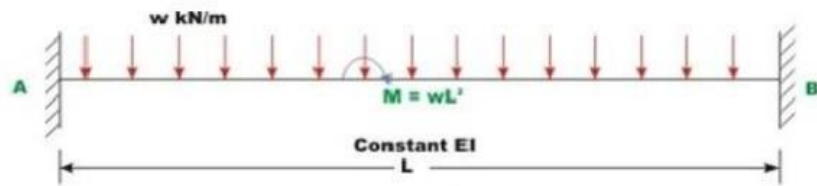


Fig 1.3 Fixed Beam

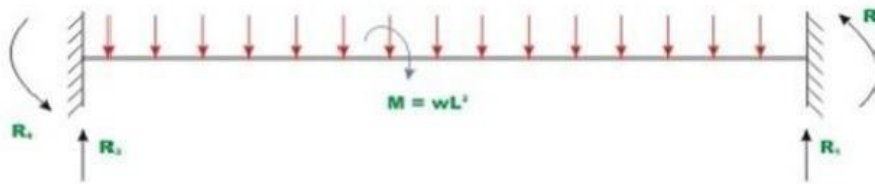


Fig 1.3 Fixed Beam with R_1 and R_2 as Redundant

Fig 1.3 Fixed Beam with R_1 and R_2 as Redundant;

Select vertical reaction (R_1) and the support moment (R_2) at B as the redundant. The primary structure in this case is a cantilever beam which could be obtained by releasing the redundant R_1 and R_2 .

The R_1 is assumed to be positive in the upward direction and R_2 is assumed to be positive in the counterclockwise direction. Now, calculate deflection at B due to only applied loading. Let (L) be the transverse deflection at B and L be the slope at B due to external loading. The positive directions of the selected redundant are shown in Fig.8.3b

The deflection (L_1) and (L_2) of the released structure can be evaluated from unit load method. Thus,

$$(L_1) = wL^4/8EI - 3wL^4/8EI = wL^4/2EI \text{ ---- (1)}$$

$$(L_2) = wL^3/6EI - wL^3/2EI = 2wL^3/3EI \text{ ---- (2)}$$

The negative sign indicates that (L) is downwards and rotation (is $1/L_2$) clockwise.

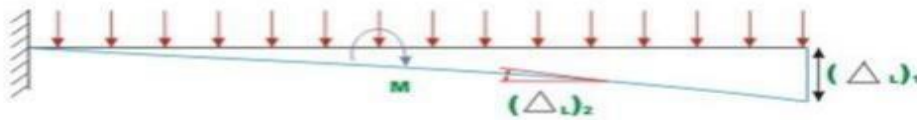


Fig 1.4 Primary Structure with external loading



Fig 1.5 Primary Structure with unit load along R_1



Fig 1.6 Primary Structure with unit Moment along R_2

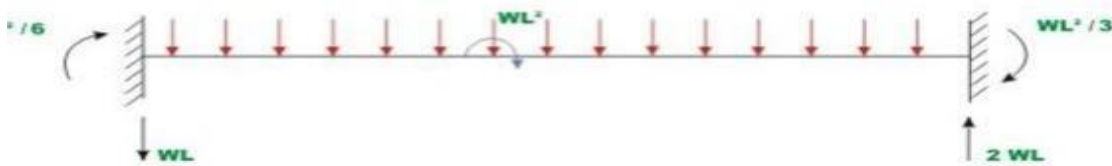


Fig 1.7 Reaction

Draw SFD & BMD:

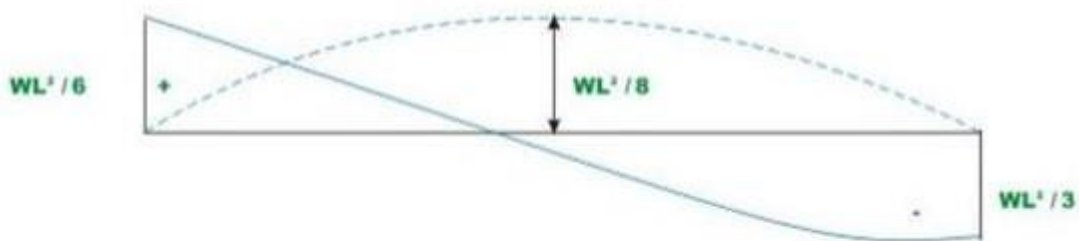


Fig1.8.Bending Moment Diagram



Fig1.9.Shear Force Diagram