2.11Derivation for Acceptance angle and Numerical aperture

Consider the light ray propagate in an optical fibre. The incident ray AO enters into core at an angle θ_0 to fibre axis. Let n_1, n_2 and n_o be the refractive indices of the core, cladding and surroundings.

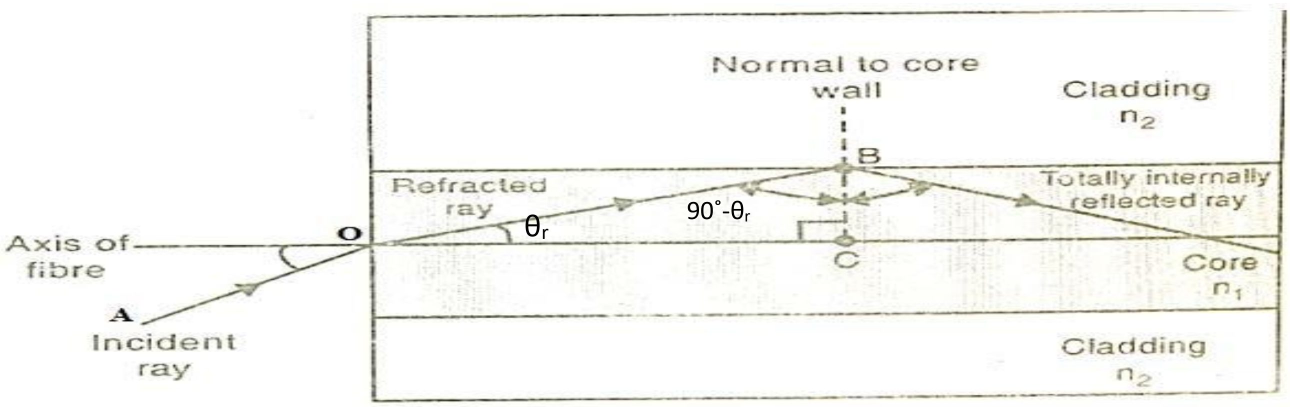

Fig 2.11.1propagation of light in fiber

Applying Snell's law of refraction at the point O we have

$$n_0 \sin \theta_0 = n_1 \sin \theta_r$$

$$\sin \theta_0 = \frac{\text{n1}}{\text{n0}} \sin \theta_r$$

$$\sin \theta_0 = \frac{\text{n1}}{\text{n0}} \sqrt{1 cos^2\theta -} \ \theta_r \text{-------------------(1)}$$

At the point B on the interface of core and cladding,

Angle of incidence $\qquad \theta_c = 90 - \theta_r$

Applying Snell's law of refraction at the point B we have

$$n_1 \sin(90^\circ - \theta_r) = n_2 \sin 90^\circ$$

$$n_1 \cos \theta_r = n_2$$

$$\cos \theta_r = \frac{\text{n2}}{\text{n1}} \ldots\ldots(2)$$

Substituting equation (2) in equation (1) we have

$$\sin\theta_0 = \frac{\mathrm{n1}}{\mathrm{n0}} \sqrt{1 - \frac{n_2^2}{n_1^2}}$$

$$\sin\theta_0 = \frac{\mathrm{n1}}{\mathrm{n0}} \sqrt{\frac{n_1^2 - n_2^2}{n1}}$$

$$\sin\theta_0 = \frac{\mathrm{n1}}{\mathrm{n0n1}} \sqrt{\frac{n_1^2 - n_2^2}{1}}$$

$$\sin\theta_0 = \frac{\sqrt{n_1^2 - n_2^2}}{n0}$$

the medium surrounding the fibre is air, then $n_o = 1$

$$\sin\theta_0 = \sqrt{n_1^2 - n_2^2}$$

$$\theta_0 = \sin^{-1}\sqrt{n_1^2 - n_2^2} \;\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

$$NA = \sin\theta_0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

This is the final expression of acceptance angle and numerical aperture.

The condition for propagation of light within the fibre is

$$\boldsymbol{\sin\theta_i < NA}$$

Acceptance angle

The maximum angle at or below which a ray of light can enter through one end of the fibre still be total internal reflection is called as acceptance angle. The cone is referred as acceptance cone.

$$\theta_0 = \sin^{-1}\sqrt{n_1^2 - n_2^2}$$

Numerical Aperture (NA)

Sine of the acceptance angle of the fibre is known as numerical aperture. It denotes the light gathering capability of the optical fibre.

$$NA = \sin \theta_0$$

Fractional Index Charge (Δ)

It is the ratio of refractive index difference in core and cladding to the refractive index of core.

$$\Delta = \frac{n1 - n2}{n1}$$