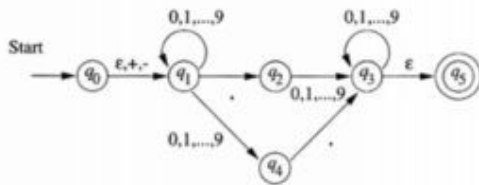


FINITE AUTOMATA WITH ϵ –TRANSITIONS

An informal treatment of ϵ -NFA's, using transition diagrams with ϵ allowed as a label. In the examples to follow, think of the automaton as accepting those sequences of labels along paths from the start state to an accepting state. However, each ϵ along a path is “invisible” i.e., it contributes nothing to the string along the path.



[Source: “J.E.Hopcroft, R.Motwani and J.D Ullman, *Introduction to Automata Theory, Languages and Computations*, Second Edition, Pearson Education, 2003]

In Fig. is an ϵ -NFA that Accepts decimal numbers consisting of:

1. An optional + or – sign,
3. A decimal point, and
4. Another string of digits. Either this string of digits, or the string (2) can be empty, but at least one of the two strings of digits must be nonempty.

Nondeterministic Finite Automata with ϵ transitions (ϵ -NFA)

- A Non-Deterministic Finite Automata with ϵ transitions is a 5-tuple $(Q, \Sigma, q_0, \delta, F)$

where – Q is a finite set (of states)

Σ is a finite alphabet of symbols

$q_0 \in Q$ is the start state

$F \subseteq Q$ is the set of accepting states

δ is a function from $Q \times (\Sigma \cup \{\epsilon\})$ to $2Q$ (transition function)

Transition function – δ is a function from

$Q \times (\Sigma \cup \{\epsilon\})$ to $2Q$

$\delta(q, a) = \text{subset of } Q \text{ (possibly empty)}$

In our example

- $\delta(q_1, 0) = \{q_1, q_4\}$
- $\delta(q_1, .) = \{q_1\}$

- $\delta(q_1, +) = \emptyset$
- $\delta(q_0, \epsilon) = \{q_1\}$

Transition function on a string

It is a function from $Q \times \Sigma^*$ to $2Q - (q, x) = \text{subset of } Q \text{ (possibly empty)}$

It Set of all states that the machine can be in, upon following all possible paths on input x

Here we need to consider all paths that include the use of ϵ transitions

E CLOSURE

- Before defining the transition function on a string $((q, x))$, it is useful to first define what is known as the ϵ closure.
- Given a set of states S, the ϵ closure will give the set of states reachable from each state in S using only ϵ transitions

ϵ closure: Recursive definition

- Let $M = (Q, \Sigma, q_0, \delta, F)$ be a ϵ -NFA
- Let S be a subset of Q
- The ϵ closure, denotes $ECLOSE(S)$ is defined:
 - For each state $p \in S$, $p \in ECLOSE(S)$
 - For any $q \in ECLOSE(S)$, every element of $\delta(q, \epsilon) \in ECLOSE(S)$
 - No other elements of Q are in $ECLOSE(S)$

ϵ -Closure

- ϵ -Closure : Algorithm
 - Since we know that $ECLOSE(S)$ is finite, we can convert the recursive definition to an algorithm.
 - To find $ECLOSE(S)$ where S is a subset of Q
 - Let $T = S$ – While (T does not change) do
 - Add all elements of $\delta(q, \epsilon)$ where $q \in T - ECLOSE(S) = T$