ENGINEERING PHYSICS

UNIT II

WAVES AND FIBRE OPTICS

2.6.PLANE PROGRESSIVE WAVES AND ITS WAVE EQUATION

Definition:

A wave which travels continuously in a medium in the same direction without any change in its amplitude is called a progressive wave or a travelling wave.

Explanation:

A Plane Progressive wave equation can be obtained to represent the displacement of a vibrating particle in a medium through which a wave passes. Each particle of a progressive wave executes simple harmonic motion of the same period and amplitude but differing in phase from each other.

(i) Displacement of Point O:

Let us assume that a progressive wave travels from the origin (O) along the x direction from left to right (Fig.). The displacement of a particle at a given instant.

$$y = A \sin \omega t \text{------(1)}$$

Where, A is the Amplitude and ω is the angular frequency of the particle, it is given by

$$\omega = \frac{2\pi}{T} \text{-----------(2)}$$

Where, T is the time period, it is defined as the total time take by the particle to complete one oscillation.

Sub eqn. (2) in (1), we get

$$y = A \sin \frac{2\pi t}{T} \text{------(3)}$$

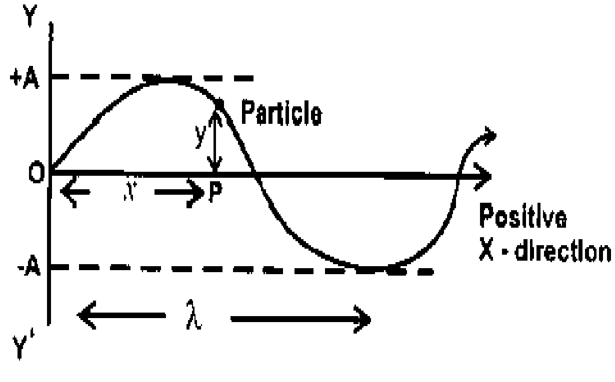

Fig 2.6.1. Plane progressive waves

(source:askiitians.com)

(ii) Displacement of Point P:

Now, assume that the particle is displaced to a distance (x) from point O to P with some time. Then the displacement of the particle at a distance x from O at a given instant is given by,

$$y = A\sin\frac{2\pi}{T}(t - \frac{x}{v})$$

$$T = \lambda/v$$

$$y = A\sin\frac{2\pi}{\lambda}(vt - x) \text{--------(4)}$$

Thus, eqn.6 shows the complete form of wave equation for plane progressive wave with respect to velocity **(v)** in the x- direction.

Definition: It is defined as the rate of change of displacement (y) of the particle with time (t). From eqn.6, we can write

$$y = A\sin\frac{2\pi}{\lambda}(vt - x)$$

Differentiating eqn.4 with respect to t, we have

$$\frac{dy}{dt} = A cos\frac{2\pi}{\lambda}(vt - x) \text{ X } \frac{2\pi}{\lambda} v \text{------(5)}$$

When particle velocity is high, then

$$cos\frac{2\pi}{\lambda}(vt - x) = 1$$

$$\frac{dy}{dt} = A\frac{2\pi}{\lambda}v$$

Differentiate (4) with respect to x

$$\frac{dy}{dx} = A cos\frac{2\pi}{\lambda}(vt - x) \text{ X } \frac{-2\pi}{\lambda} \quad \text{---------(6)}$$

$$cos\frac{2\pi}{\lambda}(vt - x) = 1$$

$$\frac{dy}{dx} = -A\frac{2\pi}{\lambda}$$

Comparing (7) & (8)

$$\frac{dy}{dt} = -v\frac{dy}{dx} \text{---------(7)}$$

Thus, from eqn. 7, the particle velocity is directly depends on the wave velocity and the slope of the displacement of the particle.

Diff (5) & (6) again

$$\frac{d^2y}{dt^2} = -Asin\frac{2\pi}{\lambda}(vt - x)\ \mathrm{X}(\frac{2\pi}{\lambda})^2 v^2$$

$$\frac{d^2y}{dx^2} = -Asin\frac{2\pi}{\lambda}(vt - x)\ \mathrm{X}(\frac{2\pi}{\lambda})^2$$

On comparing

$$\frac{d^2y}{dx^2} = \frac{d^2y}{dt^2}v^2$$

This is the differential equation for progressive waves