3.2 HYDRAULIC JUMPS

Hydraulic jump defined as, the rise of water level, which takes place due to the transformation of unstable shooting flow (super-critical) to the stable streaming flow (sub-critical flow).It frequently occurs in a canal below a regulating sluice, at the toe of a spillway, at downstream of narrow channel or at the place where a steep channel slope suddenly turns flat.

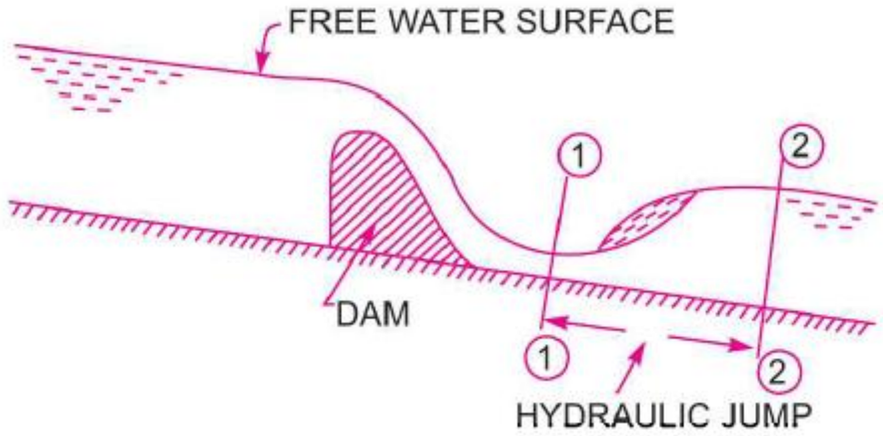

Figure 3.1 Hydraulic Jumps
[Source:Fluid Mechanics and Hydraulic Machines by Dr.R.K.Bansal,page 783]

Consider the flow of water over a dam as shown in Fig.3.1 The depth of water at the section 1-1 is small, but it increases towards downstream rapidly over a short length of the channel. This is because at the section 1-1, the flow is a shooting flow as the depth of water at section 1-1 is less than the critical depth. Shooting flow is an unstable type of flow and does not continue on the downstream side. Then this shooting will convert itself into a streaming or tranquil flow and hence depth of water will increase. This sudden increased of depth of water is called a hydraulic jump or a standing wave. When hydraulic jump take place, a loss of energy due to eddy formation and turbulence occurs.

Expression for Hydraulic Jump

Following assumption are made before deriving the expression for the depth of hydraulic jump,

1. Flow is uniform and pressure distribution is due to hydrostatic before and after the jump.

2. Losses due to friction on the surface of the bed of the channel are negligible
3. The slope of the bed of the channel is small, so that the component of the weight of the fluid in the direction of flow is negligibly small.

Consider a hydraulic jump formed in a channel of horizontal bed as shown in figure 12.2.

Consider there are two sections 1-1 and 2-2 before and after hydraulic jump.

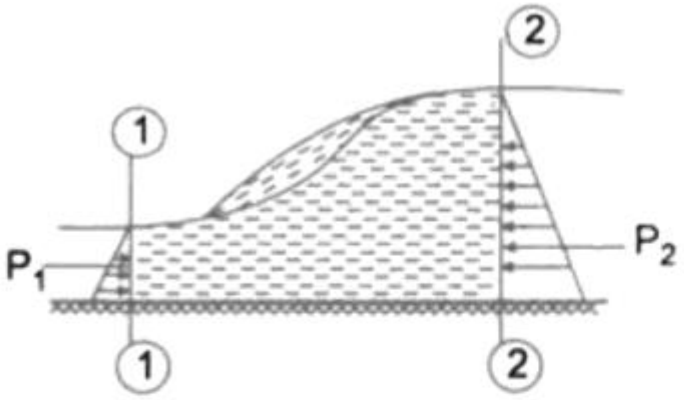

Figure 3.2 Hydraulic Jumps

[Source:Fluid Mechanics and Hydraulic Machines by Dr.R.K.Bansal,page 784]

Let

d_1 is the depth of flow at section 1-1 i.e. initial depth;

d_2 is the depth of flow at section 2-2 i.e. depth of flow after the hydraulic jump, also known as sequent depth or theoretical tail water depth. The depth pair at section 1-1 and 2-2 are called conjugate depth.

v_2 is thevelocity of flow at section 1-1;

v_2 is the velocity of flow at section 2-2;

z_1 is the depth of centroid of area at section 1-1 below free surface;

z_2 is the depth of centroid of area at section 2-2 below free surface;

A_1 is the cross sectional area at section 1-1;

A_2 is the cross sectional area at section 2-2.

Consider unit width of the channel.

The force acting on the mass of water between sections 1-1 and 2-2 are:

(i) Pressure force P_1 and P_2 on section 1-1 and 2-2 respectively.

(ii) Frictional force on the floor of the channel, which assumed to be negligible.

Let q = discharge per unit width.

$$v_1 d_1 = v_2 d_2$$

Now, pressure force on section 1-1

$$P_1 = \rho g A_1 \overline{Z_1} = \rho g (d_1 \times 1) \times \frac{d_1}{2}$$

(Since we are considering unit width hence, $A_1 = d_1 \times 1$)

$$= \frac{\rho g d_1^2}{2}$$

Similarly pressure force on section 2-2

$$P_2 = \frac{\rho g d_2^2}{2}$$

Net force acting on the mass of water between section 1-1 and 2-2

$$= P_2 - P_1$$

$$= \frac{\rho g}{2}[d_2^2 - d_1^2] \quad (12.2)$$

But from the momentum principle, the net force acting on a mass of fluid must be equal to the rate of change of momentum in the same section.

Rate of change of momentum in the direction of force

∴Rate of change of momentum in the direction of force

= mass of water per second × change of velocity in direction of force

Now,mass of water per second

= ρ × discharge per unit width × width

$$=\rho \times q \times 1 = \rho q m^3/s$$

Change of velocity in the direction of force $v_1 - v_2$

Hence, rate of change of momentum in the direction of force

$$= \rho q(v_1 - v_2)$$

Hence, according to the momentum principle, the expression given by eq. (12.2) is equal to the expression given by eq. (12.3)

or $\frac{\rho g}{2}[d_2^2 - d_1^2] \quad = \rho q(v_1 - v_2)$

But from equation (1),

$$v_1 = q/d_1 \ and \ v_2 = q/d_2$$

Substituting the value of and in eq. (12.4) and by solving we get,

$$d_2 + d_1 = \frac{2q^2}{g d_1 d_2}$$

Multiplying both sides by d_2 , we get

$$d_2^2 + d_1 d_2 = \frac{2q^2}{g d_1}$$

$$d_2^2 + d_1 d_2 - \frac{2q^2}{g d_1} = 0$$

By solving eq. (12.6) we get

$$d_2 = \frac{-d_1 \pm \sqrt{d_1^2 + \frac{2q^2}{g d_1}}}{2}$$

Neglecting the negative sign, we get

$$d_2 = -\frac{d_1}{2} + \sqrt{\frac{d_1^2}{4} + \frac{2q^2}{g d_1}}$$

$$= -\frac{d_1}{2} + \sqrt{\frac{d_1^2}{4} + \frac{2(v_1 d_1)^2}{g d_1}}$$

$$= -\frac{d_1}{2} + \frac{d_1}{2}\sqrt{1 + \frac{8{v_1}^2}{g d_1}}$$

Now, depth of hydraulic jump = $(d_2 - d_1)$

Froude number on the upstream side is given by

$$F_{r1} = \frac{v_1}{\sqrt{gd_1}}$$

Substituting the value in equation (12.7) we get,

$$d_2 = \frac{d_1}{2}(-1 + \sqrt{1 + 8F_{r1}^2}$$

ENERGY DISSIPATION DUE TO HYDRAULIC JUMP

When hydraulic jump take place, a loss of energy due to eddies formation and turbulence occurs. This loss of energy is equal to the difference of specific energies at sec. 1-1 and 2-2.

Let E_1, and E_2 are the energy at section 1-1 and 2-2 respectively. Loss of energy due to hydraulic jump

$$\Delta \boldsymbol{E} = \boldsymbol{E_1} - \mathbf{E_2}$$

$$= \left(d_1 + \frac{{v_1}^2}{2g}\right) - \left(d_2 + \frac{{v_2}^2}{2g}\right)$$

$$= \left(\frac{{v_1}^2}{2g} - \frac{{v_2}^2}{2g}\right) - (d_2 - d_1)$$

Since $v_1 = q/d_1 \; and \; v_2 = q/d_2$

$$= \left(\frac{q^2}{2gd_1^2} - \frac{q^2}{2gd_2^2}\right) - (d_2 - d_1)$$

$$= \frac{q^2}{2g}\left(\frac{d_2^2 - d_1^2}{d_1^2 d_2^2}\right) - (d_2 - d_1)$$

$$q^2 = gd_1 d_2 \left(\frac{d_2 + d_1}{2}\right)$$

solving the expression, we get

$$\Delta E = \frac{(d_2 - d_1)^3}{4d_1 d_2}$$