Stoke's theorem

It states that the circulation of a $\vec{A}$ vector around a closed path is equal to the integral of $\nabla\times\vec{A}$ over the surface bounded by this path. It may be noted that this equalityholds provided $\vec{A}$ $\nabla\times\vec{A}$ are continuous on the surface.

i.e,

$$\oint_L \vec{A}\cdot d\vec{l} = \int_s \nabla\times\vec{A}\cdot d\vec{s}$$ ....(1.82)

Proof: Let us consider an area S that is subdivided into large number of cells as shown inthe figure 5.1.

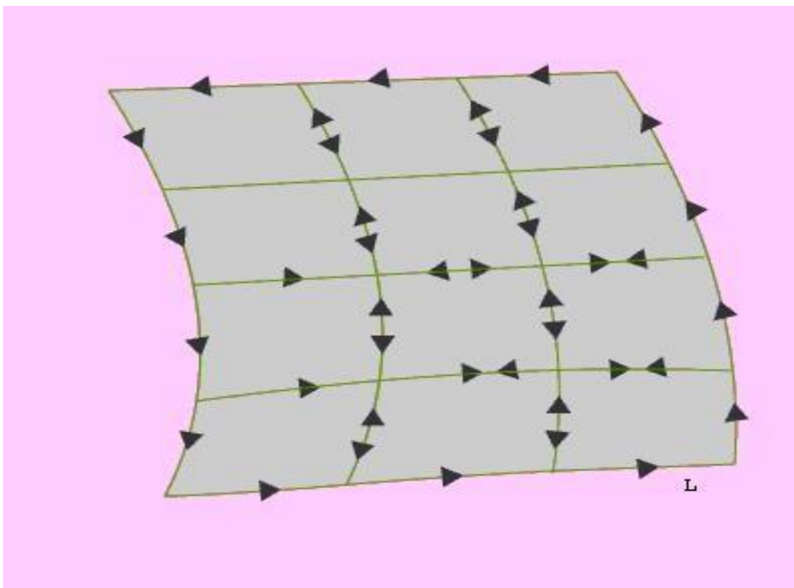

Fig 5.1: Stokes theorem
(www.brainkart.com/subject/Electromagnetic-Theory_206/)

Let k^{th}cell has surface area ΔS_k and is bounded path L_k while the total area is bounded by path L. As seen from the figure that if we evaluate the sum of the line integrals around the elementary areas, there is cancellation along every interior path and we are left the line integral along path L. Therefore we can write,

$$\oint_L \vec{A}\cdot d\vec{l} = \sum_k \oint_{L_k} \vec{A}\cdot d\vec{l} = \sum_k \frac{\oint_{L_k} \vec{A}\cdot d\vec{l}}{\Delta S_k}\Delta S_k$$ ....(1.83)

As $\Delta S_k \to 0$

$$\oint_L \vec{A} \cdot d\vec{l} = \int_S \nabla \times \vec{A} \cdot d\vec{s}$$ ....(1.84)

which is the stoke's theorem.