4.4 TRIGONOMETRIC SUBSTITUTIONS

(I) Products of powers of sines and cosines

Evaluating $\int sin^m x cos^n x dx$

Case (i) If n is odd $(n = 2k + 1)$, then

$$\int sin^m x cos^{2k+1} x dx = \int sin^m x (cos^2 x)^k cosx dx$$
$$= \int sin^m x (1 - sin^2 x)^k cosx dx$$

Here, substitute $u = sinx$

Case (ii) If m is odd $(m = 2k + 1)$, then

$$\int sin^{2k+1} x cos^n x dx = \int (sin^2 x)^k cos^n x sinx dx$$
$$= \int (1 - cos^2 x)^k cos^n x sinx dx$$

Here, substitute $u = cosx$

Note: If both m and n are odd apply case (i) or case (ii)

Case(iii) If both m and n are even, use half- angle identities

$$sin^2 x = \frac{1}{2}(1 - cos2x), \qquad cos^2 x = \frac{1}{2}(1 + cos2x)$$

$$\int_0^{\pi/2} sin^m x cos^n x dx = \frac{m-1}{m+n}\frac{m-3}{m+n-2}\dots\frac{2}{3+n}\frac{1}{1+n}$$

(if m is odd, n may be even or odd)

$$= \frac{m-1}{m+n}\frac{m-3}{m+n-2}\dots\frac{1}{2+n}\frac{n-1}{n}\frac{n-3}{n-2}\dots\frac{2}{3}.1$$

(if m is even, n is odd)

$$= \frac{m-1}{m+n}\frac{m-3}{m+n-2}\dots\frac{1}{2+n}\frac{n-1}{n}\frac{n-3}{n-2}\dots\frac{1}{2}\frac{\pi}{2}$$

(if m is even, n is even)

(II) Products of powers of $secx$ and $tanx$

Evaluating $\int tan^m x sec^n x dx$

Case (i) If m is odd $(m = 2k + 1)$, then

$$\int tan^{2k+1} x sec^n x dx = \int (tan^2 x)^k sec^{n-1} x secx tanx dx$$
$$= \int (sec^2 x - 1)^k sec^{n-1} x secx tanx dx$$

Here, substitute $u = secx$

Case (ii) If n is even $(n = 2k)$, then

$$\int tan^m x sec^{2k} x dx = \int tan^m x (sec^2 x)^{k-1} sec^2 x dx$$

$$= \int tan^m x(1 + tan^2 x)^{k-1} sec^2 x dx$$

Here, substitute $u = tanx$

(III) Products of sines and cosines of multiples of x

Evaluating $\int sinmx\ sinnxdx, \quad \int sinmx\ cosnxdx\ \ and \int cosmx\ cosnx\ dx$

Use the following identities

$$sinmxsinnx = \frac{1}{2}[\cos(m-n)\,x - \cos(m+n)\,x]$$

$$sinmxcosnx = \frac{1}{2}[\sin(m-n)\,x + \sin(m+n)\,x]$$

$$cosmxcosnx = \frac{1}{2}[\cos(m-n)\,x + \cos(m+n)\,x]$$

Example:

(i)Evaluate $\int \boldsymbol{sin^6 xcos^3 xdx}$

Solution:

Given $\int sin^6 xcos^3 xdx$ Here m = 6, n = 3 (odd)

$$= \int sin^6 xcos^2 xcosxdx$$

$$= \int sin^6 x(1 - sin^2 x)cosxdx \ldots.(1)$$

$$Put \quad u = sinx; \qquad du = cosxdx$$

$$(1) \Rightarrow \int u^6(1-u^2)\,du \quad = \int (u^6 - u^8)du$$

$$= \frac{u^7}{7} - \frac{u^9}{9} + C$$

$$= \frac{sin^7 x}{7} - \frac{sin^9 x}{9} + C$$

(ii) Evaluate $\int \boldsymbol{sin^2 (\pi x) cos^5 (\pi x) dx}$

Solution:

Given $\int sin^2 (\pi x)cos^5(\pi x)dx$ (Here m = 2, n = 5 (odd))

$$= \int sin^2 (\pi x)cos^4(\pi x)\cos(\pi x)\,dx$$

$$= \int sin^2 (\pi x)[1 - sin^2(\pi x)]^2 \cos(\pi x)\,dx \ldots (1)$$

$$Put \ \ u = sin\pi x; \qquad du = \pi cos\pi xdx$$

$$(1) \Rightarrow \int u^2(1-u^2)^2 \frac{du}{\pi} \quad = \frac{1}{\pi}\int u^2(1 - 2u^2 + u^4)du$$

$$= \frac{1}{\pi}\int (u^2 - 2u^4 + u^6)du$$

$$= \frac{1}{\pi}\left[\frac{u^3}{3} - \frac{2u^5}{5} + \frac{u^7}{7}\right] + C$$

$$= \frac{1}{3\pi} sin^3(\pi x) - \frac{2}{5\pi} sin^5(\pi x) + \frac{1}{7\pi} sin^7(\pi x) + C$$

Example:

Evaluate $\int \boldsymbol{sin^5 xcos^2 xdx}$

Solution:

Given $\int sin^5 xcos^2 xdx$ (Here m = 5 (odd), n = 2)

$$= \int sin^4 xcos^2 xsinxdx$$

$$= \int (1 - cos^2 x)^2 cos^2 xsinxdx \ldots .(1)$$

$$Put \ \ u = cosx; \qquad du = -sinxdx$$

$$(1) \Rightarrow \int (1-u^2)^2 u^2(-du) \ = \ -\int (1 - 2u^2 + u^4)u^2 du$$

$$= \ -\int (u^2 - 2u^4 + u^6) du$$

$$= -\left[\frac{u^3}{3} - \frac{2u^5}{5} + \frac{u^7}{7}\right] + C$$

$$= -\frac{1}{3} cos^3(x) + \frac{2}{5} cos^5(x) - \frac{1}{7} cos^7(x) + C$$

Example:

Evaluate $\int \boldsymbol{cos^2 xsin2xdx}$

Solution:

Given $\int cos^2 xsin2xdx$

$$= \int 2sinxcosxcos^2 xdx$$

$= \ \int 2sinxcos^3 xdx$ (Here, m = 1, n = 3)

$$= 2 \ \int sinxcos^3 xdx \ldots (1)$$

$$Put \ \ u = cosx; \qquad du = -sinxdx$$

$$(1) \Rightarrow \ 2\int u^3(-du) \ = \ -2\int u^3 du$$

$$= \ -2\frac{u^4}{4} + C = -\frac{1}{2} cos^4 x + c$$

Example:

Evaluate $\int \boldsymbol{sin^2 xcos^4 xdx}$

Solution:

Given $\int sin^2 xcos^4 xdx$ (Here, m = 2, n = 4)

$$= \int \left(\frac{1-cos2x}{2}\right)\left(\frac{1+cos2x}{2}\right)^2 dx$$

$$= \frac{1}{8}\int (1 - cos2x)(1 + 2cos2x + cos^2 2x)dx$$

$$= \frac{1}{8}\int (1 + cos2x - cos^2 2x - cos^3 2x)dx$$

$$= \frac{1}{8}\left[x + \frac{1}{2}sin2x - \int (cos^2 2x + cos^3 2x)dx\right] \dots (1)$$

$$\int cos^2 2xdx = \int \frac{1+cos4x}{2}dx = \frac{1}{2}\left(x + \frac{1}{4}sin4x\right)$$

$$\int cos^3 2xdx = \int cos^2 2xcos2xdx = \int (1 - sin^2 2x)\, cos2xdx$$

$$Put\ \ u = sin2x; \qquad du = 2cos2xdx$$

$$\therefore \int cos^3 2xdx = \int (1 - u^2)\frac{du}{2} = \frac{1}{2}\left[u - \frac{u^3}{3}\right] = \frac{1}{2}\left[sin2x - \frac{1}{3}sin^3 2x\right]$$

$$(1) \Rightarrow \frac{1}{8}\left[x + \frac{1}{2}sin2x - \frac{1}{2}x - \frac{1}{8}sin4x - \frac{1}{2}sin2x + \frac{1}{6}sin^3 2x\right] + C$$

$$= \frac{1}{8}\left[\frac{1}{2}x - \frac{1}{8}sin4x + \frac{1}{6}sin^3 2x\right] + C$$

$$= \frac{1}{16}\left[x - \frac{1}{4}sin4x + \frac{1}{3}sin^3 2x\right] + C$$

Example:

(i)Evaluate $\int tan\, xsec^3 xdx$

Solution:

Given $\int tan\, xsec^3 xdx$ (Here m =1 (odd))

$$= \int sec^2 x(secx\, tanx)dx$$

$$Put\ u = secx; \qquad du = secx\, tanxdx$$

$$= \int u^2\, du = \frac{u^3}{3} + C = \frac{sec^3 x}{3} + C$$

(ii)Evaluate $\int_0^{\pi/3} tan^5 xsec^4 xdx$

Solution:

Given $\int_0^{\pi/3} tan^5 xsec^4 xdx$ (Here m = 5 (odd))

$$= \int_0^{\pi/3} tan^4 x\, sec^3 x\, (secx\, tanx)dx$$

$$= \int_0^{\pi/3} (sec^2 x - 1)^2 sec^3 x(secx\, tanx)\, dx \quad \dots (1)$$

$Put\ u = secx$ when $x = 0 \Rightarrow u = 1$

$du = secx\, tanx\, dx$ $\quad x = \frac{\pi}{3} \Rightarrow u = 2$

$$\therefore (1) \Rightarrow \int_1^2 (u^2-1)^2\, u^3\, du \;=\; \int_1^2 (u^4-2u^2+1)u^3\, du$$

$$= \int_1^2 (u^3-2u^5+u^7)\, du$$

$$= \left[\frac{u^4}{4}-\frac{2u^6}{6}+\frac{u^8}{8}\right]_1^2$$

$$= \left(4-\frac{64}{3}+32\right)-\left(\frac{1}{4}-\frac{1}{3}+\frac{1}{8}\right) \;=\; \frac{117}{8}$$

Example:

(i)Evaluate $\int tan^2 x sec^4 x d\,x$

Solution:

Given $\int tan^2 x sec^4 x d\,x$

$$= \int tan^2 x sec^2 x\, sec^2 x\, d\,x$$

$$= \int tan^2 x(1+\ tan^2 x)sec^2 x\, dx \ldots (1)$$

$$Put\ u = tanx\ ; \qquad du = sec^2 x\ \ dx$$

$$(1) \Rightarrow \int u^2(1+u^2)\, du \;=\; \int (u^2+u^4)du$$

$$= \left[\frac{u^3}{3}+\frac{u^5}{5}\right]+C$$

$$= \frac{1}{3}tan^3(x)+\frac{1}{5}tan^5(x)+C$$

(ii)Evaluate $\int tanx sec^2 x d\,x$

Solution:

Given $\int tanxsec^2 x d\,x$

$$Put\ u = tanx\ ; \quad du = sec^2 x\ \ dx$$

$$= \int udu$$

$$= \left[\frac{u^2}{2}\right]+C = \frac{1}{2}tan^2(x)+C$$

Example:

(i) Evaluate$\int sec^3 x\, dx$

Solution:

Given I = $\int sec^3 x\, dx \;= \int sec^2 x\, secx\, dx$

$$Put\ u = secx \qquad\qquad dv = \ sec^2 x\, dx$$

$$du = secx\, tanx\, dx \qquad\qquad v = \ \int sec^2 x\, dx = \ tanx$$

$$\int u\,dv$$

$$= uv - \int v\,du$$

$$I = (secx)tanx - \int tanx\,(secx\,tanx)dx$$

$$= (secx)tanx - \int tan^2x\,secx\,dx$$

$$= (secx)tanx - \int (sec^2x - 1)\,secx\,dx$$

$$= (secx)tanx - \int sec^3x\,dx + \int secx\,dx$$

$$= secx\,tanx - I + \log(secx + tanx)$$

$$2I = secx\,tanx + \log(secx + tanx)$$

$$I = \frac{1}{2}secx\,tanx + \frac{1}{2}\log(secx + tanx) + C$$

(ii)Evaluate $\int tan^2x\,secx\,dx$

Solution:

Given $\int tan^2x\,secx\,dx = \int (sec^2x - 1)secx\,dx$

$$= \int sec^3x\,dx - \int secx\,dx$$

$$= \frac{1}{2}secx\,tanx + \frac{1}{2}\log(secx + tanx) - \log(secx + tanx) + C$$

Using example (3.53(i))

$$= \frac{1}{2}secx\,tanx - \frac{1}{2}\log(secx + tanx) + C$$

Example:

(i) Evaluate $\int_0^{\pi/2} sin^7x\,cos^5x\,dx$

Solution:

Given $\int_0^{\pi/2} sin^7x\,cos^5x\,dx$ (Here m = 7 , n = 5)

$\int_0^{\pi/2} sin^mx cos^nx dx = \frac{m-1}{m+n}\frac{m-3}{m+n-2}\dots\frac{2}{3+n}\frac{1}{1+n}$ (m is odd, n even or odd)

$$= \frac{7-1}{7+5}\frac{7-3}{7+5-2}\dots\frac{2}{3+5}\frac{1}{1+5}$$

$$= \left(\frac{6}{12}\right)\left(\frac{4}{10}\right)\left(\frac{2}{8}\right)\left(\frac{1}{6}\right) = \frac{1}{120}$$

(ii)Evaluate $\int_0^{\pi/2} sin^7x\,dx$

Solution:

Given $\int_0^{\pi/2} sin^7 x\, dx$ (Here m = 7 (odd) , n = 0)

$\int_0^{\pi/2} sin^m x cos^n x dx = \frac{m-1}{m+n}\frac{m-3}{m+n-2}\dots\frac{2}{3+n}\frac{1}{1+n}$ (m is odd, n even or odd)

$$= \frac{7-1}{7+0}\frac{7-3}{7+0-2}\dots\frac{2}{3+0}\frac{1}{1+0} = \left(\frac{6}{7}\right)\left(\frac{4}{5}\right)\left(\frac{2}{3}\right)(1) \quad = \frac{16}{35}$$

Example:

i)Evaluate $\int sin\ 4x\ cos\ 5x\ dx$

Solution:

Given $\int sin\ 4x\ cos\ 5x\ dx$

We know that, $sinAxcosBx = \frac{1}{2}[\sin(A-B)\,x + \sin(A+B)\,x]$

$$= \frac{1}{2}\int[\sin(-x) + sin9x]dx$$

$$= \frac{1}{2}\int(-sinx + sin9x)dx$$

$$= \frac{1}{2}\left[cosx - \frac{1}{9}cos9x\right] + C$$

ii)Evaluate $\int cos\ 3x\ cos\ 4x\ dx$

Solution:

Given $\int cos\ 3x\ cos\ 4x\ dx$

We know that $cosAxcosBx = \frac{1}{2}[\cos(A-B)\,x + \cos(A+B)\,x]$

$$= \frac{1}{2}\int(cosx + cos7x)dx$$

$$= \frac{1}{2}\left[sinx + \frac{1}{7}sin7x\right] + C$$

$$= \frac{1}{2}sinx + \frac{1}{14}sin7x + C$$

iii) Evaluate $\int sin\ 5x\ sinx\ dx$

Solution:

Given $\int sin\ 5x\ sinx\ dx$

We know that $sinAxsinBx = \frac{1}{2}[\cos(A-B)\,x - \cos(A+B)\,x]$

$$= \frac{1}{2}\int(cos4x - cos6x)dx$$

$$= \frac{1}{2}\left[\frac{1}{4}sin4x - \frac{1}{6}sin6x\right] + C$$

$$= \frac{1}{2}sin4x - \frac{1}{12}sin6x + C$$

Trigonometric Substitutions

Expression	Substitution	Identity Used
$\sqrt{a^2 - x^2}$	x = a$\sin\theta$	$cos^2\theta = 1 - sin^2\theta$
$\sqrt{a^2 + x^2}$	x = a$\tan\theta$	$sec^2\theta = 1 + tan^2\theta$
$\sqrt{x^2 - a^2}$	x = a$\sec\theta$	$tan^2\theta = sec^2\theta - 1$

Example:

Evaluate $\int \frac{x^2}{\sqrt{9-x^2}} dx$

Solution:

Put $x = 3sin\theta, -\frac{\pi}{2} \le \theta \le \frac{\pi}{2};\qquad dx = 3cos\theta d\theta$

$$= \int \frac{9sin^2\theta}{\sqrt{9-9sin^2\theta}} 3cos\theta d\theta$$

$$= \int \frac{9sin^2\theta}{3cos\theta} 3cos\theta d\theta \qquad \left[\because \sqrt{1-sin^2\theta} = \sqrt{cos^2\theta} = cos\theta\right]$$

$$= 9\int sin^2\theta d\theta$$

$$= 9\int \frac{1-cos2\theta}{2} d\theta$$

$$= \frac{9}{2}\left[\theta - \frac{sin2\theta}{2}\right] + C$$

$$= \frac{9}{2}\theta - \frac{9}{4}sin2\theta + C$$

$$= \frac{9}{2}\theta - \frac{9}{4}(2sin\theta cos\theta) + C$$

$$= \frac{9}{2}\theta - \frac{9}{2}sin\theta cos\theta + C$$

$$= \frac{9}{2}sin^{-1}\left(\frac{x}{3}\right) - \frac{9}{2}\left(\frac{x}{3}\right)\left(\frac{\sqrt{9-x^2}}{3}\right) + C$$

$$= \frac{9}{2}sin^{-1}\left(\frac{x}{3}\right) - \frac{1}{2}x\sqrt{9-x^2} + C$$

Example:

Evaluate $\int \frac{1}{\sqrt{a^2-x^2}} dx$ by using trigonometric substitution.

Solution:

Put $x = asin\theta, -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$; $dx = acos\theta d\theta$

$$= \int \frac{1}{\sqrt{a^2 - a^2 sin^2\theta}} acos\theta d\theta$$

$$= \int \frac{1}{acos\theta} acos\theta d\theta$$

$$= \int d\theta$$

$$= \theta + C$$

$$= sin^{-1}\left(\frac{x}{a}\right) + C$$

Example:

Evaluate $\int \frac{x}{\sqrt{3-2x-x^2}} dx$

Solution:

Let $3 - 2x - x^2 = -(x^2 + 2x) + 3$

$$= -[(x+1)^2 - 1] + 3$$

$$= -(x+1)^2 + 4$$

Put $u = x + 1 \quad \Rightarrow x = u - 1 \quad \Rightarrow du = dx$

$$\Rightarrow \int \frac{x}{\sqrt{3-2x-x^2}} dx = \int \frac{u-1}{\sqrt{4-u^2}} du$$

$Put \quad u = 2sin\theta$; $du = 2cos\theta d\theta$

$$= \int \frac{2sin\theta - 1}{\sqrt{4 - 4sin^2\theta}} 2cos\theta d\theta$$

$$= \int \frac{2sin\theta - 1}{2cos\theta} 2cos\theta d\theta$$

$$= \int (2sin\theta - 1) d\theta$$

$$= 2(-cos\theta) - \theta + C$$

$$= -2cos\theta - \theta + C$$

$$= -2\frac{\sqrt{4-u^2}}{2} - sin^{-1}\left(\frac{u}{2}\right) + C$$

$$= -\sqrt{4 - (x+1)^2} - sin^{-1}\left(\frac{x+1}{2}\right) + C$$

$$= -\sqrt{3 - 2x - x^2} - sin^{-1}\left(\frac{x+1}{2}\right) + C$$

Example:

Evaluate $\int \frac{dx}{\sqrt{4+x^2}}$

Solution:

Put $x = 2tan\theta, -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}; \qquad dx = 2sec^2\theta d\theta$

$$= \int \frac{1}{\sqrt{4+4tan^2\theta}} 2sec^2\theta d\theta$$

$$= \int \frac{1}{2sec\theta} 2sec^2\theta d\theta \qquad [\because 1 + tan^2\theta = sec^2\theta]$$

$$= \int sec\theta d\theta$$

$$= \log(sec\theta + tan\theta) + C$$

$$= log\left[\frac{\sqrt{4+x^2}}{2} + \frac{x}{2}\right] + C$$

Example:

Evaluate $\int \frac{x^2}{(a^2-x^2)^{3/2}} dx$

Solution:

Given $\int \frac{x^2}{(a^2-x^2)^{3/2}} dx$

$Put\ \ x = asin\theta \Rightarrow \theta = sin^{-1}\left(\frac{x}{a}\right) \quad \Rightarrow dx = acos\theta d\theta$

$$\int \frac{x^2}{(a^2-x^2)^{3/2}} dx = \int \frac{a^2sin^2\theta}{(a^2-a^2sin^2\theta)^{3/2}} acos\theta d\theta$$

$$= \int \frac{a^2sin^2\theta}{(a^2cos^2\theta)^{3/2}} acos\theta d\theta$$

$$= \int \frac{a^2sin^2\theta}{(a^3cos^3\theta)} acos\theta d\theta$$

$$= \int \frac{sin^2\theta}{cos^2\theta} d\theta$$

$$= \int \frac{1-cos^2\theta}{cos^2\theta} d\theta = \int (sec^2\theta - 1) d\theta$$

$$= \int sec^2\theta\, d\theta - \int d\theta$$

$$= tan\theta - \theta + C$$

$$= \frac{x}{\sqrt{a^2-x^2}} - sin^{-1}\left(\frac{x}{a}\right)$$

Example:

Evaluate $\int \frac{1}{x^2\sqrt{x^2+2^2}} dx$

Solution:

Put $x = 2tan\theta, -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}; \qquad dx = 2sec^2\theta d\theta$

$$\int \frac{1}{x^2\sqrt{x^2+2^2}}dx = \int \frac{1}{4tan^2\theta\sqrt{4tan^2\theta+4}}2sec^2\theta d\theta$$

$$= \int \frac{1}{4tan^2\theta 2sec\theta}2sec^2\theta d\theta$$

$$= \int \frac{1}{4}\frac{1}{tan^2\theta}sec\theta d\theta$$

$$= \frac{1}{4}\int \frac{\cos\theta}{sin^2\theta}d\theta$$

$$= \frac{1}{4}\int cosec\theta cot\theta d\theta$$

$$= -\frac{1}{4}cosec\theta + C$$

$$= -\frac{1}{4}\frac{\sqrt{x^2+4}}{x} + c$$

$$= -\frac{\sqrt{x^2+4}}{4x} + C$$

Example:

Evaluate $\int_0^{3\sqrt{3}/2} \frac{x^3}{(4x^2+9)^{3/2}}dx$

Solution:

Given $\int_0^{3\sqrt{3}/2} \frac{x^3}{(4x^2+9)^{3/2}}dx$

$$= \int_0^{3\sqrt{3}/2} \frac{x^3}{8\left(x^2+\frac{9}{4}\right)^{3/2}}dx$$

Put $x = \frac{3}{2}tan\theta$; $\quad x = 0 \Rightarrow \theta = 0$

$dx = \frac{3}{2}sec^2\theta d\theta$; $\quad x = \frac{3\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{3}$

$$= \int_0^{\pi/3} \frac{1}{8}\frac{\frac{27}{8}tan^3\theta}{\left(\frac{9}{4}tan^2\theta+\frac{9}{4}\right)^{3/2}}\frac{3}{2}sec^2 d\theta$$

$$= \int_0^{\pi/3} \left(\frac{1}{8}\right)\left(\frac{27}{8}\right)\left(\frac{3}{2}\right)\frac{tan^3\theta sec^2\theta}{\left(\frac{9}{4}\right)^{3/2}(1+tan^2\theta)^{3/2}}d\theta$$

$$= \int_0^{\pi/3} \left(\frac{1}{8}\right)\left(\frac{27}{8}\right)\left(\frac{3}{2}\right)\left(\frac{4}{9}\right)^{3/2}\frac{tan^3\theta sec^2\theta}{(sec^2\theta)^{3/2}}d\theta$$

$$= \int_0^{\pi/3} \left(\frac{1}{8}\right)\left(\frac{27}{8}\right)\left(\frac{3}{2}\right)\left(\frac{2}{3}\right)^3\frac{tan^3\theta}{sec\theta}d\theta$$

$$= \int_0^{\pi/3} \left(\frac{1}{8}\right)\left(\frac{3}{2}\right)\frac{tan^3\theta}{sec\theta}d\theta$$

$$= \frac{3}{16}\int_0^{\pi/3} \frac{tan^2\theta}{sec\theta}tan\theta d\theta$$

$$= \frac{3}{16}\int_0^{\pi/3}\left(\frac{sec^2\theta-1}{sec\theta}\right) tan\theta d\theta$$

$$= \frac{3}{16}\int_0^{\pi/3}(sec\theta - cos\theta)\, tan\theta d\theta$$

$$= \frac{3}{16}\int_0^{\pi/3}(sec\theta tan\theta - sin\theta)\, d\theta$$

$$= \frac{3}{16}\left[(sec\theta + cos\theta)\right]_0^{\pi/3}$$

$$= \frac{3}{16}\left[\left(2+\frac{1}{2}\right) - (1+1)\right] \;= \frac{3}{32}$$

$$= sin\theta + C$$

$$= \frac{\sqrt{x^2-1}}{x} + C$$