2.1 WEBER - FECHNER LAW

(Relation between loudness and intensity of sound)

According to Weber - Fechner law, the loudness of sound varies with intensity of sound.

Statement

The law states that the **loudness (L) produced is directly proportional to logarithm of intensity.**

i.e., $L \propto k \log_{10} I$

$$L = k \log_{10} I$$

where L – loudness

I – intensity

k – constant

$$\frac{dL}{dI} = \frac{k}{I}$$

SABINES FORMULA FOR REVERBERATION TIME

$$T = \frac{0.167\, V}{\Sigma\, as}$$

It is given by

$$T = \frac{0.167\, V}{a_1 s_1 + a_2 s_2 + \dots}$$

where *V* – Volume of the room or hall in **m³**

a – Absorption coefficients of surface areas of different materials present in the hall in **O.W.U.**

s – Surface areas of the different surfaces in **m²**

Σ *as* – Total absorption of sound i.e., sum of the product of absorption coefficients and surface areas of the different surfaces present in the hall in **O.W.U.** m² **or sabine**

It is popularly known as ***Sabine's formula for reverberation time***.

DERIVATION USING GROWTH AND DECAY METHOD

Let us assume that the sound energy is uniformly distributed throughout the hall. It does not depend on frequency.

We shall calculate the rate at which the sound energy is incident upon the walls and hence the rate at which the sound energy is being absorbed.

Consider a small element *ds* on a plane wall *AB* in the hall as shown in fig. 2.4.

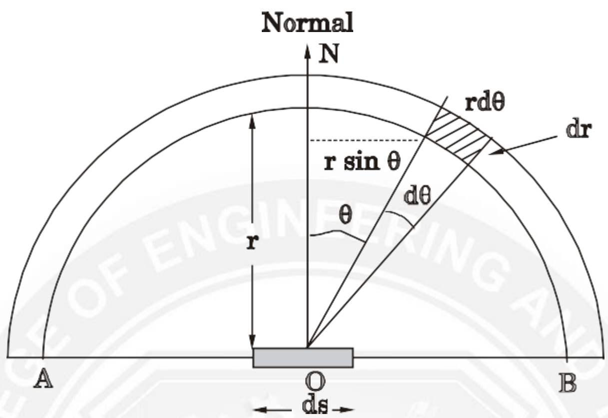

Fig. 2.1.1 Sound absorption on a plane wall

It is assumed that the element *ds* receives sound energy. Taking O as a mid point on *ds*, two semicircles are drawn with radii r and $r + dr$.

Now, consider a small shaded portion between the circles lying between two radii r and $r + dr$ drawn at angles θ and $\theta + d\theta$ with normal ON as shown in fig. 2.4.(a).

Radial length of the shaded portion $= dr$

Arc length of the shaded portion $= r\ d\theta$

Area of this shaded portion $= r\ d\theta\ dr$ ...(1)

Imagine, the whole figure is rotated about the normal through an angle $d\phi$ (radius of the rotating shaded portion being $r \sin\theta$).

The shaded portion travels through a small distance *dx* (circumferential length) and thus, traces out an elemental volume dV (Fig. 2.5).

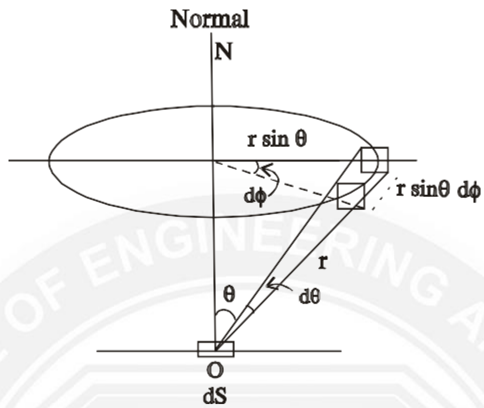

Fig. 2.1.2 - circumferential length

Distance travelled by this shaded portion,

$$dx = r \sin\theta \, d\phi$$

$\therefore$ Volume traced by the shaded portion,

$$dV = \text{area} \times \text{distance travelled}$$

$$dV = r\, d\theta\, dr \times r \sin\theta\, d\phi$$

$$dV = r^2 \sin\theta\, d\theta\, dr\, d\phi \quad \text{...(2)}$$

If E is the sound energy density i.e., sound energy per unit volume, then,

Sound energy present within the elemental volume dV

$$= E \times dV$$

On substituting eqn (2), we have

$$= Er^2 \sin\theta\, dr\, d\theta\, d\phi \qquad \text{...(3)}$$

This sound energy from elemental volume is travelling equally in all directions in total solid angle of 4 π.

$\therefore$ Sound energy travels the volume dV per unit solid angle

$$= \frac{EdV}{4\pi} = \frac{Er^2 \sin\theta \, dr \, d\theta \, d\phi}{4\pi} \quad ...(4)$$

In this case, the solid angle subtended by the area ds at this elemental volume dV

$$= \frac{ds \cos\theta}{r^2}$$

Hence, sound energy from the elemental volume dV towards 'ds' is given by

$$\frac{Er^2 \sin\theta \, d\theta \, dr \, d\phi}{4\pi} \quad \frac{ds \cos\theta}{r^2}$$

$= \frac{Eds}{4\pi} \sin\theta \cos\theta \, d\theta \, d\phi \, dr$ ...(5)

Since sound energy is falling on *ds* from all directions,

θ changes from 0 to π/2 and ϕ changes from 0 to 2π.

Further, to get total sound energy received per second, *r* changes from 0 to *v*, where *v* is the velocity of sound (since sound existing within the distance of 0 to *v* metre from ds reaches *ds* in one second).

$$=\frac{Eds}{4\pi}\int_0^{\pi/2} \sin\theta \cos\theta \, d\theta \int_0^{2\pi} d\emptyset \int_0^{v} dr$$

$$=\frac{Eds}{4\pi}\int_0^{\pi/2} \sin\theta \cos\theta \, d\theta \times 2\pi \times v$$

$$= \frac{Eds}{4\pi} \times \frac{1}{2} \times 2\pi \times v$$

$$= \frac{Evds}{4\pi} \quad \ldots\ldots\ldots\ldots\ldots(6)$$

The total sound energy falling on ds per second = E.v.ds/4, where v is the velocity of sound.

If a is the absorption coefficient of the wall AB of which ds is a part, then the sound energyabsorbed by ds in one second

$$= \frac{E.v.ds.a}{4}$$

Total energy absorbed per second by the whole enclosure = $\frac{E.v.£ads}{4}$

$= EvA/4$ ……………..(7)

Where $A = \Sigma$ a.ds is total absorption of sound in all the surfaces on which sound energy is incident.

Growth of sound decay

If P is the sound power output and V is the total volume of the hall, then total sound energy in the hall at a given instant ‘t’ = EV.

Therefore, rate of growth per second = $V.dE/dt$ …….(8)

Rate of emission of sound energy = Rate of growth of sound energy in room + Rate of absorption of sound by the walls.

i.e. $P = V.dE/dt + EvA/4$ ……………. (9)

When steady state if reached, $dE/dt = 0$, and if steady state energy density is denoted as E_m,

$P = E_m vA/4$

Therefore, $E_m = 4P/vA$

Dividing equation (9) by V, we get

$(dE/dt) + (EvA/4V) = P/V$ …(10)

Putting $vA/4V = \alpha$,

$(dE/dt) + E\alpha = 4P\alpha /vA$

Multiplying with $e^{\alpha t}$ on both sides,

$(dE/dt + E\alpha)\, e^{\alpha t} = 4P\alpha e^{\alpha t}/vA$ $d/dt(Ee^{\alpha t}) = 4P\alpha e^{\alpha t}/vA$

Integrating on both sides,

$$\int \frac{d}{dt}(Ee^{\alpha t}) = \int \frac{4P\alpha e^{\alpha t}}{vA} = \frac{4P\alpha}{vA}\int e^{\alpha t} \int e^{\alpha t} = e^{\alpha t}/\alpha$$

$Ee^{\alpha t} = (4P\alpha e^{\alpha t}/vA) + K$ ……….......... (11)

where K is a constant of integration. The value of K is determined by considering the boundary conditions.

Growth of Sound Energy

Sound energy grows from the instant the source begin to emit sound at t = 0 and E = 0

Applying this in equation (11), we get

$K = -4P/vA$ ……………….(12)

Therefore, using eq 11 and 12

$Ee^{\alpha t} = (4Pe^{\alpha t}/vA) - (4P/vA)$

$E = (4P/vA) - (4Pe^{-\alpha t}/vA)$

$E = 4P/vA\,(1 - e^{-\alpha t})$

Therefore, $\mathbf{E = E_m(1 - e^{-\alpha t})}$

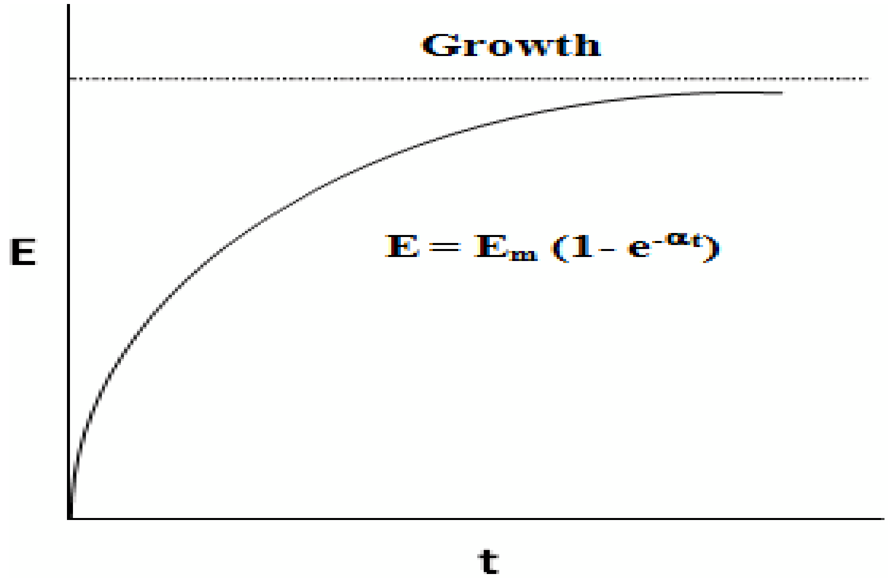

Fig:2.1.3- Growth of Sound Energy

This equation expresses the growth of sound energy density 'E' with time 't'. This indicated that E increases with t, and when $t \to \infty$, $E = E_m$.

Decay of sound energy

Assume that, when sound energy has reached its steady (maximum value) state Em, sound energy is cut off. Then the rate of emission of sound energy, P = 0.

Therefore, equation (11) can be written as $Ee^{\alpha t} = K$

Substituting the boundary conditions $E = E_m$ at t = 0 and P = 0, we get $E_m e^0 = 0 + K$

$K = E_m$ …………….(14)

Therefore, from eq (11) & 14 we get

$Ee^{\alpha t} = E_m$

$E = E_m/e^{\alpha t}$

Therefore, $\mathbf{E = E_m . e^{-\alpha t}}$……(15)

This equation represents the decay of sound energy density with time after the source is cut off.

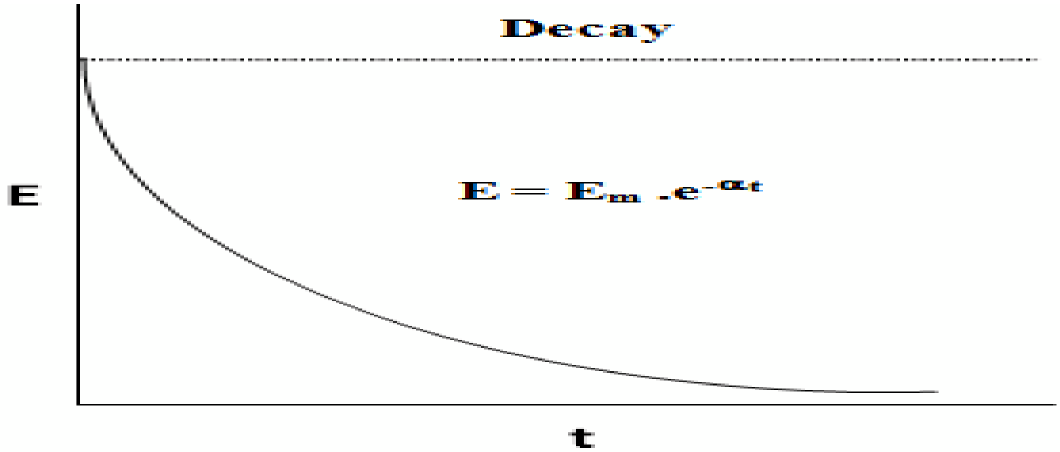

Fig:2.1.3- Decay of sound energy

Expression for reverberation time

The standard reverberation time is the time taken by thesound to fall of its intensity to one-millionth of its initial value after the source is cut off. Now, the value of sound energy density before cut off is E_m, at standard reverberation time, it reduces to$E= E_m/10^6$

To calculate T, we put $E = E_m.10^{-6}$ and $t = T$,

$E_m.10^{-6} = E_m.\ e^{-\alpha T}$

$e^{-\alpha T} = 10^{-6}$

$e^{\alpha T} = 10^6$

Taking log on both sides, we have

$\alpha T = 6\log_e 10$

$T = (6x2.3026x1)/\alpha$

$T = (6x2.3026x1) / (vA/4V)$

By using velocity of sound, v = 340 m/s

T = 0.165 V / A
or
T = 0.165 V / Σas

This equation is in agreement with the experimental values obtained by Sabine.