Unit – III

MAGNETIC AND DIELECTRIC PROPERTIES OF MATERIALS

DIELECTRIC PROPERTIES OF MATERIALS

3.9 Internal field (or) Lorentz field & Clausius Mosotti equation

When a dielectric material is kept in an external field it exerts a dipole moment in it. Therefore two fields are produced,

1. Due to external field
2. Due to dipole moment

These long range fields are called internal field (or) local field.

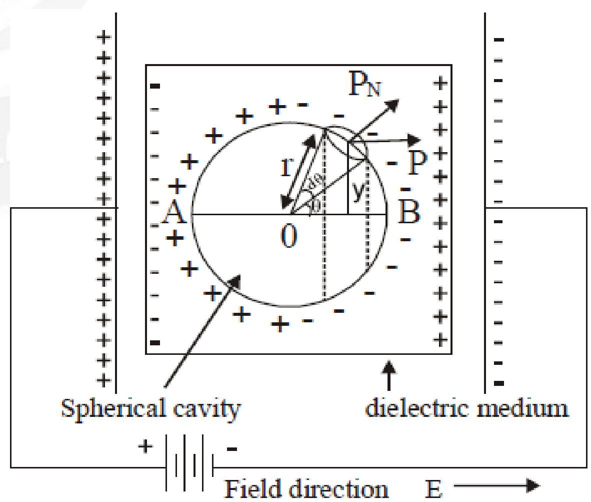
Lorentz method to finding internal field

Let us assume a dielectric material is kept in an external electric field. Consider an imaginary sphere in the solid dielectric of radius 'r'.

Here the radius of the sphere is greater than the radius of the atom, because there are many dipoles within the sphere. An elemental ring is cut with thickness 'ds' and 'y' is the radius of the ring.

The electric field at the center of the sphere is called internal field, which is arises due to the following four factors,

$E_{int} = E_1 + E_2 + E_3 + E_4$.............. (1)



Where,

E_1- Electrical field intensity due to the charges on the electrodes

E_2- Electric field due to polarization charges on the plane surface of the dielectric.

E_3- Electric field due to polarized charges induced at the spherical surface.

E_4- Electric field due to atomic dipoles inside the sphere.

Macroscopically we can take intensity E_1 and E_2 is equal to the external field

$$E = E_1 + E_2$$

If we consider the dielectric is highly symmetric, the short range forces due to dipole moments inside the cavity becomes zero

i.e. $E_4 = 0$

Now the equation (1) can be rewritten as

$$E_{int} = E + E_3 \ldots\ldots\ldots\ldots.. (2)$$

Calculation of E_3

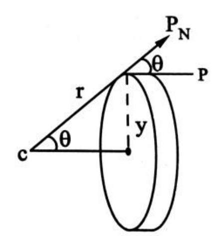
In the elemental ring, let q' be the charge on the area 'dA'. Polarization is defined as the surface charges per unit area. Polarization component is perpendicular to the area.

$$P_N = P\cos\theta = q'/dA$$

$$q' = P\cos\theta\, dA \ldots. (3)$$

The electric field at the center of cavity due to charge $q'$is

$$dE_3 = \frac{q'}{4\pi\varepsilon_0 r^2} \text{-----------} (4)$$

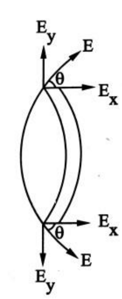


Substituting equation (3) in equation (4) we get

$$dE_3 = \frac{P\cos\theta\, dA}{4\pi\varepsilon_0 r^2} \ldots\ldots\ldots\ldots\ldots (5)$$

The above intensity is along the radius 'r'. Resolving the intensity into two components.

Component parallel to the field direction is



$$E_x = dE_3\cos\theta$$

$$E_x = \frac{P\cos^2\theta\, dA}{4\pi\varepsilon_0 r^2} \ldots... (6)$$

Component perpendicular to the field direction is

$$E_y = dE_3\sin\theta$$

$$E_y = \frac{P\cos\theta\,\sin\theta\, dA}{4\pi\varepsilon_0 r^2} \text{----------} (7)$$

The perpendicular components are in opposite directions and hence cancel each other. So the parallel components are alone taken into consideration.

$$dE_3 = E_x = \frac{Pcos^2\,\theta\, dA}{4\pi\varepsilon_0 r^2} \text{---------- (8)}$$

Consider a ring of area dA

$$dA = circumference\ X\ thickness$$

$$= 2\pi r \sin\theta\,.r\,d\theta$$

$$dA = 2\pi r^2 \sin\theta\, d\theta \text{ .... (9)}$$

Substituting equation(9) in (8), we get

$$dE_3 = \frac{Pcos^2\theta}{4\pi\varepsilon_0 r^2} \text{ x } 2\pi r^2 \sin\theta\, d\theta$$

$$dE_3 = \frac{P\cos^2\theta\sin\theta d\theta}{2\varepsilon_0} \text{ --------------- (10)}$$

Electrical field intensity due to the whole sphere can be derived by integrating equation (10) within the limits 0 to π.

$$\int dE_3 = E_3 = \int_0^{\pi} \frac{P\cos^2\theta\sin\theta d\theta}{2\varepsilon_0} \text{ -------- (11)}$$

$$\int_0^{\pi} \cos^2\theta \sin\theta\, d\theta = 2/3$$

Substituting the above value in equation (11) we get

$$E_3 = \frac{2}{3} . \frac{P}{2\varepsilon_0}$$

$$E_3 = \frac{P}{3\varepsilon_0} \text{ .... (12)}$$

Substitute equation (12) in (2) we get,

$$E_{int} = E + E_3$$

$$E_{int} = E + \frac{P}{3\varepsilon_0} \text{ ------------ (13)}$$

Where E_{int} is known as the internal field or Lorentz field.

Clausius-Mosotti Equation

Then total polarization,

$$P = N\alpha E_{int}$$

$$E_{int} = \frac{P}{N\alpha} \ldots\ldots\ldots\ldots (1)$$

We know that

$$D = \varepsilon E = \varepsilon_0 \text{E} + \text{P}$$

By rearranging we get,

$$(\varepsilon - \varepsilon_0)E = P$$

$$E = \frac{P}{(\varepsilon - \varepsilon_0)} \ldots\ldots\ldots\ldots (2)$$

Internal field is given by

$$E_{int} = E + \frac{P}{3\varepsilon_0} \ldots\ldots\ldots\ldots (3)$$

Sub equation (2) in (3), we get

$$E_{int} = \frac{P}{(\varepsilon - \varepsilon_0)} + \frac{P}{3\varepsilon_0}$$

$$= \frac{P[3\varepsilon_0 + (\varepsilon - \varepsilon_0)]}{3\varepsilon_0(\varepsilon - \varepsilon_0)}$$

$$E_{int} = \frac{P(2\varepsilon_0 + \varepsilon)}{3\varepsilon_0(\varepsilon - \varepsilon_0)} \ldots\ldots\ldots\ldots (4)$$

Comparing equation (1) and (4) we get,

$$\frac{P}{N\alpha} = \frac{P(2\varepsilon_0 + \varepsilon)}{3\varepsilon_0(\varepsilon - \varepsilon_0)}$$

$$\frac{N\alpha}{3\varepsilon_0} = \frac{\varepsilon - \varepsilon_0}{2\varepsilon_0 + \varepsilon}$$

$$\frac{N\alpha}{3\varepsilon_0} = \frac{\varepsilon_0(\varepsilon/\varepsilon_0 - 1)}{\varepsilon_0(\varepsilon/\varepsilon_0 + 2)}$$

$$\frac{N\alpha}{3\varepsilon_0} = \frac{\varepsilon_r - 1}{\varepsilon_r + 2} \text{---------------- (5)}$$

Where ε_r *i*s dielectric constant. The above equation is known as Clausius-Mosotti Equation. This equation gives us relation between dielectric constant **(ε_r)** and polarizability**(α).**