UNIT IV

INITIAL VALUE PROBLEMS FOR ORDINARY DIFFERENTIAL EQUATIONS

PROBLEMS BASED ON

Milne's Predictor and Corrector Method

Milne's Predictor and Corrector Method

$$y_{n+1}, p = y_{n-3} + \frac{4h}{3}\left[2y'_{n-2} - y'_{n-1} + 2y'_{n}\right]$$

$$y_{n+1}, c = y_{n-1} + \frac{h}{3}\left[y'_{n-1} + 4y'_{n} + y'_{n+1}\right]$$

1. ***Using Milne's Method Given*** $\frac{dy}{dx} = x^3 + y$

with $y(0) = 2, y(0.2) = 2.073, y(0.4) = 2.452, y(0.6) = 3.023$

find $y(0.8)$

solution:

Given: $y' = f(x,y) = x^3 + y$

$x_0 = 0$	$y_0 = 2$
$x_1 = 0.2$	$y_1 = 2.073$
$x_2 = 0.4$	$y_2 = 2.452$

$x_3 = 0.6$	$y_3 = 3.023$
$x_3 = 0.8$	$y_4 = ?$

$$h = x_1 - x_0 = 0.2 - 0 = 0.2$$

By Milne's Predictor Method

$$y_{n+1}, p = y_{n-3} + \frac{4h}{3}[2y'_{n-2} - y'_{n-1} + 2y'_n]$$

$$y_4, p = y_0 + \frac{4h}{3}[2y'_1 - y'_2 + 2y'_3]$$

We have $y' = x^3 + y$

$y'_1 = x_1^3 + y_1$	$y'_1 = (0.2)^3 + 2.073 = 0.004 + 2.073 = 2.081$
$y'_2 = x_2^3 + y_2$	$y'_2 = (0.4)^3 + 2.452 = 0.064 + 2.452 = 2.516$
$y'_3 = x_3^3 + y_3$	$y'_3 = (0.6)^3 + 3.023 = 0.216 + 3.023 = 3.239$

$$y_4, p = y_0 + \frac{4h}{3}[2y'_1 - y'_2 + 2y'_3]$$

$$y_4, p = 2 + \frac{4(0.2)}{3}[2(2.081) - 2.516 + 2(3.239)]$$

$$y_4, p = 2 + \frac{0.8}{3}[8.124]$$

$$= 2 + 2.1664 = 4.1664$$

By Milne's Corrector Method

$$y_{n+1}, c = y_{n-1} + \frac{h}{3}[y'_{n-1} + 4y'_n + y'_{n+1}]$$

$$y_4, c = y_2 + \frac{h}{3}[y'_2 + 4y'_3 + y'_4]$$

$$y'_4 = x_4^3 + y_4 = (0.8)^3 + 4.1664$$

$$= 0.512 + 4.1664 = 4.6784$$

$$y_4, c = 2.452 + \frac{0.2}{3}[2.513 + 4(3.239) + 4.6784]$$

$$= 2.452 + \frac{0.2}{3}[20.1504]$$

$$= 3.79536$$

2.Using Milne's Method Given $y' = x - y^2$ and with $y(0) = 0, y(0.2) = 0.02, y(0.4) = 0.0795, y(0.6) = 0.1762$ find $y(0.8)$

solution:

$$Given: y' = f(x, y) = x - y^2$$

$x_0 = 0$	$y_0 = 0$
$x_1 = 0.2$	$y_1 = 0.02$
$x_2 = 0.4$	$y_2 = 0.0795$
$x_3 = 0.6$	$y_3 = 0.1762$
$x_3 = 0.8$	$y_4 = ?$

$$h = x_1 - x_0 = 0.2 - 0 = 0.2$$

By Milne's Predictor Method

$$y_{n+1}, p = y_{n-3} + \frac{4h}{3}[2y'_{n-2} - y'_{n-1} + 2y'_n]$$

$$y_4, p = y_0 + \frac{4h}{3}[2y'_1 - y'_2 + 2y'_3]$$

$We\ have\ \ y' = x - y^2$

$y'_1 = x_1 - y_1^2$	$y'_1 = 0.2 - (0.02)^2 = 0.2 - 0.0004 = 0.1996$
$y'_2 = x_2 - y_2^2$	$y'_2 = 0.4 - (0.0795)^2 = 0.4 - 0.0063 = 0.3937$
$y'_3 = x_3 - y_3^2$	$y'_3 = 0.6 - (0.0795)^2 = 0.4 - 0.0063 = 0.3937$

$$y_4, p = y_0 + \frac{4h}{3}[2y'_1 - y'_2 + 2y'_3]$$

$$y_4, p = 2 + \frac{4(0.2)}{3}[2(2.081) - 2.516 + 2(3.239)]$$

$$y_4, p = 2 + \frac{0.8}{3}[8.124]$$

$$= 2 + 2.1664 = 4.1664$$

By Milne's Corrector Method

$$y_{n+1}, c = y_{n-1} + \frac{h}{3}[y'_{n-1} + 4y'_n + y'_{n+1}]$$

$$y_4, c = y_2 + \frac{h}{3}[y'_2 + 4y'_3 + y'_4]$$

$$y'_4 = x_4^3 + y_4 = (0.8)^3 + 4.1664$$

$$= 0.512 + 4.1664 = 4.6784$$

$$y_4, c = 2.452 + \frac{0.2}{3}[2.513 + 4(3.239) + 4.6784]$$

$$= 2.452 + \frac{0.2}{3}[20.1504]$$

$$= 3.7953$$

3. $Using$ $Milne's\ \ Method$ $to\ Find\ y(0.4)$ $Given\ y' = \frac{(1+x^2)y^2}{2}, y(0) = 1, y(0.1) = 1.06, y(0.2) = 1.12$ $and\ y(0.3) = 1.21$

Solution :

$$Given\ y' = \frac{(1+x^2)y^2}{2}$$

$x_0 = 0$	$y_0 = 1$
$x_1 = 0.1$	$y_1 = 1.06$
$x_2 = 0.2$	$y_2 = 1.12$
$x_3 = 0.3$	$y_3 = 1.21$
$x_4 = 0.4$	$y_4 = ?$

$$y' = \frac{(1+x^2)y^2}{2}$$

$$y'_0 = \frac{(1+{x_0}^2){y_0}^2}{2} = \frac{1}{2}(1+0)(1) = \frac{1}{2} = 0.5$$

$$y'_1 = \frac{(1+{x_1}^2){y_1}^2}{2} = \frac{1}{2}(1+(0.1)^2)(1.06)^2 = 0.5674$$

$$y'_2 = \frac{(1+{x_2}^2){y_2}^2}{2} = \frac{1}{2}(1+(0.2)^2)(1.12)^2 = 0.6522$$

$$y'_3 = \frac{(1+{x_3}^2){y_3}^2}{2} = \frac{1}{2}(1+(0.3)^2)(1.21)^2 = 0.7979$$

By milne's method

$$y_4, p = y_0 + \frac{4h}{3}[2y'_1 - y'_2 + 2y'_3]$$

$$= y_0 + \frac{4(0.1)}{3}[2(0.5674) - 0.6522 + 2(0.7979)] = 1.2771$$

By Milne's Corrector Method

$$y_{n+1}, c = y_{n-1} + \frac{h}{3}[y'_{n-1} + 4y'_n + y'_{n+1}]$$

$$y'_4 = \frac{(1+{x_4}^2){y_4}^2}{2} = \frac{1}{2}(1+(0.4)^2)(1.2771)^2 = 0.9460$$

$$y_4, c = 1.12 + \frac{(0.1)}{3}[0.6522 + 4(0.7979) + 0.9460]$$

$$= 1.2797$$

$4.$ $\boldsymbol{Using\ Runge-Kutta\ \ fourth\ order\ Method}$

$\boldsymbol{find\ y\ at\ x = 0.1, 0.2, 0.3\ if\ \frac{dy}{dx} = xy + y^2\ \ with\ y(0) = 1}$ **and alsofind the solution at** $\boldsymbol{x = 0.4}$ **using Milne's method -**

$\boldsymbol{solution}$:

$\boldsymbol{Given\ y' = f(x, y) = xy + y^2\ \ \ and}$

$$\boldsymbol{x_0 = 0\ and\ y_0 = 1}$$

$$\boldsymbol{h = \ x_1 - \ x_0 = 0.1 - 0 = 0.}$$

$\boldsymbol{By\ Runge-Kutta\ Method}$

$\boldsymbol{k_1 = hf(x_n, y_n)}$

$\boldsymbol{k_2 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)}$

$\boldsymbol{k_3 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)}$

$\boldsymbol{k_4 = hf(x_n + h, y_n + k_3)}$

$$\boldsymbol{\Delta y = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]}$$

$\boldsymbol{k_1 = hf(x_0, y_0)}$

$$= (0.1)f(0,1)$$

$$= (0.1)[0 + 1^2] = 0.1$$

$$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$$

$$= (0.1)f\left(0 + \frac{0.1}{2}, 1 + \frac{0.1}{2}\right)$$

$$= (0.1)f(0.05, 1.05)$$

$$= (0.1)[(0.05)(1.05) + (1.05)^2] = 0.1[0.0525 + 1.1025]$$

$$= 0.1[1.155]$$

$$= 0.1155$$

$$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$$

$$= (0.1)f\left(0 + \frac{0.1}{2}, 1 + \frac{0.1155}{2}\right)$$

$$= (0.1)f(0.05, 1.05775)$$

$$= (0.1)[(0.05)(1.05775) + (1.05775)^2] = 0.1[0.0528875 + 1.118835]$$

$$= 0.1[1.1717225]$$

$$= 0.11717$$

$$k_4 = hf(x_0 + h, y_0 + k_3)$$

$$= 0.1f(0 + 0.1, 1 + 0.11717)$$

$$= hf(0.1, 1.11717)$$

$$= (0.1)[(0.1)(1.11717) + (1.11717)^2] = 0.1[0.111717 + 1.24807]$$

$$= 0.13598$$

$$\Delta y = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$$

$$= \frac{1}{6}[0.1 + 2(0.1155) + 2(0.11717) + 0.13598]$$

$$= \frac{1}{6}[0.70132] = 0.11689$$

$$y_1 = y_0 + \Delta y = 1 + 0.11689 = 1.11689$$

To find $y(0.2)$

$$x_1 = 0.1 \qquad y_1 = 1.11689$$

$$k_1 = hf(x_1, y_1) = (0.1)f(0.1, 1.11689)$$

$$= (0.1)[(0.1)(1.11689) + (1.11689)^2]$$

$$= (0.1)[0.111689 + 1.24744]$$

$$=0.1359$$

$$k_2 = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{k_1}{2}\right)$$

$$= (0.1)f\left(0.1 + \frac{0.1}{2}, 1.11689 + \frac{0.1359}{2}\right)$$

$$= (0.1)f(0.15, 1.18484)$$

$$= (0.1)[(0.15)(1.18484) + (1.18484)^2]$$

$$= (0.1)[0.177726 + 1.403846]$$

$$=0.1582$$

$$k_3 = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2}\right)$$

$$= 0.1f\left(0.1 + \frac{0.1}{2}, 1.1689 + \frac{0.1582}{2}\right)$$

$$= 0.1f(0.15, 1.19599)$$

$$=(0.1)[(0.15)(1.19599) + (1.19599)^2]$$

$$= (0.1)[0.1793985 + 1.43039208]$$

$$=0.16098$$

$$k_4 = hf(x_1 + h, y_1 + k_3)$$

$$= 0.1f(0.1 + 0.1, 1.11689 + 0.16098)$$

$$= 0.1f(0.2, 1.27787)$$

$$=(0.1)[(0.2)(1.27787) + (1.27787)^2]$$

$$=(0.1)[0.255574 + 1.63295]$$

$$=0.1889$$

$$\Delta y = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$$

$$= \frac{1}{6}[0.1359 + 2(0.1582) + 2(0.16098) + 0.1889]$$

$$= 0.16053$$

$$y_2 = y(0.2) = y_1 + \Delta y = 1.11689 + 0.16053 = 1.2774$$

To find $y(0.3)$

$$x_2 = 0.2 \qquad y_2 = 1.2774$$

$$k_1 = hf(x_2, y_2) = (0.1)f(0.2, 1.2774)$$

$$= (0.1)[(0.2)(1.2774) + (1.2774)^2]$$

$$= (0.1)[(0.25548) + (1.63175)]$$

$$= 0.1887$$

$$k_2 = hf\left(x_2 + \frac{h}{2}, y_2 + \frac{k_1}{2}\right)$$

$$= (0.1)f\left(0.2 + \frac{0.1}{2}, 1.2774 + \frac{0.1887}{2}\right)$$

$$= (0.1)f(0.25, 1.37175)$$

$$= (0.1)[(0.25)(1.37175) + (1.37175)^2]$$

$$=(0.1)[(0.3429375) + (1.881698)^2]$$

$$=0.22246$$

$$k_3 = hf\left(x_2 + \frac{h}{2}, y_2 + \frac{k_2}{2}\right)$$

$$= (0.1)f\left(0.2 + \frac{0.1}{2}, 1.2774 + \frac{0.22246}{2}\right)$$

$$= (0.1)f(0.25, 1.38863)$$

$$= (0.1)[(0.25)(1.38863) + (1.38863)^2]$$

$$= 0.2275$$

$$k_4 = hf(x_2 + h, y_2 + k_3)$$

$$= 0.1f(0.2 + 0.1, 1.2774 + 0.2275)$$

$$= 0.1f(0.3, 1.5049)$$

$$=(0.1)[(0.3)(1.5049) + (1.5049)^2]$$

$$=0.2716$$

$$\Delta y = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$$

$$= \frac{1}{6}[0.1887 + 2(0.22246) + 2(0.2275) + 0.2716]$$

=0.2267

$$y(0.3) = y_2 + \Delta y$$

=1.2774+0.2267

= 1.5041

$x_0 = 0$	$y_0 = 1$	$y'_0 = 0$
$x_1 = 0.1$	$y_1 = 1.11689$	$y'_1 = (0.1)(1.11689) + (1.11689)^2 = 1.3591$
$x_2 = 0.2$	$y_2 = 1.2774$	$y'_2 = (0.2)(1.2774) + (1.2774)^2 = 1.8872$
$x_3 = 0.3$	$y_3 = 1.5041$	$y'_3 = (0.3)(1.5041) + (1.5041)^2 = 2.7136$

By Milne's Predictor formula

$$y_{4}, p = y_0 + \frac{4h}{3}[2y'_1 - y'_2 + 2y'_3]$$

$$= 1 + \frac{4(0.1)}{3}[2(1.3591) - 1.8872 + 2(2.7136)]$$

=1.8344

$x_4 = 0.4$	$y_4 = 1.8344$	$y'_4 = (0.4)(1.8344) + (1.8344)^2$ $= 4.09878$

By Milne's Corrector Method

$$y_{n+1}, c = y_{n-1} + \frac{h}{3}[y'_{n-1} + 4y'_n + y'_{n+1}]$$

$$y_4, c = y_2 + \frac{h}{3}[y'_2 + 4y'_3 + y'_4]$$

$$= 1.2774 + \frac{0.1}{3}[1.8872 + 4(2.7136) + 4.09878]$$

=1.8387

5. *Using Taylors Serious Method find y at x = 0.1*

$if\ y'' + xy' + y = 0 with\ y(0) = 1, y'(0) = 0$ **Obtain y for x=0.1,0.2,0.3 by Taylor's series method and find the solution for y(0.4) by Milne's method**

Given : $y'' + xy' + y = 0$

$$y'' = -xy' - y$$

$$x_0 = 0 \text{ and } y_0 = 1$$

$$y'_0 = 0$$

$y'' = -xy' - y$	$y''_0 = -x_0y'_0 - y_0$ $= -(0)(0) - 1 = -1$
$y''' = -xy'' - y' - y'$ $= -xy'' - 2y'$	$y'''_0 = -x_0y''_0 - 2y'_0$ =-(0)(-1)-2(0)=0
$y'^v = -3y'' - xy''$	$y'^v = -3y''_0 - x_0y''_0$

	=-3(-1)-(0)(0)=3

$$y(x) = y_0 + \frac{x}{1!}y'_0 + \frac{x^2}{2!}y''_0 + \frac{x^3}{3!}y'''_0 + \cdots \ldots \ldots \ldots \ldots \ldots \ldots \ldots ..$$

$$y(x) = 1 + x(o) + \frac{(x)^2}{2}(-1) + 0 + \frac{(x)^4}{4!}(3) + \cdots \ldots \ldots \ldots \ldots \ldots \ldots \ldots ..$$

$$y(x) = 1 - \frac{(x)^2}{2!} + \frac{(x)^4}{8} - \frac{(x)^6}{48} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots ..$$

$$y(0.1) = 1 - \frac{(0.1)^2}{2!} + \frac{(0.1)^4}{8} + \cdots \ldots \ldots \ldots \ldots \ldots \ldots \ldots ..$$

$$y(0.1) = 1 - 0.005 + 0.0000125 = 0.995$$

$$y(0.2) = 1 - \frac{(0.2)^2}{2!} + \frac{(0.2)^4}{8} + \cdots \ldots \ldots \ldots \ldots \ldots \ldots \ldots ..$$

$$= 1 - 0.02 + 0.0002 = 0.9802$$

$$y(0.3) = 1 - \frac{(0.3)^2}{2!} + \frac{(0.3)^4}{8} + \cdots \ldots \ldots \ldots \ldots \ldots \ldots \ldots ..$$

$$= 1 - 0.045 + 0.0010125 = 0.956$$

$x_0 = 0$	$y_0 = 1$	$y'_0 = 0$
$x_1 = 0.1$	$y_1 = 0.995$	$y'_1 = -0.0995$
$x_2 = 0.2$	$y_2 = 0.9802$	$y'_2 = -0.196$
$x_3 = 0.3$	$y_3 = 0.956$	$y'_3 = -0.2865$

$$y(x) = 1 - \frac{(x)^2}{2!} + \frac{(x)^4}{8} - \frac{(x)^6}{48} \dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$y'(x) = \frac{-2x}{2!} + \frac{4(x)^3}{8} - \frac{6(x)^5}{48} \dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$y'(x) = -x + \frac{1(x)^3}{2} - \frac{1(x)^5}{8} \dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$y'(0.1) = -0.1 + \frac{1(0.1)^3}{2} - \frac{1(0.1)^5}{8} \dots\dots\dots\dots\dots\dots\dots\dots$$

$$= -0.1 + 0.0005$$

$$= 0.0995$$

$$y'(0.2) = -0.2 + \frac{1(0.2)^3}{2}$$

$$= -0.196$$

$$y'(0.3) = -0.3 + \frac{1(0.3)^3}{2}$$

$$= -0.2865$$

By milne's method

$$y_4, p = y_0 + \frac{4h}{3}\left[2y'_1 - y'_2 + 2y'_3\right]$$

$$= 1 + \frac{4(0.1)}{3}[2(-0.0995) - (-0.196) + 2(-0.2865)]$$

=0.9232

By Milne's Corrector Method

$$y_{n+1}, c = y_{n-1} + \frac{h}{3}\left[y'_{n-1} + 4y'_n + y'_{n+1}\right]$$

$$y_4, c = y_2 + \frac{h}{3}\left[y'_2 + 4y'_3 + y'_4\right]$$

$$y'(0.4) = -0.4 + \frac{1(0.4)^3}{2}$$

$$= -0.368$$

$$y_4, c = -0.9802 + \frac{0.1}{3}[-0.196 + 4(-0.2865) + (-0.368)]$$

$$= 0.9232$$

Adam's Bash Forth Predictor and Corrector Method

$$y_{n+1}, p = y_n + \frac{h}{24}\left[55y'_n - 59y'_{n-1} + 37y'_{n-2} - 9y'_{n-3}\right]$$

$$y_{n+1}, c = y_n + \frac{h}{24}\left[9y'_{n+1} + 19y'_n - 5y'_{n-1} + y'_{n-2}\right]$$

1. *Using Adam's Bash Method Given* $\frac{dy}{dx} = x^2(1 + y)$

with $y(1) = 1, y(1.1) = 1.233, y(1.2) = 1.548, y(1.3) = 1.979$

$find\ y(1.4)$

$solution:$

$$Given: y' = x^2(1+y)$$

$x_0 = 1$	$y_0 = 1$
$x_1 = 1.1$	$y_1 = 1.233$
$x_2 = 1.2$	$y_2 = 1.548$
$x_3 = 1.3$	$y_3 = 1.979$
$x_4 = 1.4$	$y_4 = ?$

$$h = x_1 - x_0 = 1.1 - 1 = 0.1$$

By Adam's Bash Forth Predictor Method

$$y_{n+1},p = y_n + \frac{h}{24}[55y'_n - 59y'_{n-1} + 37y'_{n-2} - 9y'_{n-3}]$$

$$y_4,p = y_3 + \frac{h}{24}[55y'_3 - 59y'_2 + 37y'_1 - 9y'_0]$$

$We\ have\ y' = x^2(1+y)$

$y'_0 = x_0^2(1+y_0)$	$y'_0 = (1)^2(1+1) = 1(2) = 2$
$y'_1 = x_1^2(1+y_1)$	$y'_1 = (1.1)^2(1+1.233) = 1.21 + 2.233 = 3.443$
$y'_2 = x_2^2(1+y_2)$	$y'_2 = (1.2)^2(1+1.548) = 1.44 + 2.548 = 3.988$
$y'_3 = x_3^2(1+y_3)$	$y'_3 = (1.3)^2(1+1.979) = 1.69 + 2.979 = 4.669$

$$y_4,p = y_3 + \frac{h}{24}[55y'_3 - 59y'_2 + 37y'_1 - 9y'_0]$$

$$y_4,p = 1.979 + \frac{0.1}{24}[55(4.669) - 59(3.988) + 37(3.443) - 9(2)]$$

$$y_4, p = 1.979 + \frac{0.1}{24}[256.795 - 235.292 + 127.391 - 18]$$

$$y_4, p = 1.979 + \frac{0.1}{24}[130.894]$$

$$y_4, p = 1.979 + 0.5454 = 2.5244$$

By Adam's Bash Forth Corrector Method

$$y_{n+1}, c = y_n + \frac{h}{24}[9y'_{n+1} + 19y'_n - 5y'_{n-1} + y'_{n-2}]$$

$$y_4, c = y_3 + \frac{h}{24}[9y'_4 + 19y'_3 - 5y'_2 + y'_1]$$

$$y'_4 = x_4^2(1 + y_4) = (1.4)^2(1 + 2.5244)$$

$$= 1.96 + 3.5244 = 5.4844$$

$$y_4, c = 1.979 + \frac{0.1}{24}[9(5.4844) + 19(4.669) - 5(3.988) + 3.443]$$

$$= 2.452 + \frac{0.2}{3}[49.3596 + 88.711 - 19.94 - 3.443]$$

$$= 2.452 + \frac{0.2}{3}[114.6876]$$

$$= 2.452 + 0.4779$$

$$= 2.9299$$

2. ***Using Adam's Bash Method Given*** $\frac{dy}{dx} = xy + y^2$

$with\ y(0) = 1, y(0.1) = 1.1169\ , y(0.2) = 1.2774\ , y(0.3) = 1.5041$

$find\ y(0.4)$

solution:

$$Given: y' = xy + y^2$$

$x_0 = 0$	$y_0 = 1$
$x_1 = 0.1$	$y_1 = 1.1169$
$x_2 = 0.2$	$y_2 = 1.2774$
$x_3 = 0.3$	$y_3 = 1.5041$
$x_4 = 0.4$	$y_4 = ?$

$$h = x_1 - x_0 = 0.1 - 0 = 0.1$$

By Adam's Bash Forth Predictor Method

$$y_{n+1}, p = y_n + \frac{h}{24}[55y'_n - 59y'_{n-1} + 37y'_{n-2} - 9y'_{n-3}]$$

$$y_4, p = y_3 + \frac{h}{24}[55y'_3 - 59y'_2 + 37y'_1 - 9y'_0]$$

$We\ have\ y' = xy + y^2$

$y'_0 = x_0y_0 + {y_0}^2$	$y'_0 = (0)(1) + (1)^2 = 0 + 1 = 1$
$y'_1 = x_1y_1 + {y_1}^2$	$y'_1 = (0.1)(1.1169) + (1.1169)^2 = 1.3592$
$y'_2 = x_2y_2 + {y_2}^2$	$y'_2 = (0.2)(1.2774) + (1.2774)^2 = 1.8872$
$y'_3 = x_3y_3 + {y_3}^2$	$y'_3 = (0.3)(1.5041) + (1.5041)^2 = 2.7135$

$$y_4, p = y_3 + \frac{h}{24}[55y'_3 - 59y'_2 + 37y'_1 - 9y'_0]$$

$$y_4, p = 1.5041 + \frac{0.1}{24}[55(2.7135) - 59(1.8872) + 37(1.3592) - 9(1)]$$

$y_4, p = 1.5041 + 0.33 = 1.8341$

By Adam's Bash Forth Corrector Method

$$y_{n+1}, c = y_n + \frac{h}{24}[9y'_{n+1} + 19y'_n - 5y'_{n-1} + y'_{n-2}]$$

$$y_4, c = y_3 + \frac{h}{24}[9y'_4 + 19y'_3 - 5y'_2 + y'_1]$$

$$y'_4 = x_4 y_4 + {y_4}^2 = (0.4)(1.8341) + (1.8341)^2 = 4.0976$$

$$y_4, c = 1.5041 + \frac{0.1}{24}[9(4.0976) + 19(2.7135) - 5(1.8872) + 1.3592]$$

$$= 1.5041 + 0.3348$$

$$= 1.8389$$

3. ***Cosider the initial value problem*** $\frac{dy}{dx} = y - x^2 + 1$

$$\textit{with } y(0) = 0.5$$

(i)***Using the*** **modified Euler method ,find y(0.2)**

(ii)***Using the*** $4th$ **order Runge kutta method ,find y(0.4) and y(0.6)**

(iii) ***Using Adam's Bash   Method*** **find y(0.8)**

Solution :

(i) $(x, y) = y - x^2 + 1$, $x_0 = 0$ $y_0 = 0.5$, **h=0.2** $x_1 = 0.2$

Modified Euler Method

$$y_{n+1} = y_0 + h\left[f\left(x_n + \frac{h}{2}, y_n + \frac{h}{2} f(x_n, y_n)\right)\right]$$

$$y_1 = y_0 + h\left[f\left(x_0 + \frac{h}{2}, y_0 + \frac{h}{2} f(x_0, y_0)\right)\right]$$

$$y_1 = 0.5 + 0.2\left[f\left(0 + \frac{0.2}{2}, 0.5 + \frac{0.2}{2} f(0, 0.5)\right)\right]$$
$$y_1 = 0.5 + 0.2[f(0.1, 0.5 + (0.1)(0.5 + 1))]$$
$$y_1 = 0.5 + 0.2[f(0.1, 0.65)]$$
$$y_1 = 0.5 + 0.2[0.65 - (0.1)^2 + 1\,]$$
$$y_1 = 0.5 + 0.328$$
$$y_1 = 0.828$$

(ii) $(x, y) = y - x^2 + 1$, $x_0 = 0$ $y_0 = 0.5$, h=0.2 $x_1 = 0.2$

$y_1 = 0.828,$ $x_2 = 0.4$

To find $y_2 = y(0.4)$

By Runge Kutta 4th order method

<u>Runge – Kutta Method</u>

$k_1 = hf(x_n, y_n)$

$k_2 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$

$k_3 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$

$k_4 = hf(x_n + h, y_n + k_3)$

$\Delta y = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$

$$y_1 = y_0 + \Delta y$$

$$k_1 = hf(x_1, y_1)$$

$=(0.2)[y_1 - x_1^2 + 1]$

$=(0.2)[0.828 - (0.2)^2 + 1]$

$=0.3576$

$$k_2 = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{k_1}{2}\right)$$

$$= (0.2)f\left((0.2) + \frac{(0.2)}{2}, 0.828 + \frac{0.3576}{2}\right)$$

$$= (0.2)f(0.3, 1.0068)$$

$=(0.2)[1.0068 - (0.3)^2 + 1]$

$=(0.2)[1.9168]$

$=0.3834$

$$k_3 = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2}\right)$$

$$= (0.2)f\left((0.2) + \frac{(0.2)}{2}, 0.828 + \frac{0.3834}{2}\right)$$

$$= (0.2)f(0.3, 1.0197)$$

$$=(0.2)[1.0197 - (0.3)^2 + 1]$$

$$=(0.2)[1.9297]$$

$$=0.3859$$

$$k_4 = hf(x_1 + h, y_1 + k_3)$$

$$k_4 = (0.2)f(0.2 + 0.2, 0.828 + 0.3859)$$

$$= (0.2)f(0.4, 1.2139)$$

$$= (0.2)[1.2139 - (0.4)^2 + 1]$$

$$= (0.2)[2.0539]$$

$$=0.4108$$

$\Delta y = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$

$= \frac{1}{6}[0.3576 + 2(0.3834) + 2(0.3859) + 0.4108]$

=0.3845

$y(0.4) = y_1 + \Delta y$

$= 0.828 + 0.3845$

$y(0.4) = y_2 = 1.2125$

To find $y_3 = y(0.6)$

By Runge Kutta 4th order method

$k_1 = hf(x_2, y_2)$

$$= (0.2)f(0.4, 1.2125)$$

$$= (0.2)[1.2125 - (0.4)^2 + 1] = 0.4105$$

$k_2 = hf\left(x_2 + \frac{h}{2}, y_2 + \frac{k_1}{2}\right)$

$k_2 = (0.2)f\left(0.4 + \frac{0.2}{2}, 1.2125 + \frac{0.4105}{2}\right)$

$k_2 = (0.2)f(0.5, 1.41775)$

$$= (0.2)[1.41775 - (0.5)^2 + 1] = 0.4336$$

$$k_3 = hf\left(x_3 + \frac{h}{2}, y_3 + \frac{k_2}{2}\right)$$

$$= (0.2)f\left(0.4 + \frac{0.2}{2}, 1.2125 + \frac{0.4336}{2}\right)$$

$$= (0.2)f(0.5, 1.4293)$$

$$= (0.2)[1.4293 - (0.5)^2 + 1] = 0.4359$$

$$k_4 = hf(x_2 + h, y_2 + k_3)$$

$$k_4 = (0.2)f(0.4 + 0.2, 1.2125 + 0.4359)$$

$$k_4 = (0.2)f(0.6, 1.6484)$$

$$k_4 = (0.2)[1.6484 - (0.6)^2 + 1] = 0.4577$$

$$\Delta y = \frac{1}{6}[0.4105 + 2(0.4336) + 2(0.4359) + 0.4577]$$

$$= 0.4345$$

$$y(0.6) = y_3 = y_2 + \Delta y = 1.2125 + 0.4345 = 1.647$$

To find $y(0.8) = y_4$ **by using Adam-Bashforth predictor –Corner method**

Adam Predictor formula

$$y_4, p = y_3 + \frac{h}{24}\left[55y'_3 - 59y'_2 + 37y'_1 - 9y'_0\right]$$

$$y'_0 = [y_0 - x_0^2 + 1] = 0.5 - 0 + 1 = 1.5$$

$$y'_1 = [y_1 - x_1^2 + 1] = 0.828 - (0.2)^2 + 1 = 1.788$$

$$y'_2 = [y_2 - x_2^2 + 1] = 1.2125 - (0.4)^2 + 1 = 2.0525$$

$$y'_3 = [y_3 - x_3^2 + 1] = 1.647 - (0.6)^2 + 1 = 2.287$$

$$y_4, p = 1.647 + \frac{0.2}{24}[55(2.287) - 59(2.0525) + 37(1.788) - 9(1.5)]$$

$=2.1249$

$$y_4' = [y_4 - x_4^2 + 1] = 2.1249 - (0.8)^2 + 1 = 2.4849$$

By Adam's Bash Forth Corrector Method

$$y_{n+1}, c = y_n + \frac{h}{24}\left[9y'_{n+1} + 19y'_n - 5y'_{n-1} + y'_{n-2}\right]$$

$$y_4, c = y_3 + \frac{h}{24}\left[9y'_4 + 19y'_3 - 5y'_2 + y'_1\right]$$

$$y_4, c = 1.647 + \frac{0.2}{24}[9(2.4849) + 19(2.287) - 5(2.0525) + 1.788]$$

=2.109955