Circle Diagram of a 3 Phase Induction Motor

The equivalent circuit of a 3 phase induction motor is shown in the Fig.1.

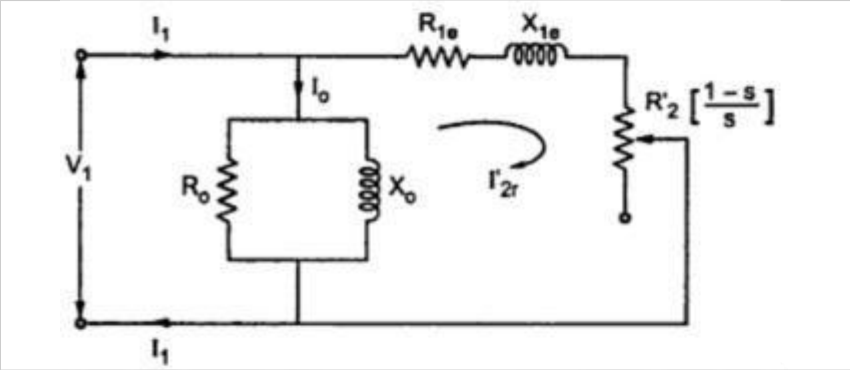

Fig. 3.33 Equivalent circuit of a 3 phase induction motor

All the values shown are per phase values. The circuit is similar to series R-L circuit. The reactance X_{1e} is fixed while the total resistance R_{1e} + (R_2'(1- s)/s) is variable. This is because the slip s varies as load varies. The voltage across the parallel exciting branch is V_1. Hence we can write the expression for the rotor current referred to stator as,

$$I_{2r}' = V_1/\sqrt{(R_{1e} + R_L)^2 + X_{1e}^2}$$

Where

$R_L' = R_2'(1-s)/s$ = Variable equivalent load resistance

$R_{1e} = R_1 + R_2'$ = Equivalent resistance of motor referred to stator

$X_{1e} = X_1 + X_2'$ = Equivalent reactance of motor referred to stator

Dividing and multiplying by,

$$I'_{2r} = \frac{V_1}{X_{1e}} \times \frac{X_{1e}}{\sqrt{(R_{1e} + R'_L)^2 + (X_{1e})^2}}$$

$\therefore$ $I_{2r}' = I_{max}$ sin .... (1)

where $\sin\Phi = X/Z = X_{1e}/\sqrt{(R_{1e} + R_L')^2 + X_{1e}^2}$

and $I_{max} = V_1/X_{1e}$

The I_{2r}' will be at its maximum when $R_{1e} + R_L' = 0$ i.e., there exists an ideal short circuit. Hence current I_{max} is called ideal short circuit current of an induction motor.

The equation (1) represents equation of a circle with as its diameter. Thus locus of extremely of I_{2r}' is a circle, as shown in the Fig.2.

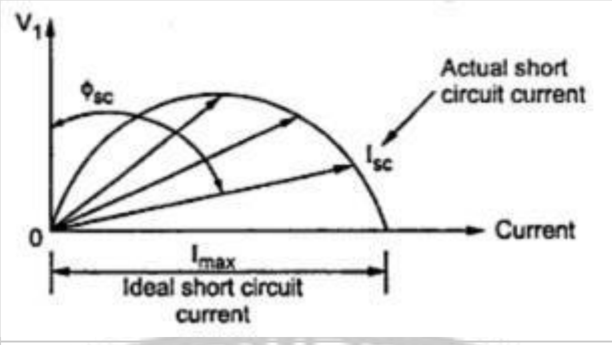

Fig. 3.34 Circle diagram

I_{2r}'.

But the total stator current I_1 per phase is a vector addition of current I_o and

$I_1 = I_o + I_{2r}'$.. Vector addition

For and induction motor, I_o has a fixed value and phase angle Φ_o which is decided by its active component and magnetising component I_m.

$\bar{I}_o = \bar{I}_c + \bar{I}_m$

As I_o has fixed magnitude and phase, the locus of extremities of I_1, which is $I_o + I_{2r}'$ is also a circle with a diameter still as V_1/X_{1e}. The only charge will be that the diameter V_1/X_{1e} will no longer be along X-axis i.e. current axis but will get shifted at the tip of the I_o phasor. All the I_{2r}' phasors are to be drawn from I_o phasor to get I_1, as has fixed magnitude and phase angle Φ_o.

Key Point : Thus the current locus for a stator current is also a semicircle which is truly called circle diagram of a three phase induction motor. This diagram once obtained can be used to predict the performance of an induction motor under variable load conditions. The circle diagram is shown in the Fig. 3.

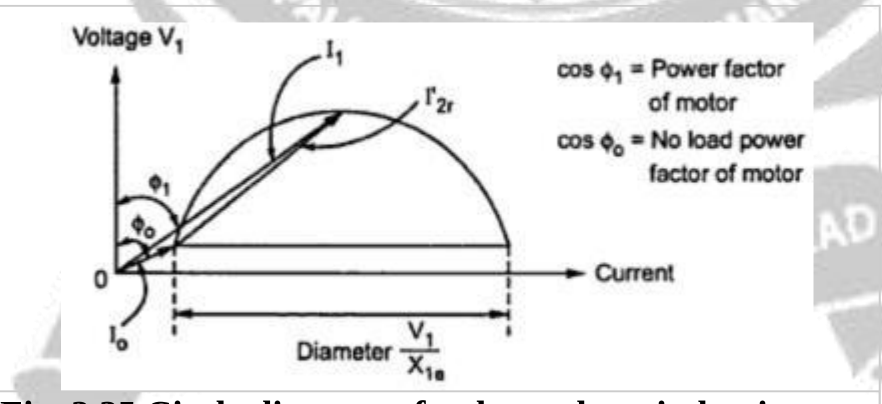

Fig. 3.35 Circle diagram of a three phase induction mo tor

Let us see, how to obtain the data for plotting the circle diagram.

Obtaining Data to Plot Circle Diagram

The data required to draw the circle diagram is obtained by conducting two testes which are,

1. No load test or open circuit test
2. Blocked rotor test or short circuit test

No LoadTest

In this test, the motor is made to run without any load i.e. no load condition. The speed of the motor is very close to the synchronous speed but less than the synchronous speed. The rated voltage is applied to the stator. The input line current and total in put power is measured. The two wattmeter method is used to measure the total input power. The circuit diagram for the test is shown in the Fig. 3.36.

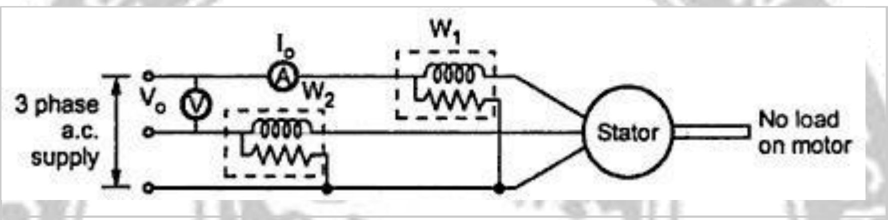

Fig. 3.36 No load test

As the motor is on no load, the power factor is very low which is less than 0.5 and one of the two wattmeters reads negative. It is necessary to reverse the current coil or pressure coil connections of such a wattmeter to get the positive reading. This reading must be taken negative for the further calculations.

The total power input W_o is the algebraic sum of the two wattmeter readings. The observation table is,

V_o volts Rated line voltage	I_o Amp No load current	$W_o = W_1 + W_2$ (Algebraic sum) in watts

The calculations are,

$$W_o = \sqrt{3} V_o I_o \cos\Phi_o$$

$$\therefore \quad \cos\phi_o = \frac{W_o}{\sqrt{3} V_o I_o} \quad \text{where } V_o, I_o \text{ are line values}$$

This is no load power factor.

Thus we are now in a position to obtain magnitude and phase angle of no load current I_o, which is required for the circle diagram.

From the knowledge of I_o and Φ_o, the parameters of the equivalent circuit can be obtained as,

$I_c = I_o \cos\Phi_o$ = Active component of no load current

$I_m = I_o \sin\Phi_o$ = Magnetising component of no load current

$R_o = V_o$ (per phase) / I_c (per phase) = No load branch resistance $X_o = V_o$

(per phase)/ I_m (per phase) = No load branch resistance

The power input W_o consists of following losses,

1. Stator copper loss i.e. $3 I^2 R^2$ where I is no load per phase current
2. Stator core loss i.e. iron loss.
3. Friction and windage loss.

The no load rotor current is very small and hence rotor copper loss is negligibly small. The rotor frequency is s times supply frequency and on no load it is very small. Rotor iron losses are proportional to this frequency and hence are negligibly small.

Key Point : Under no load condition, I_o is also very small and in many practical cases it is also neglected.

Thus W_o consists of stator iron loss and friction and windage loss which are consists for all load conditions. Hence W_o is said to give fixed losses of the motor.

$\therefore$ W_o = No load power input

$\therefore$ W_o = Fixed Loss ... Neglecting stator copper loss

Separation No Load Losses

The no load losses are the constant losses which include core loss and friction and windage loss. The separation between the two can be carried out by the no load test conducted from variable voltage, rated frequency supply.

When the voltage is decreased below the rated value, The core loss reduces as nearly square of voltage. The slip does not increase significantly the friction and windage loss almost remains constant.

The voltage is continuously decreased till the machine slip suddenly begins to increase and the motor tends to stall. At no load, this takes place at a sufficiently reduced voltage. The graph showing no load losses $P_{N.L.}$ versus as shown in the Fig. is extrapolated to V = 0 which gives friction and windage loss as iron or core loss is zero at zero voltage.

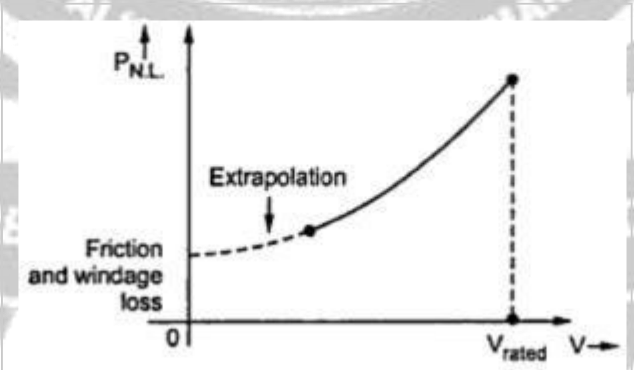

Fig. 3.37 characteristics curve

Blocked Rotor Tests

In this test, the rotor is locked and it is not allowed to rotate. Thus the slip s = 1 and $R_L' = R_2' (1-s)/s$ is zero. If the motor is slip ring induction motor then the windings are short circuited at the slip rings.

The situation is exactly similar to the short circuit test on transformer. If under short circuit condition, if primary is excited with rated voltage, a large shot circuit current can flow which is dangerous from the windings point of view. So similar to the transformer short circuit test, the reduced voltage (about 10 to 15 % of rated voltage) just enough such that stator carries rated current is applied. Now the applied voltage V_{sc}, the input power W_{sc} and a short circuit current I_{sc} are measured.

As $R_L' = 0$, the equivalent circuit is exactly similar to that of a transformer and hence the calculations are similar to that of short circuit test on a transformer.

V_{sc} = Short circuit reduced voltage (line value) I_{sc} = Short circuit current (line value)

W_{sc} = Short circuit input power

Now $W_{sc} = \sqrt{3} V_{sc} I_{sc} \cos\Phi_{sc}$ .... Line values

$$\therefore \quad \cos\phi_{sc} = \frac{W_{sc}}{\sqrt{3}\, V_{sc}\, I_{sc}}$$

This gives us short circuit power factor of a motor.

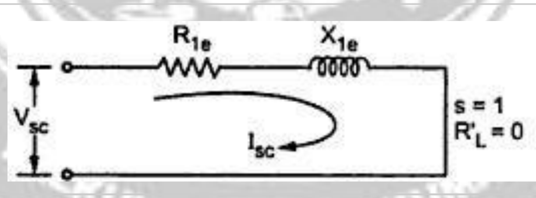

Fig. 3.38 Equivalent Circuit

Now the equivalent circuit is as shown in the Fig. 1.

$$\therefore \quad W_{sc} = 3\,(I_{sc})^2 R_{1e}$$

where I_{sc} = Per phase value

$$\therefore \quad R_{1e} = \frac{W_{sc}}{3\,(I_{sc})^2}$$

This is equivalent resistance referred to stator.

$Z_{1e} = V_{sc}$ (per phase)/ I_{sc} (per phase) = Equivalent impedance referred to stator.

$$\therefore \quad R_{1e} = \frac{W_{sc}}{3(I_{sc})^2}$$

$$\therefore \quad X_{1e} = \sqrt{Z_{1e}^2 - R_{1e}^2}$$

= Equivalent reactance referred to stator

During this test, the stator carries rated current hence the stator copper loss is also dominant. Similarly the rotor also carries short circuit current to produce dominant rotor copper loss. As the voltage is reduced, the iron loss which is proportional to voltage is negligibly small. The motor is at standstill hence mechanical loss i.e. friction and windage loss is absent. Hence we can write,

W_{sc} = Stator copper loss + Rotor copper loss

But it is necessary to obtain short circuit current when normal voltage is applied to the motor. This is practically not possible. But the reduced voltage test results can be used to find current I_{SN} which is short circuit current if normal voltage is applied.

If V_L = Normal rated voltage (line value)

V_{sc} = Reduced short circuit voltage (line voltage)

then $$I_{SN} = \left(\frac{V_L}{V_{sc}}\right) \times I_{sc}$$

where I_{sc} = Short circuit current at reduced voltage Thus,

I_{SN} = Short circuit current at normal voltage Now power input is proportional to square of the current. So W_{SN} = Short circuit input power at normal voltage

This cab be obtained as,

$$W_{SN} = \left(\frac{I_{SN}}{I_{sc}}\right)^2 W_{sc}$$

But at normal voltage core loss can not be negligible hence, W_{SN} = Core loss + Stator and rotor copper loss

Construction of Circle Diagram

By using the data obtained from the no load test and the blocked rotor test, the circle diagram can be drawn using the following steps :

Step 1 : Take reference phasor V as vertical (Y-axis).

Step 2 : Select suitable current scale such that diameter of circle is about 20 to 30 cm.

Step3 : From no load test, I_o and are Φ_o obtained. Draw vector I_o, lagging V by angle Φ_o. This is the line OO' as shown in the Fig. 1.

Step 4 : Draw horizontal line through extremity of I_o i.e. O', parallel to horizontal axis.

Step 5 : Draw the current I_{SN} calculated from I_{sc} with the same scale, lagging V by angle Φ_{sc}, from the origin O. This is phasor OA as shown in the Fig. 1.

Step 6 : Join O'A is called output line.

Step 7 : Draw a perpendicular bisector of O'A. Extend it to meet line O'B at point C. This is the centre of the circle.

Step 8 : Draw the circle, with C as a center and radius equal to O'C. This meets the horizontal line drawn from O' at B as shown in the Fig. 1.

Step 9 : Draw the perpendicular from point A on the horizontal axis, to meet O'B line at F and meet horizontal axis at D.

Step 10 : Torque line.

The torque line separates stator and rotor copper losses.

Note that as voltage axis is vertical, all the vertical distances are proportional to active components of currents or power inputs, if measured at appropriate scale.

Thus the vertical distance AD represents power input at short circuit i.e. W_{SN}, now which consists of core loss and stator, rotor copper losses.

Now FD = O'G

= Fixed loss

Where O'G is drawn perpendicular from O' on horizontal axis. This represents power input on no load i.e. fixed loss.

Hence AF α Sum of stator and rotor copper losses Then point E can be located as,

AE/EF = Rotor copper loss / Stator copper loss

The line O'E under this condition is called torque line.

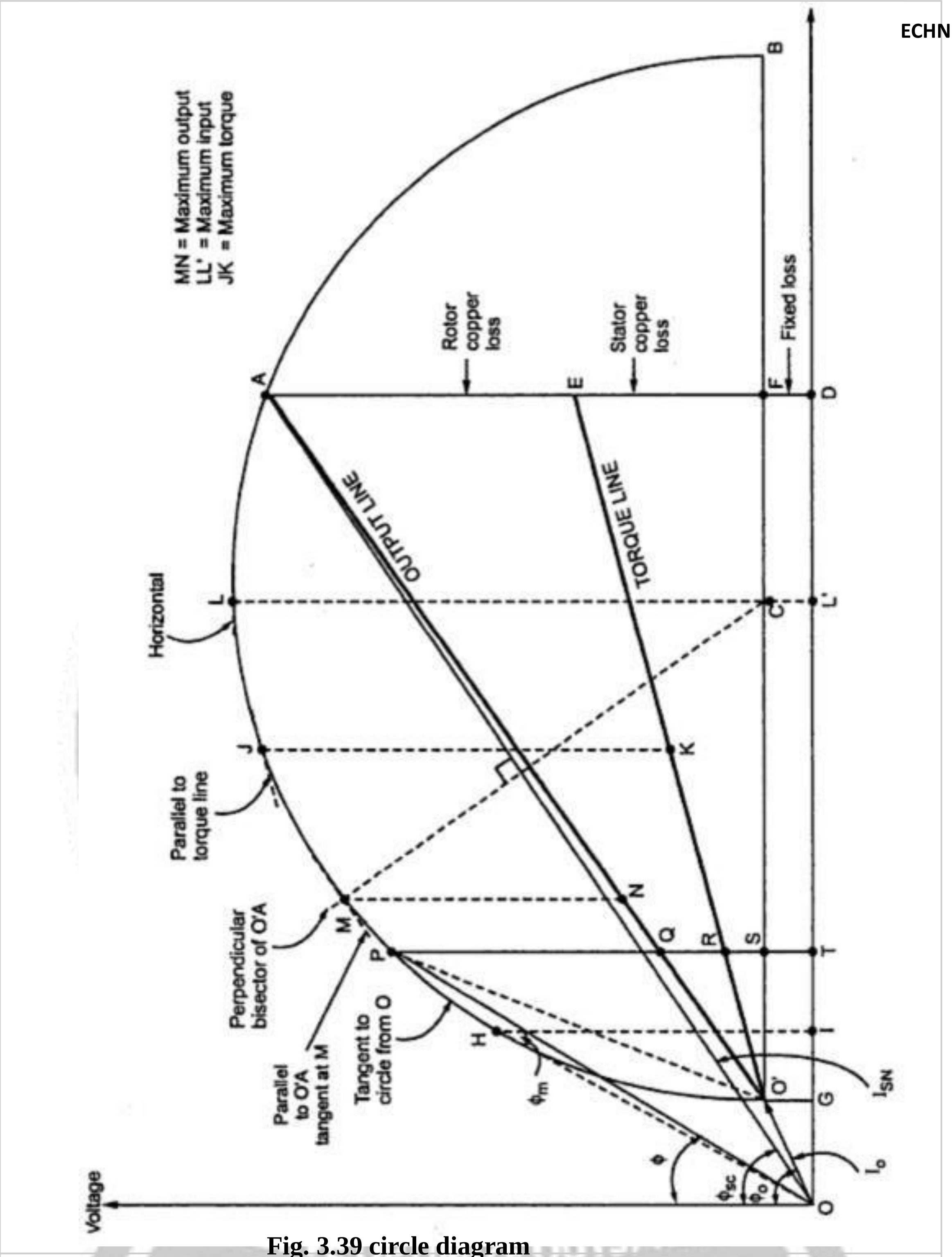

Fig. 3.39 circle diagram

Power scale : As AD represents W_{SN} i.e. power input on short circuit at normal voltage, the power scale can be obtained as,

Power scale = $W_{SN}/l(AD)$ W/cm

where l(AD) = Distance AD in cm

Location of Point E : In a slip ring induction motor, the stator resistance per phase R_1 and rotor resistance per phase R_2 can be easily measured. Similarly by introducing ammeters in stator and rotor circuit, the currents I_1 and I_2 also can be measured.

∴ $K = I_1/I_2$ = Transformation ratio

Now AF/EF = Rotor copper loss / Stator copper loss = $(I_2^2R_2)/(I_1^2R_1)$ =

$(R_2/R_2)(I^2/I_1^2) = (R_2/R_2).(1/K^2)$

But $R_2' = R_2/K^2$ = Rotor resistance referred to stator

∴ $AE/EF = R_2'/R_1$

Thus point E can be obtained by dividing line AF in the ratio R_2' to R_1.

In **a squirrel cage motor**, the stator resistance can be measured by conducting resistance tset.

∴ Stator copper loss = $3I_{SN}^2 R_1$ where I_{SN} is phase value.

Neglecting core loss, W_{SN} = Stator Cu loss + Rotor Cu loss

∴ Rotor copper loss = $W_{SN} - 3I^2 R$

Dividing line AF in this ratio, the point E can be obtained and hence O'E represents torque line.

Predicting Performance Form Circle Diagram

Let motor is running by taking a current OP as shown in the Fig. 1. The various performance parameters can be obtained from the circle diagram at that load condition.

Draw perpendicular from point P to meet output line at Q, torque line at R, the base line at S and horizontal axis at T.

We know the power scale as obtained earlier.

Using the power scale and various distances, the values of the performance parameters can be obtained as,

Total motor input = PT x Power scale Fixed loss = ST x power scale

Stator copper loss = SR x power scale Rotor copper loss = QR x power scale Total loss = QT x power scale

Rotor output = PQ x power scale

Rotor input = PQ + QR = PR x power scale Slip s = Rotor Cu loss = QR/PR

Power factor cos = PT/OP

Motor efficiency = Output / Input = PQ/PT

Rotor efficiency = Rotor output / Rotor input = PQ/PR Rotor output / Rotor input = 1 - s = N/N_s = PQ/PR

Maximum Quantities

The maximum values of various parameters can also be obtained by using circle diagram.

1. Maximum Output : Draw a line parallel to O'A and is also tangent to the circle at point M. The point M can also be obtained by extending the perpendicular drawn from C on O'A to meet the circle at M. Then the maximum output is given by l(MN) at the power scale. This is shown in the Fig. 1.

2. Maximum Input : It occurs at the highest point on the circle i.e. at point L. At this point, tangent to the circle is horizontal. The maximum input given l(LL') at the power scale.

3. Maximum Torque : Draw a line parallel to the torque line and is also tangent to the circle at point J. The point J can also be obtained by drawing perpendicular from C on torque line and extending it to meet circle at point J. The l(JK) represents maximum torque in synchronous watts at the power scale. This torque is also called stalling torque or pull out torque.

4. Maximum Power Factor : Draw a line tangent to the circle from the origin O, meeting circle at point H. Draw a perpendicular from H on horizontal axis till it meets it at point I. Then angle OHI gives angle corresponding to maximum power factor angle.

∴ Maximum p.f. = cos ∟{OHI}

= HI/OH

5. Starting Torque : The torque is proportional to the rotor input. At s = 1, rotor input is equal to rotor copper loss i.e. l(AE).

∴ T_{start} = l(AE) x Power scale.. in synchronous watts

Full load Condition

The full load motor output is given on the name plates in watts or h.p. Calculates the distance corresponding to the full load output using the power scale.

Then extend AD upwards from A onwards, equal to the distance corresponding to full load output, say A'. Draw parallel to the output line O'A from A' to meet the circle at point P'. This is the point corresponding to the full load condition, as shown in the Fig. 2.

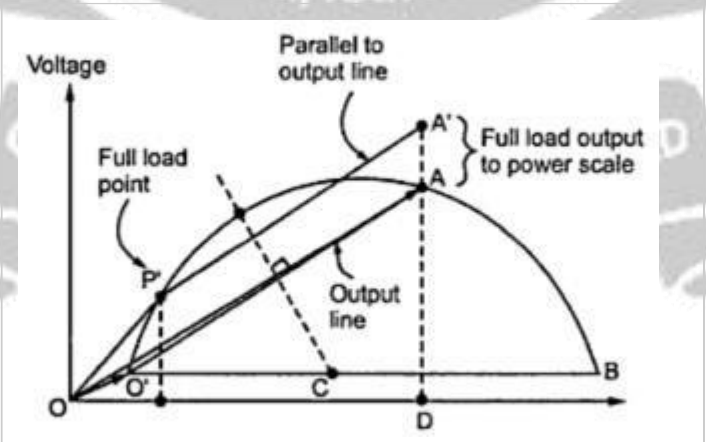

Fig. 3.40 Locating full load point

Once point P' is known, the other performance parameters can be obtained easily as discussed above.