3.2 Analogy between electrical and mechanical oscillating systems:

The analogy of mechanical oscillations with electrical and magnetic oscillations are so called electromagnetic oscillations in LCR circuit.

In spring system there exists mechanical oscillations and in LCR circuits there exists electromagnetic oscillations.

In spring mass system there exists two forms of energy they are

i)Potential energy of the compressed (or) extended spring

ii) Kinetic energy of the mass vibrating with respect to the mean position.

Similarly ,there are two forms of energy involved in LCR circuits namely

i)Magnetic energy of Tnductor carrying current.

ii)Electrical energy of the charged capacitor.

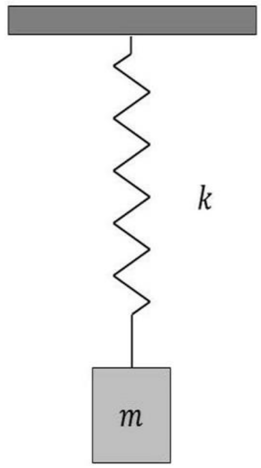
<u>Analogies:</u>

1) The kinetic energy in spring mass system is analogous to magnetic energy of the inductor carrying current in LCR circuit.
2) The potential energy restored by the spring in spring mass system is analogous to the electrical energy restored by the capacitor in LCR circuit.
3) The damping force which slow down the oscillations in spring mass system is analogous to the resistance which restrics the flow of current in LCR circuit.

Proof:

Mechanical oscillations in spring mass system:

Consider a spring mass system in which a mass is located to a spring.



Two forces act on the system they are

i) Restoring force and
ii) Damping force,that acts in opposite direction

Restoring force F_1 = - ky ---------------------(1)

Damping force F_2 = -bv ----------------------(2)

Where K- Force Constant

y-displacement

b-damping constant

v-velocity

If "F" is the force required to accelerate the mass "m" in order to make mechanical oscillations,

Force = mass X Acceleration

F = ma

Accelerating Force = Restoring force + Damping force

ma = - ky + -bv

ma + ky + bv = 0 ---------------------(4)

here, $v = \frac{dy}{dt}$ (velocity)

$$a = \frac{d^2 y}{dt^2} \quad \text{(acceleration)}$$

(4) can be written as

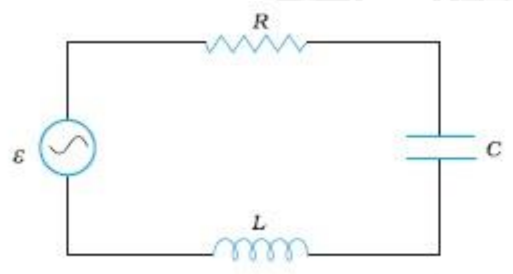
$$m\frac{d^2 y}{dt^2} + b\frac{dy}{dt} + ky = o \text{ ---------------------(5)}$$

Equation (5) represents the second order differential equation for mechanical oscillations.

ii)Electromagnetic oscillations in LCR circuit:

Let ‘q’ be the charge restored in the capacitor ‘c’ and I be the current flowing through the circuit,

According to Loop’s Law



LCR circuit connected to ac source

$$\frac{q}{c} + IR + L\frac{dI}{dt} = 0$$

$$L\frac{dI}{dt} + \frac{q}{c} + IR = 0 \text{ ----------------(6)}$$

$$\text{Current } I = \frac{dq}{dt} \text{ -----------------(7)}$$

$$\text{Rate of flow of current } \frac{dI}{dt} = \frac{d^2 q}{dt^2} \text{ -------------(8)}$$

Sub (7) and (8) in (6) we get

$$L\frac{d^2 q}{dt^2} + \frac{dq}{dt}R + \frac{q}{c} = 0 \text{ -------------------(9)}$$

Equation (9) represents the second order differential equation for electromagnetic oscillations in LCR circuit.

Now by comparing all the terms in eqn(5) and (9) we get

m= L (i.e) mass (m) = Inductance (L)

b= R (i.e) Damping Constant(b) = Resistance (R)

k = $\frac{1}{c}$ (i.e) Force constant k = Reciprocal of Capacitor ($\frac{1}{c}$)

ANALOGIES AND CONCLUSION:

i) The mass in spring mass system (mechanical oscillations) is analogous to the inductor 'L' in LCR circuit (electromagnetic oscillations).

ii) Damping Constant(b) in spring mass system (mechanical oscillations) is analogous to the Resistance (R) in LCR circuit (electromagnetic oscillations).

iii) Force constant k in spring mass system (mechanical oscillations) is analogous to the Capacitor ($\frac{1}{c}$) in LCR circuit (electromagnetic oscillations).