THERMAL CONDUCTION THROUGH COMPOUND MEDIA

BODIES IN SERIES

Let us consider a compound slab made of three different materials A, B and C of thickness d_1, d_2 and d_3 as shown in fig. Let K_1, K_2 and K_3 be their thermal conductivities respectively. Let the temperatures of the faces be θ_1, θ_2, θ_3 and θ_4 respectively and heat flows from A to C. If Q is the quantity of heat flowing through each material per second and A is the surface area then,

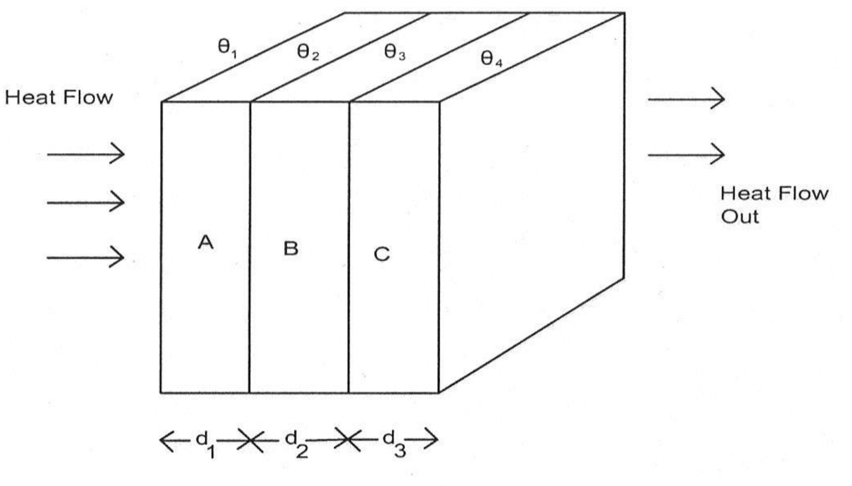

Fig. Bodies in series

$$Q = \frac{K_1 A(\theta_1 - \theta_2)}{d_1} = \frac{K_2 A(\theta_2 - \theta_3)}{d_2} = \frac{K_3 A(\theta_3 - \theta_4)}{d_3}$$

$$\theta_1 - \theta_2 = Q\frac{d_1}{K_1 A} \quad \ldots\ldots\ldots\ldots\ldots.. \quad (1)$$

$$\theta_2 - \theta_3 = Q\frac{d_2}{K_2 A} \quad \ldots\ldots\ldots\ldots\ldots.. \quad (2)$$

$$\theta_3 - \theta_4 = Q\frac{d_3}{K_3 A} \quad \ldots\ldots\ldots\ldots\ldots.. \quad (3)$$

Adding (1), (2) & (3)

$$\theta_1 - \theta_4 = \frac{Q}{A}\left[\frac{d_1}{K_1} + \frac{d_2}{K_2} + \frac{d_3}{K_3}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

Quantity of heat conducted per unit time is given by

$$Q = \frac{A(\theta_1 - \theta_4)}{\left[\frac{d_1}{K_1} + \frac{d_2}{K_2} + \frac{d_3}{K_3}\right]}$$

If there are *n* different materials in series with θ_1 and θ_{n+1} as the temperature of first and last face then quantity of heat conducted per unit time is given by

$$Q = \frac{A(\theta_1 - \theta_{n+1})}{\sum\left(\frac{d}{K}\right)}$$

Note: The amount of heat flowing through the material A, B and C is equal under steady state conditions.

BODIES IN PARALLEL

Let us consider a compound slab made of three different materials A, B & C of cross sectional areas A_1, A_2 and A_3 all with thickness *d* stacked together in parallel as shown in fig. Let K_1, K_2 & K_3 be their thermal conductivities respectively. Let θ_1 and θ_2 be the temperatures of two faces and heat flows from face at temperature θ_1 to face at temperature θ_2.

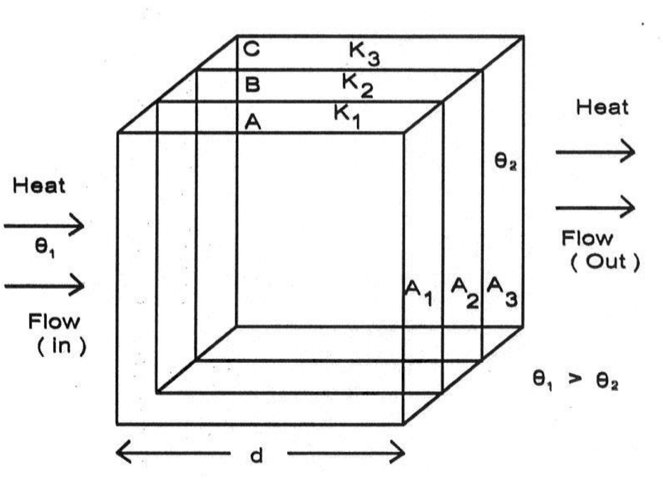

Fig . Bodies in parallel

Let Q_1 be the quantity of heat flowing through material A per second, then

$$Q_1 = \frac{K_1 A_1 (\theta_1 - \theta_2)}{d}$$

Similarly considering materials B & C

$$Q_2 = \frac{K_2 A_2 (\theta_1 - \theta_2)}{d}$$

$$Q_3 = \frac{K_3 A_3 (\theta_1 - \theta_2)}{d}$$

Hence total heat flow/sec is given by

$$Q = Q_1 + Q_2 + Q_3$$

$$Q = \frac{(K_1 A_1 + K_2 A_2 + K_3 A_3)(\theta_1 - \theta_2)}{d}$$

$$Q = \frac{(\theta_1 - \theta_2)}{d} \sum KA$$

This is applicable when all materials are of the same thickness 'd'. But in practice it may vary. For example when we consider the Walls of our houses, the entire wall is not of concrete. The wall may have glass panel or wooden panel etc. In such cases when the thickness of the materials vary, then

$$Q = \left[\frac{K_1 A_1}{d_1} + \frac{K_2 A_2}{d_2} + \frac{K_3 A_3}{d_3} \right] (\theta_1 - \theta_2)$$