## 1.2 PARAMETERS OF SINGLE AND THREE PHASE TRANSMISSION LINES WITH SINGLE AND DOUBLE CIRCUITS

### 1.2.1 CONSTANTS OF A TRANSMISSION LINE

A transmission line has resistance, inductance and capacitance uniformly distributed along the whole length of the line. Before we pass on to the methods of finding these constants for a transmission line, it is profitable to understand them thoroughly.

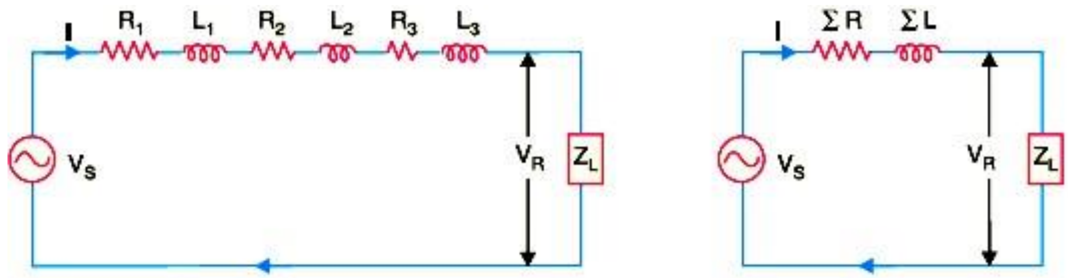


**Figure 1.2.1 Resistance and Inductance**

*[Source: "Principles of Power System" by V.K.Mehta Page: 203]*

**( *i*) Resistance.** It is the opposition of line conductors to current flow. The resistance is distributed uniformly along the whole length of the line as shown in Fig. However, the performance of a transmission line can be analysed conveniently if distributed resistance is considered as lumped as shown in Fig.1.2.1

**( *ii*) Inductance.** When an alternating current flows through a conductor, a changing flux is set up which links the conductor. Due to these flux linkages, the conductor possesses inductance. Mathematically, inductance is defined as the flux linkages per ampere *i.e.*,

$L = \frac{\varphi}{I}$ henry

where $\varphi$ = flux linkages in weber-turns

$I$ = current in amperes

The inductance is also uniformly distributed along the length of the line as show in Fig. Again for the convenience of analysis, it can be taken to be lumped as shown in Fig1.2.1

**(*iii*) Capacitance.** We know that any two conductors separated by an insulating material consti-tute a capacitor. As any two conductors of an overhead transmission line are separated by air which acts as an insulation, therefore, capacitance exists between any two overhead line conductors. The capacitance between the conductors is the charge per unit potential difference *i.e.*,

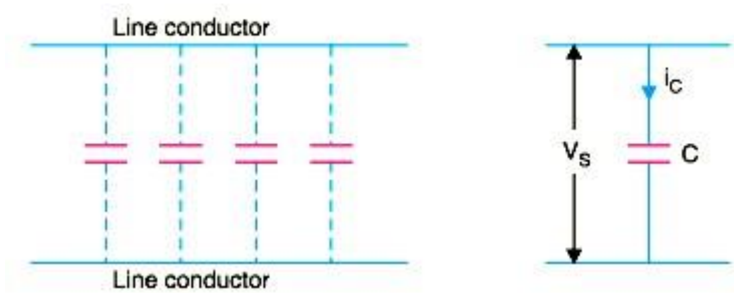


**Figure 1.2.2 Capacitance**

*[Source: "Principles of Power System" by V.K.Mehta Page: 203]*

C = q/v

q = charge on the line in coulomb

v = p.d. between the conductors in volts

The capacitance is uniformly distributed along the whole length of the line and may regarded as a uniform series of capacitors connected between the conductors as shown in Fig. 9.2( *i*). When an alternating voltage is impressed on a transmission line, the charge on the conductors at any point increases and decreases with the increase and decrease of the instantaneous value of the voltage between conductors at that point. The result is that a current (known as *charging current*) flows between the conductors [See Fig. 1.2.2]. This charging current flows in the line even when it is open-circuited *i.e.*, supplying no load. It affects the voltage drop along the line as well as the efficiency and power factor of the line.

### 1.2.2 RESISTANCE OF A TRANSMISSION LINE

The resistance of transmission line conductors is the most important cause of power loss in a transmission line. The resistance *R* of a line conductor having resistivity ρ, length *l* and area of cross-section *a* is given by ;

$R = \rho\, l/a$

The variation of resistance of metallic conductors with temperature is practically linear over the normal range of operation. Suppose $R_1$ and $R_2$ are the resistances of a conductor at $t_1$ ºC and $t_2$ ºC

( $t_2 > t_1$ ) respectively. If $\alpha_1$ is the temperature coefficient at $t_1$ °C, then,

$$R_2 = R_1[1 + \alpha_1(t_2 - t_2)]$$

$$\alpha_1 = \frac{\alpha_0}{1 + \alpha_0 t_1}$$

$$\alpha_0 = temperature\ coefficient\ at\ 0^oC$$

### 1.2.3 INDUCTANCE OF A SINGLE PHASE TWO-WIRE LINE

A single phase line consists of two parallel conductors which form a rectangular loop of one turn.

When an alternating current flows through such a loop, a changing magnetic flux is set up. The changing flux links the loop and hence the loop (or single phase line) possesses inductance. It may appear that inductance of a single phase line is negligible because it consists of a loop of one turn and the flux path is through air of high reluctance. But as the X -sectional area of the loop is very large, even for a small flux density, the total flux linking the loop is quite large and hence the line has appreciable inductance.

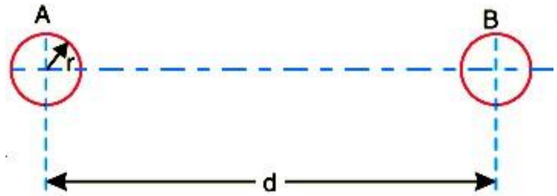


**Figure 1.2.3 Single phase Line**

*[Source: "Principles of Power System" by V.K.Mehta Page: 208]*

Consider a single phase overhead line consisting of two parallel conductors A and B spaced d metres apart as shown in Fig. 9.7. Conductors A and B carry the same amount of current ( i.e. $I_A = I_B$ ), but in the opposite direction because one forms the return circuit of the other.

$$I_A + I_B = 0$$

In order to find the inductance of conductor A (or conductor B), we shall have to consider the flux linkages with it. There will be flux linkages with conductor A due to its own current $I_A$ and also A due to the mutual inductance effect of current $I_B$ in the conductor B Flux linkages with conductor A due to its own current

$$= \frac{\mu_0 I_A}{2\pi}\left(\frac{1}{4} + \int_r^{\infty} \frac{dx}{x}\right)$$

Flux linkages with conductor A due to current $I_B$

$$= \frac{\mu_0 I_B}{2\pi}\int_d^{\infty} \frac{dx}{x}$$

Total flux linkages with conductor A is

$$\psi_A = \text{exp. }(i) + \text{exp }(ii)$$

$$= \frac{\mu_0 I_A}{2\pi}\left(\frac{1}{4} + \int_r^{\infty}\frac{dx}{x}\right) + \frac{\mu_0 I_B}{2\pi}\int_d^{\infty}\frac{dx}{x}$$

$$= \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} + \int_r^{\infty}\frac{dx}{x}\right)I_A + I_B\int_d^{\infty}\frac{dx}{x}\right]$$

$$= \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} + \log_e \infty - \log_e r\right)I_A + \left(\log_e \infty - \log_e d\right)I_B\right]$$

$$= \frac{\mu_0}{2\pi}\left[\left(\frac{I_A}{4} + \log_e \infty\left(I_A + I_B\right) - I_A \log_e r - I_B \log_e d\right)\right]$$

$$= \frac{\mu_0}{2\pi}\left[\frac{I_A}{4} - I_A \log_e r - I_B \log_e d\right] \quad (\because I_A + I_B = 0)$$

Now, $I_A + I_B = 0 \quad \text{or} \quad -I_B = I_A$

$\therefore \quad -I_B \log_e d = I_A \log_e d$

$$\therefore \quad \psi_A = \frac{\mu_0}{2\pi}\left[\frac{I_A}{4} + I_A \log_e d - I_A \log_e r\right] \text{ wb-turns/m}$$

$$= \frac{\mu_0}{2\pi}\left[\frac{I_A}{4} + I_A \log_e \frac{d}{r}\right]$$

$$= \frac{\mu_0 I_A}{2\pi}\left[\frac{1}{4} + \log_e \frac{d}{r}\right] \text{ wb-turns/m}$$

$$\text{Inductance of conductor } A, L_A = \frac{\Psi_A}{I_A}$$

$$= \frac{\mu_0}{2\pi}\left[\frac{1}{4} + \log_e \frac{d}{r}\right] \text{H/m} = \frac{4\pi \times 10^{-7}}{2\pi}\left[\frac{1}{4} + \log_e \frac{d}{r}\right] \text{H/m}$$

$$L_A = 10^{-7}\left[\frac{1}{2} + 2\log_e \frac{d}{r}\right] \text{H/m}$$

$$\text{Loop inductance} = 2 L_A \text{ H/m} = 10^{-7}\left[1 + 4\log_e \frac{d}{r}\right] \text{H/m}$$

$$\text{Loop inductance} = 10^{-7}\left[1 + 4\log_e \frac{d}{r}\right] \text{H/m}$$

Note that eqn. is the inductance of the two-wire line and is sometimes called loop inductance. However, inductance given by eqn. is the inductance per conductor and is equal to half the loop inductance.

### 1.2.4 INDUCTANCE OF A 3-PHASE OVERHEAD LINE

Fig. shows the three conductors A, B and C of a 3-phase line carrying currents $I_A$ , $I_B$ and $I_C$ respectively. Let $d_1$ , $d_2$ and $d_3$ be the spacings between the conductors as shown. Let us further assume that the loads are balanced i.e. $I_A + I_B + I_C = 0$. Consider the flux linkages with conductor There will be flux linkages with conductor A due to its own current and also due to the mutual inductance effects of $I_B$ and $I_C$

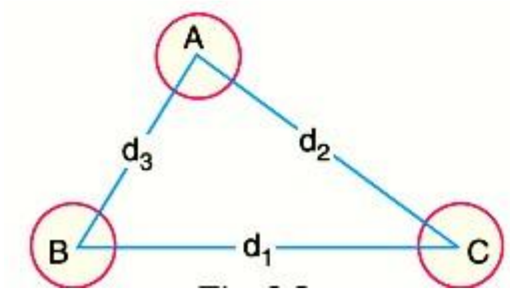


**Figure 1.2.4 Three phase Line**

*[Source: "Principles of Power System" by V.K.Mehta Page: 208]*

Flux linkages with conductor A due to its own current

$$= \frac{\mu_0 I_A}{2\pi}\left(\frac{1}{4} + \int_r^{\infty} \frac{dx}{x}\right) \qquad ...(i)$$

Flux linkages with conductor A due to current $I_B$

$$= \frac{\mu_0 I_B}{2\pi} \int_{d_3}^{\infty} \frac{dx}{x}$$

Flux linkages with conductor A due to current $I_C$

$$= \frac{\mu_0 I_C}{2\pi} \int^{\infty} \frac{dx}{x}$$

Total flux linkages with conductor A is

$$\psi_A = (i) + (ii) + (iii)$$

$$= \frac{\mu_0 I_A}{2\pi}\left(\frac{1}{4} + \int_r^{\infty} \frac{dx}{x}\right) + \frac{\mu_0 I_B}{2\pi} \int_{d_3}^{\infty} \frac{dx}{x} + \frac{\mu_0 I_C}{2\pi} \int_{d_2}^{\infty} \frac{dx}{x}$$

$$= \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} + \int_r^{\infty} \frac{dx}{x}\right) I_A + I_B \int_{d_3}^{\infty} \frac{dx}{x} + I_C \int_{d_2}^{\infty} \frac{dx}{x}\right]$$

$$= \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right) I_A - I_B \log_e d_3 - I_C \log_e d_2 + \log_e \infty \left(I_A + I_B + I_C\right)\right]$$

As $I_A + I_B + I_C = 0,$

$$\therefore \quad \psi_A = \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right) I_A - I_B \log_e d_3 - I_C \log_e d_2\right]$$

## 1.2.4.1 SYMMETRICAL SPACING

If the three conductors A, B and C are placed symmetrically at the corners of an equilateral triangle of side d, then, d1 = d2 = d3 = d. Under such conditions, the flux Derived in a similar way, the expressions for inductance are the same for conductors B and C.

$$\psi_A = \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right) I_A - I_B \log_e d - I_C \log_e d\right]$$

$$= \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right) I_A - (I_B + I_C)\log_e d\right]$$

$$= \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right) I_A + I_A \log_e d\right] \qquad (\because I_B + I_C = -I_A)$$

$$= \frac{\mu_0 I_A}{2\pi}\left[\frac{1}{4} + \log_e \frac{d}{r}\right] \text{ werber-turns/m}$$

$$L_A = \frac{\psi_A}{I_A} \text{ H / m} = \frac{\mu_0}{2\pi}\left[\frac{1}{4} + \log_e \frac{d}{r}\right] \text{ H/m}$$

$$= \frac{4\pi \times 10^{-7}}{2\pi}\left[\frac{1}{4} + \log_e \frac{d}{r}\right] \text{ H/m}$$

$$L_A = 10^{-7}\left[0 \cdot 5 + 2\log_e \frac{d}{r}\right] \text{ H/m}$$

### 1.2.4.2 UNSYMMETRICAL SPACING

When 3-phase line conductors are not equidistant from each other, the conductor spacing is said to be unsymmetrical. Under such conditions, the flux linkages and inductance of each phase are not the same. A different inductance in each phase results in unequal voltage drops in the three phases even if the currents in the conductors are balanced. Therefore, the voltage at the receiving end will not be the same for all phases. In order that voltage drops are equal in all conductors, we generally interchange the positions of the conductors at regular intervals along the line so that each conductor occupies the original position of every other conductor over an equal distance. Such an exchange of positions is known as transposition. Fig. shows the transposed line. The phase conductors are designated as A, B and C and the positions occupied are numbered 1, 2 and 3. The effect of transposition is that each conductor has the same average inductance.

Fig. shows a 3-phase transposed line having unsymmetrical spacing. Let us assume that each of the three sections is 1 m in length. Let us further assume balanced conditions i.e.,

$$I_A + I_B + I_C = 0$$

Let the line currents be :

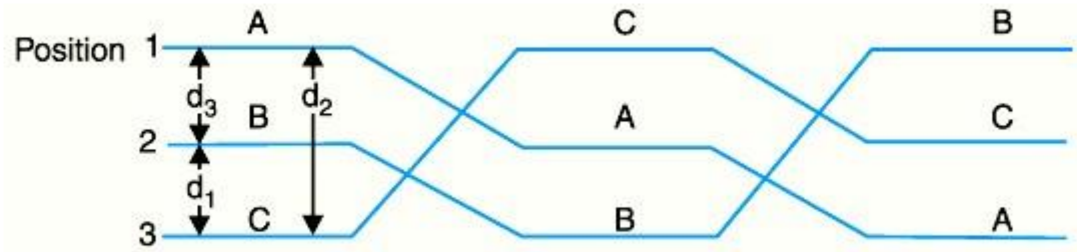


**Figure 1.2.5 Unsymmetrical Spacing**

*[Source: "Principles of Power System" by V.K.Mehta Page: 211]*

$$I_A = I(1 + j\,0)$$
$$I_B = I(-0{\cdot}5 - j\,0{\cdot}866)$$
$$I_C = I(-0{\cdot}5 + j\,0{\cdot}866)$$

As proved above, the total flux linkages per metre length of conductor A is

$$\psi_A = \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right)I_A - I_B \log_e d_3 - I_C \log_e d_2\right]$$

Putting the values of $I_A$, $I_B$ and $I_C$, we get,

$$\psi_A = \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right)I - I(-0\cdot5 - j\,0\cdot866)\log_e d_3 - I(-0\cdot5 + j\,0\cdot866)\log_e d_2\right]$$

$$= \frac{\mu_0}{2\pi}\left[\frac{1}{4}I - I\log_e r + 0\cdot5\,I\log_e d_3 + j\,0\cdot866\log_e d_3 + 0\cdot5\,I\log_e d_2 - j\,0\cdot866\,I\log_e d_2\right]$$

$$= \frac{\mu_0}{2\pi}\left[\frac{1}{4}I - I\log_e r + 0\cdot5\,I\left(\log_e d_3 + \log_e d_2\right) + j\,0\cdot866\,I\left(\log_e d_3 - \log_e d_2\right)\right]$$

$$= \frac{\mu_0}{2\pi}\left[\frac{1}{4}I - I\log_e r + I^*\log_e\sqrt{d_2 d_3} + j\,0\cdot866\,I\log_e\frac{d_3}{d_2}\right]$$

Ac

$$= \frac{\mu_0 I}{2\pi}\left[\frac{1}{4} + \log_e \frac{\sqrt{d_2 d_3}}{r} + j\,0\cdot 866 \log_e \frac{d_3}{d_2}\right]$$

$\therefore$ Inductance of conductor $A$ is

$$L_A = \frac{\psi_A}{I_A} = \frac{\psi_A}{I}$$

$$= \frac{\mu_0}{2\pi}\left[\frac{1}{4} + \log_e \frac{\sqrt{d_2\, d_3}}{r} + j\,0\cdot 866 \log_e \frac{d_3}{d_2}\right]$$

$$= \frac{4\pi \times 10^{-7}}{2\pi}\left[\frac{1}{4} + \log_e \frac{\sqrt{d_2\, d_3}}{r} + j\,0\cdot 866 \log_e \frac{d_3}{d_2}\right] \text{ H/m}$$

$$= 10^{-7}\left[\frac{1}{2} + 2\log_e \frac{\sqrt{d_2\, d_3}}{r} + j\,1\cdot 732 \log_e \frac{d_3}{d_2}\right] \text{ H/m}$$

Similarly inductance of conductors B and C will be :

$$L_B = 10^{-7}\left[\frac{1}{2} + 2\log_e \frac{\sqrt{d_3\, d_1}}{r} + j\,1\cdot 732 \log_e \frac{d_1}{d_3}\right] \text{ H/m}$$

$$L_C = 10^{-7}\left[\frac{1}{2} + 2\log_e \frac{\sqrt{d_1\, d_2}}{r} + j\,1\cdot 732 \log_e \frac{d_2}{d_1}\right] \text{ H/m}$$

Inducance of each line conductor

$$= \frac{1}{3}(L_A + L_B + L_C)$$

$$= {}^{*}\left[\frac{1}{2} + 2\log_e \frac{\sqrt[3]{d_1\, d_2\, d_3}}{r}\right] \times 10^{-7} \text{ H/m}$$

$$= \left[0\cdot 5 + 2\log_e \frac{\sqrt[3]{d_1\, d_2\, d_3}}{r}\right] \times 10^{-7} \text{ H/m}$$

If we compare the formula of inductance of an un symmetrically spaced transposed line with that of symmetrically spaced line, we find that inductance of each line conductor in the two cases will be equal if $d = \sqrt[3]{d_1 d_2 d_3}$ The distance d is known as equivalent equilateral spacing for un symmetrically transposed line.

1. A single phase transmission line has two parallel conductors 3 m apart, the radius of each conductor being 1 cm. Calculate the loop inductance per km length of the line if the material of the conductor is (i) copper (ii) steel with relative permeability of 100.

   Spacing of conductors, d = 300 cm

   Radius of conductor, r = 1 cm

   Loop inductance = $10^{-7}$ ($\mu_r$ + 4 loge d/r) H/m

   (i) With copper conductors, $\mu_r = 1$

   ∴ Loop inductance/m = $10^{-7}$ (1 + 4 loge d/r) H = $10^{-7}$ (1 + 4 loge 300/1) H

   = $23{\cdot}8 \times 10^{-7}$ H

   Loop inductance/km = $23{\cdot}8 \times 10^{-7} \times 1000 = 2{\cdot}38 \times 10^{-3}$ H = 2·38 mH

   (ii) With steel conductors, $\mu r = 100$

   ∴ Loop inductance/m = $10^{-7}$ (100 + 4 loge 300/1) H = $122{\cdot}8 \times 10^{-7}$ H

   Loop inductance/km = $122{\cdot}8 \times 10^{-7} \times 1000 = 12{\cdot}28 \times 10^{-3}$ H = 12·28 mH

2. The three conductors of a 3-phase line are arranged at the corners of a triangle of sides 2 m, 2·5 m and 4·5 m. Calculate the inductance per km of the line when the conductors are regularly transposed. The diameter of each conductor is 1·24 cm.

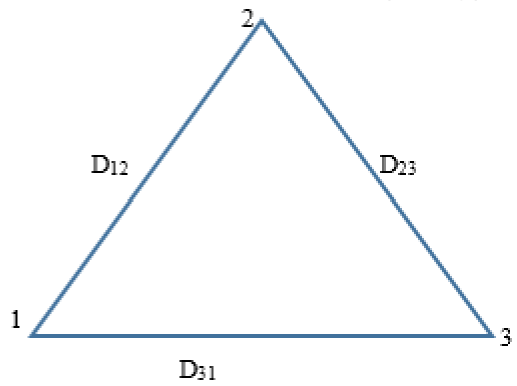


**Figure 1.2.6**

Fig.1.2.6 shows three conductors of a 3-phase line placed at the corners of a triangle of sides $D_{12}$ = 2 m, $D_{23}$ = 2·5 m and $D_{31}$ = 4·5 m.

The conductor radius r = 1·24/2 = 0·62 cm.

Equivalent equilateral spacing, $D_{eq} = (D_{12}\, D_{23}\, D_{31})^{1/3}$

$= (2 \times 2 \cdot 5 \times 4 \cdot 5)^{1/3}$

$= 2 \cdot 82$ m

$= 282$ cm

Inductance/phase/m $= 10^{-7}(0 \cdot 5 + 2 \log_e D_{eq}/r)$ H

$= 10^{-7}(0 \cdot 5 + 2 \log_e 282/0 \cdot 62)$ H

$= 12 \cdot 74 \times 10^{-7}$ H

Inductance/phase/km $= 12 \cdot 74 \times 10^{-7} \times 1000$

$= 1 \cdot 274 \times 10^{-3}$ H

= **1·274 mH**