Unit-III AVAILABILITY AND APPLICATIONS OF II LAW.

Ideal gases undergoing different processes - principle of increase of entropy - Applications of II law. High and low grade energy. Availability and irreversibility for open and closed system processes - I and II law efficiency.

Course Objective :- Impart knowledge on availability and applications of second law of thermodynamics.

1. Define available energy (AE) and unavailable energy (UE) of a system.

Let us consider a heat reservoir capable of suppling heat Q at a temperature T. It is the total heat energy (TE) available for conversion into work. From the second law of thermodynamics, a maximum amount of work that can be produced by a reversible heat engine, when the rejection of heat takes place to the sink at the surrounding temperature T_0 can be calculated.

Now, the total energy (TE) available for conversion

$$Q = AE + UE$$

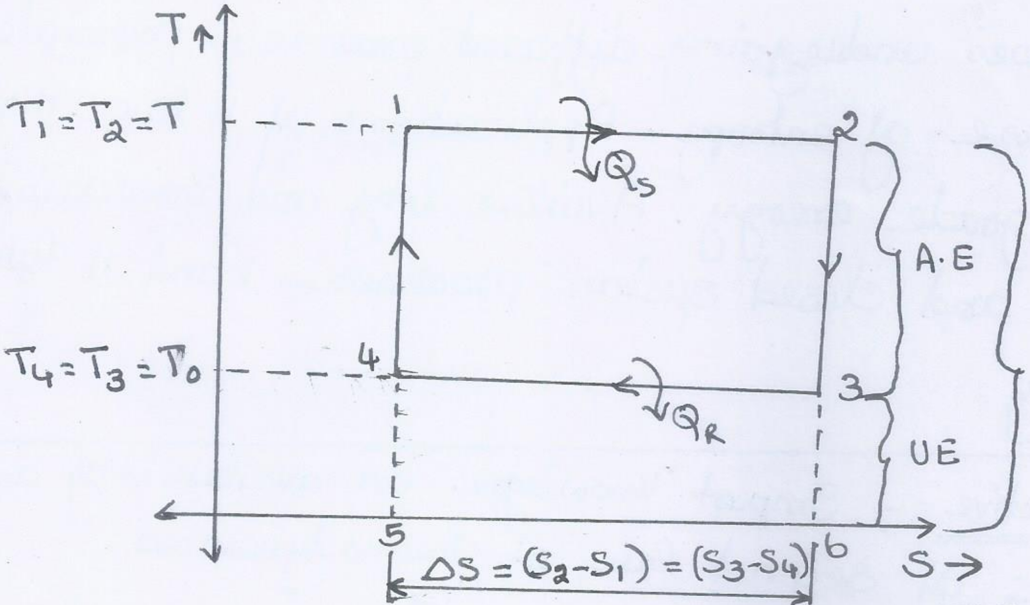
For reversible heat engine,

$$\eta_{max} = \frac{W_{max}}{Q} = \frac{T_{max} - T_{min}}{T_{max}} = \frac{T - T_0}{T}$$

$$W_{max} = Q\left(1 - \frac{T_0}{T}\right)$$

Since this theoritical maximum work is also known as available energy (AE)

$$AE = W_{max} = Q\left(1 - \frac{T_0}{T}\right)$$



TE, AE and UE of a source on T-S diagram

The unavailable energy is the difference between the total energy (TE) and the available energy (AE).

$$UE = TE - AE$$

TE is the total heat supplied to the Carnot engine and it is indicated by the area 1-2-6-5-1. AE (or W_{max}) is the available energy indicated by the area 1-2-3-4-1. The area 4-3-6-5-4 indicates the UE.

$$\Delta S = \frac{Q}{T}$$

TE = Area 1-2-6-5-1 $= T.\Delta S$

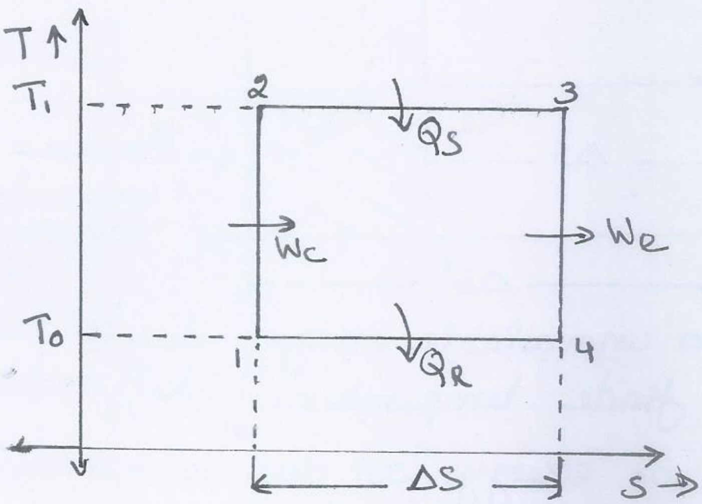
AE = Area 1-2-3-4-1 $= (T - T_0)\,\Delta S$

UE = Area 4-3-6-5-4 $= T_0.\Delta S$

It may be noted that the AE is the particular reversible work developed by the reversible heat engine invariably using the surrounding as sink. Actual work produced by a heat engine is always less than the AE, because i) the actual heat rejection occurs at a temperature higher than that of surrounding and ii) the irreversibility in the processes of the cycle due to lack of equillibrium.

2. Prove that there is decrease in available energy when heat is transfered through a finite temperature difference.

Let us consider a reversible heat engine operating between T_1 and T_0. Then,



Carnot cycle

$$Q_S = T_1 \Delta S$$
$$Q_R = T_0 \Delta S$$
$$W = A.E = (T_1 - T_0) \Delta S$$

Let us assume that heat Q_S is transferred through a finite temperature difference from the reservoir or source at T_1 to the engine, but the engine absorbs heat at T_1', lower than T_1. The availability of Q_s as recived by the engine at T_1' can be found by allowing the engine to operate reversibily in a cycle between T_1' and T_0.

Now $\quad Q_s = T_1 \Delta S = T_1' \Delta S'$

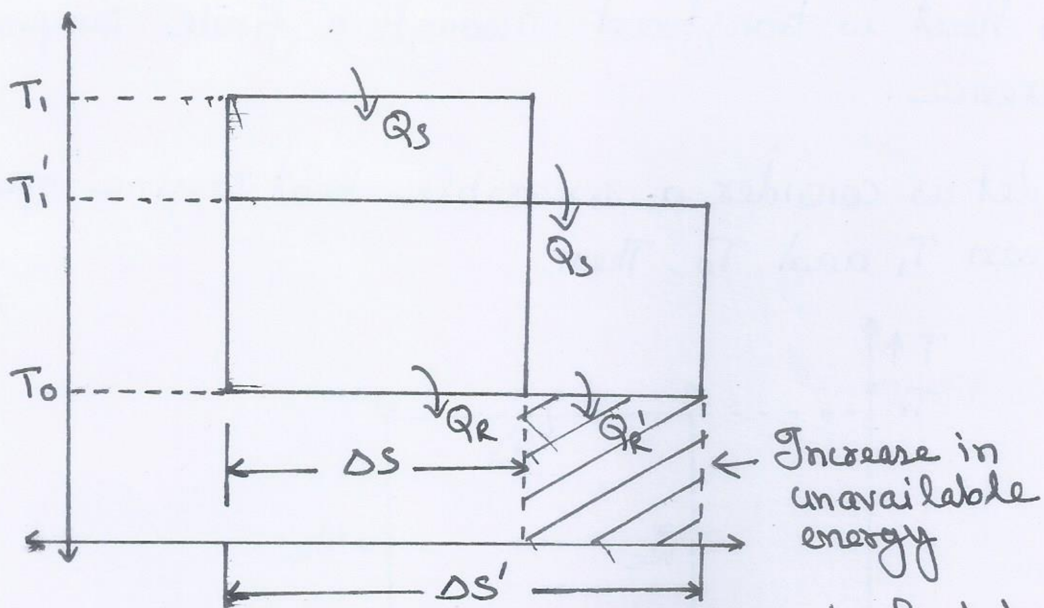
Since $\quad T_1 > T_1'$, $\Delta S' > \Delta S$.

$\quad Q_R = T_0 \Delta S, Q_R' = T_0 . \Delta S'$

Since $\quad \Delta S' > \Delta S$, $Q_R' > Q_R$

$\quad W' = Q_S - Q_R' = T_1' \Delta S' - T_0 \Delta S'$

$W > W'$, because $Q_R' > Q_R$



Increase in unavailable energy due to heat transfer through a finite temperature difference.

Available energy or exergy lost due to irreversible heat transfer through finite temperature difference between the source and working fluid during the process is given by

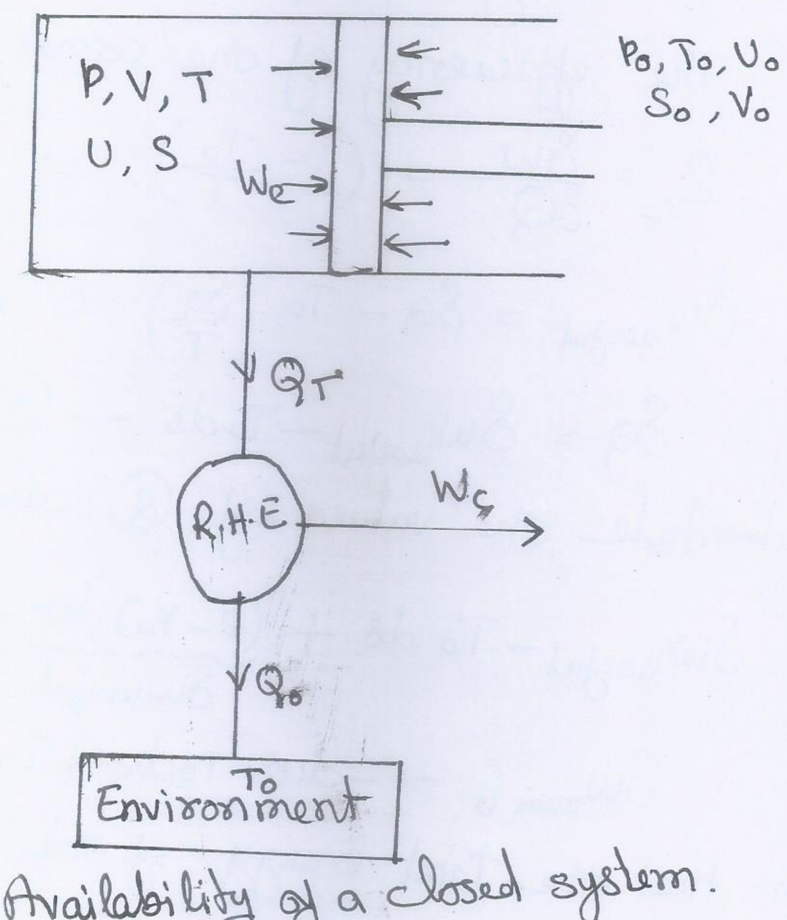
$$W - W' = (Q_S - Q_R) - (Q_S - Q_R') = Q_R' - Q_R$$
$$= T_0 \Delta S' - T_0 \Delta S = T_0 (\Delta S' - \Delta S)$$

The decrease in available energy or exergy is thus the product of the lowest feasible temperature of heat rejection and the additional entropy change in the system while receiving heat irreversibly, compared to the case of reversible heat transfer from the same source.

The greater the temperature difference $(T_1 - T_1')$ the greater is the heat rejection Q_R' and the greater will be the unavailable part of the energy supplied or anergy. Energy is said to be degraded each time it flows through a finite temperature difference.

3. Derive the expression for availability of a process in a closed system.

Consider a system containing a fluid at a pressure of P, volume V and a temperature of T. Let it expand from its present condition to the final atmospheric condition P_0, and T_0. During this process, the fluid generates the work of W_e and it rejects Q amount of heat to the surrounding. To extract work from the rejected heat, a reversible heat engine is connected and it produces a work of W_{carnot}.



Availability of a closed system.

Applying the energy balance equation.

$\cancel{E_{in}}^{0} - E_{out} = \Delta E$ [In this process energy is coming out of the system as work and heat only]

$-E_{out} = \Delta E$ [Since it is closed system the energy change take place in internal energy]

$-(\delta q + \delta w) = du$

$\delta q + \delta w = -du$ — ①

The work done by the system is piston expansion and the opposing force exerted on the piston by atmosphere.

$\delta w = (P - P_0)\,dv + P_0\,dv$ — ②

where $(P - P_0)\,dv$ is the useful work done by the piston and $P_0 dv$ is the opposing work done on the piston by atmosphere.

The efficiency of the carnot engine is given by

$$\eta_c = \frac{\delta w}{\delta Q} = \left(1 - \frac{T_0}{T}\right) \Rightarrow \delta w = \left(1 - \frac{T_0}{T}\right)\delta q$$

$$\delta w_{useful} = \delta q - T_0 \cdot \left(\frac{-\delta q}{T}\right) \Rightarrow \delta q + T_0 ds$$

$$\delta q = \delta w_{useful} - T_0 ds \quad — ③$$

Substitute the values of ② and ③ in ①.

$$\delta w_{useful} - T_0\, ds + \underbrace{(P - P_0)\, dv}_{\delta w_{useful}} + P_0\, dv = -du$$

$$\delta w_{max\, u} = -du - P_0 dv + T_0 ds$$

To find the total energy of the process integrate the above equation.

$$\int \delta w_{max,u} = \int_{u}^{u_0} -du - P_0 \int_{v}^{V_0} dv + T_0 \int_{s}^{S_0} ds$$

$$W_{max\,u} = -[U_0 - U] - P_0[V_0 - V] + T_0[S_0 - S]$$

$$W_{max\,useful} = [U - U_0] + P_0[V - V_0] - T_0[S - S_0]$$

$$W_{max\,u} = [U + P_0 V - T_0 S] - [U_0 + P_0 V_0 - T_0 S_0]$$

Let non-flow availability function $\phi = U + P_0 V - T_0 S$

$|||^{ly}$ $\phi_0 = U_0 + P_0 V_0 - T_0 S_0$

Now availability or Exergy or $W_{max,useful} = (\phi - \phi_0)$

The non flow availability function, is a composite property as it consists of three extensive properties (U, V and S) of the system and two intensive properties (P_0 and T_0) of the surroundings. It indicates the maximum possible useful work at the given state of the system. As it is only a function of properties (i.e. the point function), it is also a point function.

When the system of gas expands, from state 1 to 2, it does work on the surroundings and its availability decreases or decrease in availability during expansion is the maximum possible useful work.

$$(W_{rev})_{1-2} = (W_{max,u})_{1-2} = (\phi_1 - \phi_0) - (\phi_2 - \phi_0)$$

$$= (\phi_1 - \phi_2)$$

$$(W_{rev})_{1-2} = (W_{max,u}) = (U_1 + P_0 V_1 - T_0 S_1) - (U_2 + P_0 V_2 - T_0 S_2)$$

$$= (U_1 - U_2) + P_0 (V_1 - V_2) - T_0 (S_1 - S_2)$$

Helmholtz free energy function :-

The difference between the internal energy and the absolute temperature - entropy product is known as Helmholtz free energy function or Helmholtz function (F)

$$F = U - TS$$

then the availability function can be written as

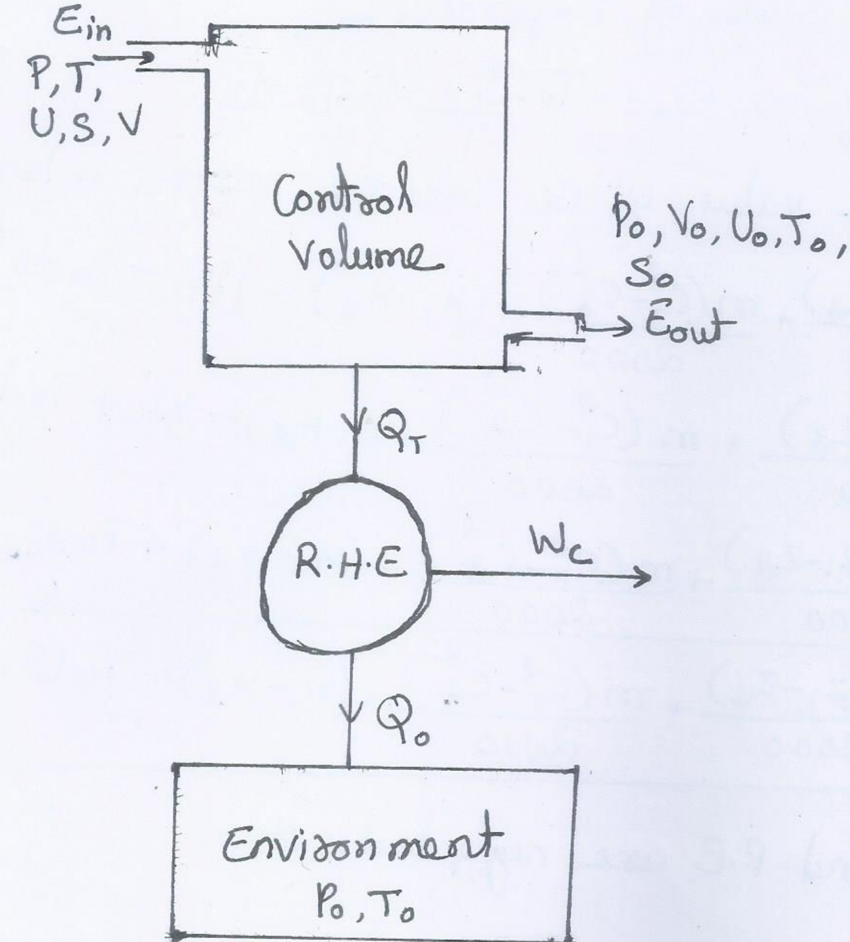
$$\phi = (U - T_0 S + P_0 V) = [F + P_0 V]$$

$$\phi_0 = (U_0 - T_0 S_0 + P_0 V_0) = [F + P_0 V_0]$$

4. Derive the expression for the availability of a process in a steady flow system.

The availability of a steady flow system can be analysed in a similar way as that of a closed system, but there is a constant flow rate of working fluid through the control volume.

Let us consider a control volume in which the working fluid enters the control volume with pressure p, velocity c and temperature T. After delivering a steady flow work rate (ie power), it leaves the control volume. During this process the system rejects heat Q_T and a reversible heat engine is connected to extract ~~heat~~ work from the heat rejected.



Availability of a open system

Applying the energy balance to the steady flow across the control volume, we get

$$E_{in} - E_{out} = \Delta E$$

$$E_{in} - E_{out} = 0 \quad \text{for steady flow}$$

$$E_{in} = E_{out}$$

$$\frac{mgz}{1000} + \frac{mc^2}{2000} + H + Q = \frac{mgz_0}{1000} + \frac{mC_0^2}{2000} + H_0 + W$$

$$W = \frac{mg(z - z_0)}{1000} + \frac{m(C^2 - C_0^2)}{2000} + (H - H_0) + Q$$

$$W = \frac{mg(z - z_0)}{1000} + \frac{m(C^2 - C_0^2)}{2000} + (H - H_0) - Q \quad \left[- \text{ sign for heat rejection}\right]$$

$$W = \frac{mg(z - z_0)}{1000} + \frac{m(C^2 - C_0^2)}{2000} + (H - H_0) - \left[W_c + Q_0\right]$$

From absolute scale of temperature

$$-\frac{Q_I}{Q_0} = \frac{T}{T_0} \Rightarrow Q_0 = -T_0 \cdot \frac{Q_I}{T} = -T_0 . ds$$

Substitute the value of Q_0 into the energy balance eqn.

$$W = \frac{mg(z - z_0)}{1000} + \frac{m(C^2 - C_0^2)}{2000} + (H - H_0) - [W_c + T_0 ds]$$

$$W = \frac{mg(z - z_0)}{1000} + \frac{m(C^2 - C_0^2)}{2000} + (H - H_0) - W_c + T_0 ds$$

$$W_{max} = \frac{mg(z - z_0)}{1000} + \frac{m(C^2 - C_0^2)}{2000} + (H - H_0) + T_0 [S_0 - S]$$

$$\boxed{W_{max} = \frac{mg(z - z_0)}{1000} + \frac{m(C^2 - C_0^2)}{2000} + (H - H_0) - T_0(S - S_0)}$$

If the K.E and P.E are neglected, then

$$W_{max} = (H - H_0) - T_0(S - S_0) \quad \text{or}$$

$$W_{max} = m\left[(h - h_0) - T_0(S - S_0)\right] \quad \text{or}$$

$$W_{max} = m\left[(h - T_0 S) - (h_0 - T_0 S_0)\right]$$

Let the availability function of open system

$$\psi = h - T_0 S + \frac{C^2}{2000} + \frac{gz}{1000}$$

$|||^{ly}$

$$\psi_0 = h_0 - T_0 S_0 + \frac{C_0^2}{2000} + \frac{gz_0}{1000}$$

Now availability or Exergy or $W_{max, useful}$

$$= (\psi - \psi_0)$$

Gibb's free energy function.

The difference between the enthalpy and temperature-entropy product is known as Gibb's free energy function (G). It is also known as Keenan's function.

$$G = H - TS$$

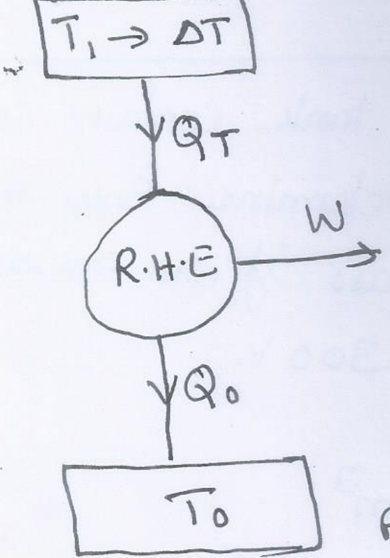
Thus $$\psi = h - T_0 s + \frac{C^2}{2000} + \frac{gz}{1000}$$

$$\psi = G + \frac{C^2}{2000} + \frac{gz}{1000}$$

$$\psi_0 = G + \frac{C_0^2}{2000} + \frac{gz_0}{1000}$$

5. Derive the expression for availability of a finite body or incompressible body.

In case of a finite body, the temperature T_1 of the reservoir changes as the heat is withdrawn from it and hence the heat is supplied to the reversible heat engine at varying temperature.



Availability of finite

Applying the energy balance equation

$$\cancel{E_{in}}^{0} - E_{out} = \Delta E$$ [There is only energy output from the system]

$$-\delta Q = mC\,dT$$

where $\delta Q = -mC\,dT$

m - mass of the finite temperature body

C - Specific heat Capacity of the body

dT - change in temperature of the body.

$$\eta_{carnot} = \frac{\delta W_u}{\delta Q} = \left(1 - \frac{T_0}{T}\right)$$

$$\delta W_{useful} = \delta Q\left(1 - \frac{T_0}{T}\right) = \delta Q - T_0 . \frac{\delta Q}{T}$$

$$= -mC\,dT + T_0 . mC\frac{dT}{T}$$

$\int$ on both sides

$$\int \delta W_{useful} = -mc\int_T^{T_0} dT + T_0 mc \int_T^{T_0} \frac{dT}{T}$$

$$W_{useful} = -mc\left[T_0 - T\right] + mcT_0 \ln\left[\frac{T_0}{T}\right]$$

$$W_{useful} = mc\left[T - T_0\right] + mcT_0 \ln\left[\frac{T_0}{T}\right]$$

6. A 200 m^3 rigid tank contains compressed air at 1 MPa and 300 K. Determine how much work can be obtained from this air if the environment conditions are 100 kPa and 300 K.

Given data :-

$$V_1 = 200\ m^3$$

$P_1 = 1MPa = 1\times10^3$ kPa $= 1000$ kPa

$T_1 = 300$ k

$P_0 = 100$ kPa

$T_0 = 300$ k.

To find :-

Available energy $(\varphi_1 - \varphi_0) = ?$

Sol :-

The system is closed one.

$$A.E = \varphi_1 - \varphi_0 = m\left[(U_1 - U_0) + P_0(V_1 - V_0) - T_0(S - S_0)\right]$$

$$U_1 - U = mC_v\left[\cancel{T_1 - T_0}\right]^{0} \qquad \because T_1 = T_0 = 300K$$

$$P_0(V_1 - V_0)$$

$$P_1V_1 = RT_1 \Rightarrow V_1 = \frac{RT_1}{P_1}$$

$$P_0V_0 = RT_0 \Rightarrow V_0 = \frac{RT_0}{P_0}$$

$$P_0(V_1 - V_0) = P_0\left[\frac{RT_1}{P_1} - \frac{RT_0}{P_0}\right] \qquad T_1 = T_0 = 300K$$

$$= RT_1\left[\frac{P_0}{P_1} - 1\right] = 0.287\times300\left[\frac{100}{1000} - 1\right]$$

$$P_0(V_1 - V_0) = 86.1\left[0.1 - 1\right] = -77.49 \text{ kJ/kg}$$

$$T_0(S - S_0) = T_0\left[C_p \ln\cancel{\left[\frac{T}{T_0}\right]}^{0} - R\ln\left[\frac{P_1}{P_0}\right]\right] \qquad T_1 = T_0 = 300$$

$$= -T_0 . R\ln\left[\frac{P_1}{P_0}\right] = -300\times0.287 \ln\left[\frac{1000}{100}\right]$$

$$T_0(S - S_0) = -198.25 \text{ kJ/kg}$$

WKT

$$P_1V_1 = mRT_1 \Rightarrow m = \frac{P_1V_1}{RT_1} = \frac{1000\times200}{0.287\times300}$$

$m = 2322.88$ kg

$$A.E(\varphi_1 - \varphi_0) = 2322.88 \left[-77.49 + 198.25\right]$$

$$A.E = (\varphi_1 - \varphi_0) = 280511.03 \text{ kJ}$$

$$\boxed{A.E = 280.511 \text{ MJ}}$$

7. Calculate the decrease in available energy when 25 kg of water at 95°C mix with 35 kg of water at 35°C, the pressure being taken as constant and the temperature of the surrounding being 15°C. Take C_p of water as 4.2 kJ/kg.K.

Given data:-

Hot water

mass $m = 25$ kg

temperature $T = 95°C = 95+273 = 368$ K

Cold water

mass $m = 35$ kg

temperature $T = 35°C = 35+273 = 308$ K

$T_0 = 15°C = 15+273 = 288$ K

$C_{of\,water} = 4.2$ kJ/kg.K

To find:-

Decrease in availability

Sol:-

Incompressible medium.

$$A.E_{(25)} = mC[T - T_0] + mCT_0 \ln\left[\frac{T_0}{T}\right]$$

$$= 25 \times 4.2 [368 - 288] + 25 \times 4.2 \ln\left[\frac{288}{368}\right] \times 288$$

$$= 8400 + 288(-25.73)$$

$$= 8400 - 7410.24$$

$$(A.E)_{25} = 987.49 \text{ kJ}$$

$$A.E_{(35)} = mC[T - T_0] + mCT_0 . \ln\left[\frac{T_0}{T}\right]$$

$$= 35 \times 4.2 [308 - 288] + 35 \times 4.2 \times 288 \ln\left[\frac{288}{308}\right]$$

$$= 2940 - 2842.40$$

$$A.E_{(3)} = 97.59 \text{ kJ}$$

Total available energy before mixing $= A.E_{(25)} + A.E_{(35)}$

$$= 987.49 + 97.59$$

$$\text{Total } A.E = 1085.08 \text{ kJ}$$

To find the final temperature of water after mixing

The heat lost by the hot water = The heat gained by the cold water

$$m_h C (T_h - T) = m_c C [T - T_c]$$

$$25 \times 4.2 [368 - T] = 35 \times 4.2 [T - 308]$$

$$105 [368 - T] = 147 [T - 308]$$

$$38640 - 105T = 147T - 45276$$

$$83916 = 252T$$

$$T = 333 \text{ K}$$

Total mass after mixing $= 25 + 35 = 60\ kg$

$$A.E_{(60)} = mc\left[T - T_0\right] + mcT_0 \ln\left[\frac{T_0}{T}\right]$$

$$= 60 \times 4.2\left[333 - 288\right] + 60 \times 4.2 \times 288 \ln\left[\frac{288}{333}\right]$$

$$= 11340 - 10536.72$$

$$A.E_{(60)} = 803.27\ kJ$$

Decrease in available energy $= 1085 - 803.27$

$$\boxed{\text{Decrease in available energy} = 281.72\ kJ}$$

8. Air expands in a turbine adiabatically from 500 kPa, 400 K and 150 m/s to 100 kPa, 300 K and 70 m/s. The environment is at 100 kPa, 17°C. Calculate per kg of air a) the maximum work output b) the actual work output and c) Irreversibility.

Given data :-

$P_1 = 500\ kPa$

$T_1 = 400\ K$

$C_1 = 150\ m/s$

$P_2 = 100\ kPa$

$T_2 = 300\ K$

$C_2 = 70\ m/s$

$P_0 = 100\ kPa$

$T_0 = 17°C = 17 + 273 = 290\ K$

To find :-

$W_{max} = ?$

$W_{act} = ?$

$I = ?$

Sol :-

Open system.

S.F.E.E

$$\frac{mgz_1}{1000} + \frac{mC_1^2}{2000} + H_1 + Q = \frac{mgz_2}{1000} + \frac{mC_2^2}{2000} + H_2 + W$$

$$W_{act} = \cancel{\frac{mg(z_1 - z_2)}{1000}}^{0} + \frac{m(C_1^2 - C_2^2)}{2000} + (H_1 - H_2) \cancel{+ Q}^{0}$$

$$= m\left[\frac{C_1^2 - C_2^2}{2000} + C_p(T_1 - T_2)\right]$$

$$= 1\left[\frac{150^2 - 70^2}{2000} + 1.005\,(400 - 300)\right]$$

$$= 8.8 + 100.5$$

$$\boxed{W_{act} = 109.3 \text{ kJ/kg}}$$

$$W_{max} = \psi_1 - \psi_2 = \left(h_1 - T_0 S_1 + \frac{C_1^2}{2000} + \frac{gz_1}{1000}\right) - \left(h_2 - T_0 S_2 + \frac{C_2^2}{2000} + \frac{gz_2}{1000}\right)$$

$$= (h_1 - h_2) - T_0(S_1 - S_2) + \frac{C_1^2 - C_2^2}{2000} + \cancel{\frac{g(z_1 - z_2)}{1000}}^{0}$$

$$= C_p(T_1 - T_2) - T_0\left[C_p \ln\left[\frac{T_1}{T_2}\right] - R \ln\left[\frac{P_1}{P_2}\right]\right] + \frac{C_1^2 - C_2^2}{2000}$$

$$= 1.005\,(400 - 300) - 290\left[1.005 \ln\left[\frac{400}{300}\right] - 0.287 \ln\left[\frac{500}{100}\right]\right] + \frac{150^2 - 70^2}{2000}$$

$$W_{max} = 100.5 - 290\left[0.289 - 0.461\right] + 8.8$$

$$= 100.5 + 50.14 + 8.8$$

$$\boxed{W_{max} = 159.41 \text{ kJ/kg}}$$

Irreversibility $I = W_{max} - W_{act}$

$I = 159.41 - 109.3$

$\boxed{I = 50.11 \text{ kJ/kg}}$

9. Air compressor recives air at 100 kPa, 315 K, 55 m/s and delivers air at 600 kPa, 550 K, 175 m/s. The delivery pipe is 0.96 m above the inlet pipe. The heat loss to the surrounding is 20 kJ/kg. If the surrounding is at 101 kPa and 300K, find a) the actual work required b) the minimum work required c) irreversibility and d) the second law efficiency.

Given data:-

$P_1 = 100$ kPa

$T_1 = 315$ K

$C_1 = 55$ m/s

$P_2 = 600$ kPa

$T_2 = 550$ K

$C_2 = 175$ m/s.

$Z_2 - Z_1 = 0.96$ m

$Q = 20$ kJ/kg.

To find:-

$W_{act} = ?$

$W_{min} = ?$

$I = ?$

$\eta_{\underline{II}} = ?$

Sol:-

open system

S.F.E.E

$$\frac{mgz_1}{1000} + \frac{mC_1^2}{2000} + H_1 + Q = \frac{mgz_2}{1000} + \frac{mC_2^2}{2000} + H_2 + W$$

$$W_{act} = \frac{mg(z_1 - z_2)}{1000} + \frac{m(C_1^2 - C_2^2)}{2000} + (H_1 - H_2) - Q$$

[- sign heat loss]

$$W_{act} = \frac{1 \times 9.81(-0.96)}{1000} + 1\left(\frac{55^2 - 175^2}{2000}\right) + C_p(T_1 - T_2) - 20$$

$$= -9.417 \times 10^{-3} - 13.8 + 1.005(315 - 550) - 20$$

$$= -9.417 \times 10^{-3} - 13.8 - 236.175 - 20$$

$W_{act} = -269.98$ kJ/kg [- sign indicates work is done on the system]

$W_{act} = 269.98$ kJ/kg.

$$W_{min} = (h_1 - h_2) - T_0(S_1 - S_0) + \frac{g(z_1 - z_2)}{1000} + \frac{C_1^2 - C_2^2}{2000}$$

$$W_{min} = C_p(T_1 - T_2) - T_0\left[C_p \ln\left[\frac{T_1}{T_2}\right] - R \ln\left[\frac{P_1}{P_2}\right]\right] + \frac{g(z_1 - z_2)}{1000} + \frac{C_1^2 - C_2^2}{2000}$$

$$= 1.005(315 - 550) - 300\left[1.005 \ln\left[\frac{315}{550}\right] - 0.287 \ln\left[\frac{100}{600}\right]\right] + \frac{9.81(-0.96)}{1000} + \frac{55^2 - 175^2}{2000}$$

$$W_{min} = -236.175 + 13.74 - 9.417 \times 10^{-3} - 13.8$$

$W_{min} = -236.24$ kJ/kg.

$W_{min} = 236.24$ kJ/kg.

Irreversibility $I = W_{act} - W_{min}$

$= 269.984 - 236.24$

$$\boxed{I = 33.74 \text{ kJ/kg.}}$$

$$\eta_{II} = \frac{W_{min}}{W_{act}} = \frac{236.24}{269.984} = 0.875$$

$$\boxed{\eta_{II} = 87.5\%}$$

10. Air expands through a turbine from 500 kPa, 520°C to 100 kPa, 300°C. During expansion 10 kJ/kg of heat is lost to the surroundings which is at 98 kPa, 20°C. Neglecting KE and PE changes, determine per kg of air a) the decrease in availability b) the maximum work c) the irreversibility and d) the second law efficiency. For air take $C_p = 1.005$ kJ/kg.K.

Given data:-

$P_1 = 500$ kPa

$T_1 = 520°C = 520+273 = 793$ K

$P_2 = 100$ kPa

$T_2 = 300°C = 300+273 = 573$ K

$Q_{L_R} = 10$ kJ/kg

$P_0 = 98$ kPa

$T_0 = 20°C = 20+273 = 293$ K.

To find:-

$\psi_1 - \psi_2$, W_{max}, I and η_{II}

Sol :-

open system.

S.F.E.E

$$\frac{mgz_1}{1000} + \frac{mC_1^2}{2000} + H_1 + Q = \frac{mgz_2}{1000} + \frac{mC_2^2}{2000} + H_2 + W$$

$$W_{act} = \cancelto{0}{\frac{mg(z_1-z_2)}{1000}} + \cancelto{0}{\frac{m(C_1^2-C_2^2)}{2000}} + (H_1-H_2) + Q$$

$$= (H_1 - H_2) - Q \quad \text{[- sign indicates heat loss]}$$

$$= C_p(T_1 - T_2) - 10$$

$$= 1.005(793 - 573) - 10$$

$$= 221.1 - 10$$

$$W_{act} = 211.1 \text{ KJ/kg}$$

$$W_{max} = \psi_1 - \psi_2 = (h_1 - h_2) - T_0(s_1 - s_2)$$

$$= C_p(T_1 - T_2) - T_0\left[C_p \ln\left(\frac{T_1}{T_2}\right) - R\ln\left(\frac{P_1}{P_2}\right)\right]$$

$$= 1.005(793 - 573) - 293\left[1.005 \times \ln\left(\frac{793}{573}\right) - 0.287 \times \ln\left(\frac{500}{100}\right)\right]$$

$$= 221.1 - 293[0.326 - 0.461]$$

$$= 221.1 + 39.5$$

$$\boxed{W_{max} = 260.6 \text{ KJ/kg} = \psi_1 - \psi_2}$$

Irreversibility $I = W_{max} - W_{act}$

$$= 260.6 - 211.1$$

$$\boxed{I = 49.5 \text{ KJ/kg}}$$

$$\eta_{II} = \frac{W_{act}}{W_{max}} = \frac{211.1}{260.6} = 0.81$$

$$\boxed{\eta_{II} = 81\,\%.}$$

11. Air enters a compressor at 1 bar, 30°c, which is also the state of the environment. It leaves at 3.5 bar, 141°c and 90 m/s. Neglecting inlet velocity and P.E effect determine a) whether the compression is adiabatic or polytropic b) if not adiabatic, the polytropic index c) the isothermal efficiency d) the minimum work input and irreversibility and e) the second law efficiency. Take C_p of air = 1.0035 kJ/kg.k.

Given data :-

$p_1 = 1$ bar

$T_1 = 30°C = 30+273 = 303\,K$

$p_2 = 3.5$ bar

$T_2 = 141°C = 141+273 = 414\,K$

$C_2 = 90$ m/s

$p_0 = 1$ bar

$T_0 = 30°C = 30+273 = 303\,K$.

$C_p = 1.0035$ kJ/kg.k.

To find :-

a) the process is adiabatic or polytropic

b) the polytropic index

c) η_{ISO}

d) W_{min}, I

e) η_{II}

Sol :-

Open system.

S.F.E.E

$$\frac{mgz_1}{1000} + \frac{mC_1^2}{2000} + H_1 + Q = \frac{mgz_2}{1000} + \frac{mC_2^2}{2000} + H_2 + W$$

$$W_{act} = \cancel{\frac{mg(z_1 - z_2)}{1000}}^{0} + \frac{m(\cancel{C_1^2}^{0} - C_2^2)}{2000} + (H_1 - H_2) \cancel{+ Q}^{0}$$

$$W_{act} = 1\left(\frac{-C_2^2}{2000}\right) + C_p(T_1 - T_2)$$

$$= \frac{-90^2}{2000} + 1.0035\,(303 - 414)$$

$$= -4.05 - 111.38$$

$W_{act} = -115.43$ kJ/kg. [- sign indicates work is done on the system]

$W_{act} = 115.43$ kJ/kg.

$$W_{min} = W_{act} - T_0\left[C_p \ln\left[\frac{T_1}{T_2}\right] - R \ln\left[\frac{p_1}{p_2}\right]\right]$$

$$= 115.43 - 303\left[1.0035 \ln\left[\frac{303}{414}\right] - 0.287 \ln\left[\frac{1}{3.5}\right]\right]$$

$$= 115.43 - 303\left[-0.313 + 359\right]$$

$$= 115.43 - 13.938$$

$$\boxed{W_{min} = 101.492 \text{ kJ/kg.}}$$

Irreversibility $I = W_{act} - W_{min}$

$$= 115.43 - 101.492$$

$$\boxed{I = 13.93 \text{ kJ/kg}}$$

$$\eta_{II} = \frac{W_{min}}{W_{act}} = \frac{101.492}{115.43} = .8792$$

$$\boxed{\eta_{II} = 87.92\%}$$

To find the process is whether adiabatic or polytropic

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_2 = T_1 \times \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_2 = 303\left(\frac{3.5}{1}\right)^{\frac{1.4-1}{1.4}} = 433.4\,K$$

Since this temperature is higher than the given temperature of 414 K, there is heat loss to the surroundings. The compression cannot be adiabatic. It must be polytropic.

To find polytropic index

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$$

$$\frac{414}{303} = \left(\frac{3.5}{1}\right)^{\left(\frac{n-1}{n}\right)}$$

$$1.366 = 3.5^{\left(\frac{n-1}{n}\right)}$$

Take log on both sides.

$$\ln 1.366 = \ln (3.5)^{\frac{n-1}{n}}$$

$$0.3118 = \frac{(n-1)}{n} . \ln(3.5)$$

$$0.3118 = \left(1 - \frac{1}{n}\right) 1.2527$$

$$\frac{0.3118}{1.2527} = 1 - \frac{1}{n}$$

$$0.248 = 1 - \frac{1}{n}$$

$$\frac{1}{n} = 1 - 0.248 = 0.7511$$

$$n = \frac{1}{0.7511} = 1.33$$

$$\boxed{n = 1.33}$$

To find isothermal work

$$W_{iso} = \int -vdp - \frac{C_2^2}{2000}$$

$$W_{iso} = \int_1^2 -\frac{RT}{p}.dp - \frac{C_2^2}{2000}$$

$PV = RT$

$V = \frac{RT}{p}$

$$= -RT \ln[P]_1^2 - \frac{C_2^2}{2000}$$

$$= -RT_1 \ln\left[\frac{P_2}{P_1}\right] - \frac{C_2^2}{2000}$$

$$= -0.287 \times 303 \times \ln\left[\frac{3.5}{1}\right] - \frac{90^2}{2000}$$

$$= -108.94 - 4.05$$

$$W_{iso} = 112.99 \text{ KJ/kg}$$

$$\eta_{Iso} = \frac{W_{iso}}{W_{act}} = \frac{112.99}{115.43} = 0.978$$

$$\boxed{\eta_{iso} = 97.8\%}$$

12. A lead storage battery is able to deliver 5.2MJ of electrical energy. This energy is available for starting a car. Suppose we wish to use compressed air for doing the equivalant amount of work in starting the car. The compressed air is stored at 7MPa, 25°C. What volume of tank would be required to have the compressed air having availability of 5.2MJ. Take $P_0 = 101.325$ KPa, $T_0 = 298$ K as atmospheric condition.

Given data:-

$$p_1 = 7MPa = 7 \times 10^3 \text{ kPa} = 7000 \text{ kPa}$$

$$T_1 = 25°C = 25 + 273 = 298 \text{ K}$$

$$\Phi_1 - \Phi_0 = 5.2 \text{ MJ} = 5200 \text{ KJ}$$

$p_0 = 101.325$ kPa

$T_0 = 298$ K

To find

Volume of tank V

Sol:-

Closed system.

Available energy $\varphi_1 - \varphi_0 = m\left[(u_1 - u_0) + p_0(v_1 - v_0) - T_0(s_1 - s_0)\right]$

$(u_1 - u_0) = m C_v [T_1 - T_0] = 0 \qquad \because T_1 = T_0$

$p_0 (v_1 - v_0) = p_0 \left[\frac{RT_1}{p_1} - \frac{RT_0}{p_0}\right] \qquad pV = RT, \quad V = \frac{RT}{p}$

$= 101.325 \left[\frac{0.287 \times 298}{7000} - \frac{0.287 \times 298}{101.325}\right]$

$= 101.325 \left[0.012218 - 0.844\right]$

$p_0(v_1 - v_0) = -84.28$ kJ/kg

$T_0 (s_1 - s_0) = T_0 \left[C_p \ln\left(\frac{T_1}{T_0}\right) - R \ln\left(\frac{p_1}{p_0}\right)\right]$

$= 298 \left[\cancel{1.005 \times \ln\left[\frac{298}{298}\right]}^{\,0} - 0.287 \times \ln\left[\frac{7000}{101.325}\right]\right]$

$= 298 \left[-1.21554\right]$

$= -362.23$ kJ/kg.

$5200 = m\left[-84.28 + 362.23\right]$

$5200 = m\left[277.95\right]$

$m = 18.70$ kg

Specific volume $v = \frac{V}{m}$

Total volume $V = v \times m = 0.012218 \times 18.70$

$$\boxed{\text{Total volume } V = 0.228 \text{ m}^3}$$

13. Two kg of air at 500 KPa, 80°C expands adiabatically in a closed system until its volume is doubled and its temperature becomes equal to that of the surroundings which is at 100 KPa, 5°C. For this process determine the

a) Maximum work.

b) Change in availability and

c) Irreversibility.

Given data :-

$m = 2$ kg

$P_1 = 500$ kPa

$T_1 = 80°C = 80 + 273 = 353$ K

$V_2 = 2V_1$

$P_0 = 100$ kPa

$T_0 = 5°C = 5 + 273 = 278$ K

To find :- W_{max}, W_{useful} or $\varphi_1 - \varphi_0$, I

Sol :-

$$W_{max} = m\left[(U_1 - U_0) - T_0(S_1 - S_0)\right]$$

$$= m\left[C_v(T_1 - T_0) - T_0\left[C_p \ln\left[\frac{T_1}{T_0}\right] - R \ln\left[\frac{P_1}{P_0}\right]\right]\right]$$

$$= 2\left[0.718[353 - 278] - 278\left[1.005 \times \ln\left[\frac{353}{278}\right] - \right.\right.$$

$$- 0.287 \ln \left[\frac{500}{100}\right]\Big]\Big]$$

$$W_{max} = 2\left[53.85 - 278\left[0.24 - 0.461\right]\right]$$

$$= 2\left[53.85 + 61.438\right]$$

$$\boxed{W_{max} = 230.576 \text{ kJ}}$$

change in availability, $\varphi_1 - \varphi_0 = m\left[(U_1 - U_0) + P_0(V_1 - V_0) - T_0(S_1 - S_0)\right]$

$$= W_{max} + P_0(V_1 - V_0)$$

$$= 230.576 + 100\left[\frac{mRT_1}{P_1} - \frac{mRT_0}{P_0}\right]$$

$$= 230.576 + 100\left[\frac{2 \times 0.287 \times 353}{500} - \frac{2 \times 0.287 \times 278}{100}\right]$$

$$= 230.576 + 100\left[0.405 - 1.595\right]$$

$$= 230.576 - 119.072$$

$$\boxed{\varphi_1 - \varphi_0 = 111.504 \text{ kJ}}$$

Irreversibility $I = T_0(S_1 - S_0) = T_0 \Delta S$

$$= T_0\left[mC_p \ln\left[\frac{T_1}{T_0}\right] - mR \ln\left[\frac{P_1}{P_0}\right]\right]$$

$$= 278\left[2 \times 1.005 \times \ln\left[\frac{353}{278}\right] - 2 \times 0.287 \times \ln\left[\frac{500}{100}\right]\right]$$

$$= 278\left[0.48 - 0.923\right]$$

$$\boxed{I = -123.35 \text{ kJ}}$$

14. Derive an expression for exergy balance of a closed system.

Like energy, exergy can be transferred across the boundary of a closed system. The change in exergy of a system during a process would not necessarily equal to the net exergy transfered because exergy would be destroyed if irreversibilities were present within the system during the process. The concepts of exergy change, exergy transfer, and exergy destruction are related by the closed system exergy balance.

The exergy balance for a closed system is developed by combaining the closed system energy and entropy balances:

According to first law of thermodynamics

$$\Delta E = Q - W$$

$$\Delta U + \Delta KE + \Delta PE = \left(\int_1^2 \delta Q\right) - W \quad \text{—} (1)$$

According to the second law of thermodynamics

$$\Delta S = \int_1^2 \frac{\delta Q}{T} + S_{gen}$$

X To on both sides, we get

$$T_o \Delta S = T_o \int_1^2 \frac{\delta Q}{T} + T_o S_{gen} \quad \text{—} (2)$$

(1) − (2) ⇒

$$(\Delta U + \Delta KE + \Delta PE) - T_o . \Delta S = \left(\int_1^2 \delta Q\right) - T_o \int_1^2 \frac{\delta Q}{T} - W - T_o S_{gen}$$

$$(\Delta U + \Delta PE + \Delta KE) - T_0 . \Delta S = \int_1^2 \left(1 - \frac{T_0}{T}\right)\delta Q - W - T_0 . \dot{S}_{gen} \quad -③$$

W.K.T

The change in exergy of a closed system is given by

$$\phi_1 - \phi_2 = \Delta U + P_0 . dv - T_0 . ds$$

- rearranging this equation

$$(\phi_1 - \phi_2) - P_0 (dv) = \Delta U - T_0 ds \quad -④$$

④ ⇒ ③

$$(\phi_1 - \phi_2) - P_0 dv = \int_1^2 \left(1 - \frac{T_0}{T}\right)\delta Q - W - T_0 \dot{S}_{gen}$$

$$\boxed{(\phi_1 - \phi_2) = \int_1^2 \left(1 - \frac{T_0}{T}\right)\delta Q - \left[W - P_0 dv\right] - T_0 . \dot{S}_{gen}}$$

or

$$\boxed{(\phi_1 - \phi_2) = E_q - E_W - E_d}$$

where

E_q = Exergy transfer accompanying heat transfer

$$E_q = \int_1^2 \left(1 - \frac{T_0}{T}\right)\delta Q$$

E_W = Exergy transfer accompanying work

$$E_W = \left[W - P_0 dv\right]$$

where W - boundary work or piston work

$P_0 dv$ - atmospheric work

E_d = Destruction of exergy due to irreversibilities within the system

In accordance with second law, the exergy destruction is positive when irreversibilities are present within the system during the process and vanishes in the limiting case where there are no irreversibilities.

$$E_d \begin{cases} > 0 & \text{irreversibilities present in the system} \\ = 0 & \text{no irreversibilities within the system.} \end{cases}$$

Change in exergy of a system can be positive, negative or zero.

$$\varphi_1 - \varphi_2 \begin{cases} > 0 \\ = 0 \\ < 0 \end{cases}$$

15. Derive an expression for exergy balance of a open system or Control volume at steady state.

The exergy balance for a open or control volume system can be obtained by combaining the energy balance and entrophy balance equation.

According to the conservation of energy principle applied to a control volume.

change of energy contained within the control volume	=	Energy being transferred by heat	−	Energy being transferred by work +

Energy transfer into the control volume accompanying mass flow.

$$dE_{cv} = Q_{cv} - W_{cv} + \left[m_i\left(h_i + \frac{V_i^2}{2000} + \frac{gz_i}{1000}\right) - m_e\left(h_e + \frac{V_e^2}{2000} + \frac{gz_2}{1000}\right)\right] \quad \text{—①}$$

Entropy balance equation

$$\Delta S = \int_1^2 \frac{\delta Q}{T} + S_{gen}$$

X. T_0 on both sides

$$T_0 \Delta S = T_0 \int_1^2 \frac{\delta Q}{T} + T_0 . S_{gen} \quad \text{—②}$$

① − ② ⇒

$$dE - T_0 \Delta S = \int_1^2 (\delta Q)_{cv} - T_0 \int_1^2 \frac{\delta Q}{T} - W_{cv} + [me_i - me_2] - T_0 . S_{gen}.$$

$$dE - T_0 \Delta S = \int_1^2 \left(1 - \frac{T_0}{T}\right) \delta Q - W_{cv} + [me_1 - me_2] - T_0 S_{gen}$$

$$\psi_1 - \psi_2 = \int_1^2 \left(1 - \frac{T_0}{T}\right) \delta Q - W_{cv} + [me_1 - me_2] - T_0 S_{gen}$$

At steady state condition $\psi_1 - \psi_2 = 0$

$$\boxed{0 = \int_1^2 \left(1 - \frac{T_0}{T}\right) \delta Q - W_{cv} + [me_1 - me_2] - T_0 S_{gen}}$$

or

$$\boxed{0 = E_q - E_W + E_m - E_d}$$

where

E_q = Exergy transfer accompanying heat transfer

$$E_q = \int_1^2 \left(1 - \frac{T_0}{T}\right) \delta Q$$

E_W = Exergy transfer accompanying work

E_m = Exergy transfer accompanying mass flow

E_d = Exergy destruction.

$$e_1 = (h_1 - h_0) - T_0(S_1 - S_0) + \frac{V_1^2}{2000} + \frac{gz_1}{1000}$$

$$e_2 = (h_2 - h_0) - T_0(S_2 - S_0) + \frac{V_2^2}{2000} + \frac{gz_2}{1000}$$

Unit-III AVAILABILITY AND APPLICATIONS OF II LAW

TWO MARKS QUESTIONS AND ANSWERS

1. Define Second law efficiency [MAY-16]

Second law efficiency is defined as the ratio between the change in available energy of the system to the change in available energy of the source.

$$\eta_{II} = \frac{\text{Change in available energy of system}}{\text{change in available energy of source}} = \frac{A_{out}}{A_{in}}$$

2. What do you understand by high grade and low grade energy? [May-17]

High grade energy :-

The energy which can be completely converted into other useful form of energy is called high grade energy.

ex: Mechanical work, electrical work.

Low grade energy :-

The energy which can not be completly converted into other useful form of energy is called low grade energy.

ex: Thermal energy.

3. Define irreversibility [May-16, May-19]

Processes that are not reversible are called irreversible processes. The factors that cause a process to be irreversible are called irreversibilities. They include friction unrestrained expansion, mixing of two fluids, heat transfer across a finite temperature difference, electric resistance, inelastic deformation of solids and chemical reaction. The presence of any of these effects renders a process irreversible.

4. What is irreversibility of a process? [May-19]

An irreversible process is one in which the system and its environment cannot return together to exactly to the initial state condition. The irreversibility of any process results from the second law of thermo-dynamics.

5. A turbine gets a supply of 5 kg/s of steam at 7 bar, 250°c and discharges it at 1 bar. Calculate the availability.

Given data:-

$m = 5 \text{ kg/s}$, $P_1 = 7 \text{ bar}$, $T_1 = 250^\circ C = 250+273 = 523 K$

$P_2 = 1 \text{ bar}$.

To find:-

Availability $\Delta\psi$

Sol:-

For open system,

$$\text{Availability} = m\left[(h_1 - h_2) - T_0(s_1 - s_2)\right]$$

In turbine flow is isentropic $s_1 = s_2 \Rightarrow \Delta s = s_1 - s_2 = 0$

corresponding to 7 bar and 250°c, both enthalpy and entropy are read from Mollier diagram.

$h_1 = 2954 \text{ kJ/kg}$.

$h_2 = 2581 \text{ kJ/kg}$.

$$\text{Availability} = m(h_1 - h_2) \Rightarrow 5(2954 - 2581)$$

$$\boxed{\Delta\psi = 1865 \text{ kW}}$$

6. What is loss of availability? How does it is related to entropy of universe? [Apr-08]

The portion of the energy supplied as heat which cannot be converted into work due to friction is called unavailable energy or loss of availability.

Unavailable energy = Total heat - Available energy

$$U.A.E = Q - [Q - T_0 ds]$$

$$\boxed{U.A.E = T_0 . ds}$$ [ds - Entropy of universe]

7. What are available energy and unavilable energy? [May-12]

Available energy :-

The portion of energy supplied as heat which can be converted into useful work by a reversible engine is called available energy.

Unavailable energy :-

The portion of energy supplied as heat which cannot be converted into work due to friction is called unavailable energy.

8. What is meant by dead state? [May-13]

The state of a system when it is in thermodynamic equillibrium with its environment is called dead state. Normally, the dead state is taken to be the environmental condition. The properties of a system at dead state are denoted by T_0, P_0, U_0, h_0, S_0 and V_0

9. Define Gouy-Stodola theorem.

The Gouy-Stodola theorem states that the rate of loss of availability or available energy or exergy in a process is proportional to the rate of entropy generation, Sgen.

$$I = W_{lost} = T_0 . \Delta S_{univ} = T_0 . S_{gen}.$$

A thermodynamically efficient process would involve minimum exergy loss with minimum rate of entropy generation.

10. How much of the 100 kJ of thermal energy at 650 K can be converted to useful work? Assume the environment to be at 25°C. [Dec-16]

Given data :-

$Q = 100\ kJ$

$T = 650\ K$

$T_0 = 25^\circ C = 25 + 273 = 298\ K$

Sol :-

Available energy or Useful energy

$$A.E = Q - T_0 . ds \Rightarrow Q - T_0 . \frac{Q}{T} \Rightarrow Q\left(1 - \frac{T_0}{T}\right)$$

$$= 100\left(1 - \frac{298}{650}\right) = 54.15\ kJ$$

$$\boxed{A.E = 54.15\ kJ}$$