UNIT IV

ENERGY STORING ELEMENTS AND ENGINE COMPONENTS

CHAPTER 2

Energy Stored in Helical Springs of Circular Wire

We know that the springs are used for storing energy which is equal to the work done on it by some external load.

Let W = Load applied on the spring, and

δ = Deflection produced in the spring due to the load W.

Assuming that the load is applied gradually, the energy stored in a spring is,

$$U = \frac{1}{2} W.\delta$$

We have already discussed that the maximum shear stress induced in the spring wire,

$$\tau = \frac{8\ W.D}{\pi\ d^3} K$$

$$W = \frac{\tau\ \pi\ d^3}{8.K.D}$$

We know that deflection of the spring,

$$\delta = \frac{8.W.D^3.n}{G\ .d^4}$$

$$\delta = \frac{8(.\frac{\tau\ \pi\ d^3}{8.K.D})\ .D^3.n}{G\ .d^4}$$

$$\delta = \frac{\pi\tau D^2.n}{K.d.G}$$

Substituting the values of W and δ in equation (i), we have

$$U = \frac{1}{2}\frac{\tau\ \pi\ d^3}{8.K.D} \times \frac{\pi\tau D^2.n}{K.d.G}$$

$$U = \frac{\tau^2}{4\ K^2.G}(\pi D.n)\left(\frac{\pi}{4} \times\ d^2\right)$$

$$U = \frac{\tau^2}{4\ K^2.G} \times V$$

where V = Volume of the spring wire

= Length of spring wire × Cross-sectional area of spring wire

$$V = (\pi D.n)\left(\frac{\pi}{4} \times d^2\right)$$

Problem 2.1

A rail wagon of mass 20 tonnes is moving with a velocity of 2 m/s. It is brought to rest by two buffers with springs of 300 mm diameter. The maximum deflection of springs is 250 mm. The allowable shear stress in the spring material is 600 MPa. Design the spring for the buffers.

Given Data:

m = 20 t = 20 000 kg

v = 2 m/s

D = 300 mm

δ = 250 mm

τ = 600 MPa = 600 N/mm^2

1. Diameter of the spring wire

Let d =Diameter of the spring wire.

We know that kinetic energy of the wagon

$$= \frac{1}{2}mv^2$$

$$= \frac{1}{2}(20000)2^2$$

$$= 40 \times 10^6 \text{ N-mm}$$

Let W be the equivalent load which when applied gradually on each spring causes a deflection of 250 mm. Since there are two springs, therefore

Energy stored in the springs

$$U = \frac{1}{2} W.\delta \times 2$$

$$U = W.\delta$$

$$U = 250 \text{ W N-mm}$$

From equations (i) and (ii), we have

$W = 40 \times 10^6 / 250$

$\therefore W = 160 \times 10^3$ N

We know that torque transmitted by the spring,

$T = W \times \frac{D}{2}$

$T = 1600 \times 10^3 \times \frac{300}{2}$

$T = 24 \times 10^6$ N-mm.

We also know that torque transmitted by the spring (T),

$24 \times 10^6 = \frac{\pi}{16} \times \tau \times d^3$

$24 \times 10^6 = \frac{\pi}{16} \times 600 \times d^3$

$24 \times 10^6 = 117.8\ d^3$

$d^3 = 24 \times 106 / 117.8$

$d^3 = 203.7 \times 10^3$ or

$d = 58.8$ say 60 mm.

2. Number of turns of the spring coil

Let n = Number of active turns of the spring coil.

We know that the deflection of the spring (δ),

$\delta = \frac{8.W.D^3.n}{G.d^4}$

$\delta = \frac{8 \times 160 \times 10^3 \times 300^3.n}{84 \times 10^3\ \ 60^4}$

... (Taking G = 84 MPa = 84×10^3 N/mm^2)

$\therefore n = 250 / 31.7$

$n = 7.88$ say 8.

Assuming square and ground ends, total number of turns,

$n' = n + 2$

$n' = 8 + 2$

$n' = 10.$

3. Free length of the spring

We know that free length of the spring,

$$L_F = n'.\ d + \delta + 0.15\ \delta$$

$$L_F = 10 \times 60 + 250 + 0.15 \times 250$$

$$L_F = 887.5 \text{ mm.}$$

4. Pitch of the coil

We know that pitch of the coil

$$p = \frac{\text{Free length}}{n' - 1}$$

$$p = \frac{887.5}{10 - 1}$$

$$p = 98.6 \text{ mm.}$$

Stress and Deflection in Helical Springs of Non-Circular Wire

The helical springs may be made of non-circular wire such as rectangular or square wire, in order to provide greater resilience in a given space. However, these springs have the following main disadvantages:

1. The quality of material used for springs is not so good.
2. The shape of the wire does not remain square or rectangular while forming helix, resulting in trapezoidal cross-sections. It reduces the energy absorbing capacity of the spring.
3. The stress distribution is not as favourable as for circular wires.

But this effect is negligible where loading is of static nature.

For springs made of rectangular wire, as shown in Fig. 2.1, the maximum shear stress is given by

$$\tau = K \times \frac{W.D\ (1.5t + 0.9b)}{b^2 t^2}$$

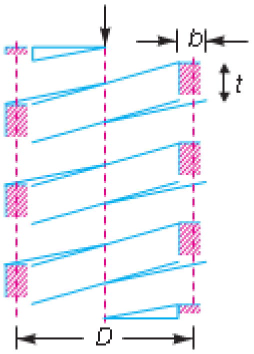

Fig 2.1 Spring of rectangular wire.

[Source: "A Textbook of Machine Design by R.S. Khurmi J.K. Gupta, Page: 852]

This expression is applicable when the longer side (i.e. t > b) is parallel to the axis of the spring. But when the shorter side (i.e. t < b) is parallel to the axis of the spring, then maximum shear stress,

$$\tau = K \times \frac{W.D\,(1.5t+0.9b)}{b^2t^2}$$

and deflection of the spring,

$$\delta = \frac{2.45W.D^3.n}{Gb^3\,(t-0.56b)}$$

For springs made of square wire, the dimensions b and t are equal. Therefore, the maximum shear stress is given by

$$\tau = K \times \frac{2.4\,W.D}{b^3}$$

and deflection of the spring,

$$\delta = \frac{5.568W.D^3.n}{Gb^4}$$

$$\delta = \frac{5.568W.C^3.n}{Gb} \quad \ldots..(C = \frac{D}{b})$$

where b = Side of the square.

<u>Helical Springs Subjected to Fatigue Loading</u>

The helical springs subjected to fatigue loading are designed by using the Soderberg line method. The spring materials are usually tested for torsional endurance strength under a repeated stress that varies from zero to a maximum. Since the springs are ordinarily loaded in one direction only (the load in springs is never reversed in nature), therefore a modified Soderberg diagram is used for springs, as shown in Fig. 2.2.

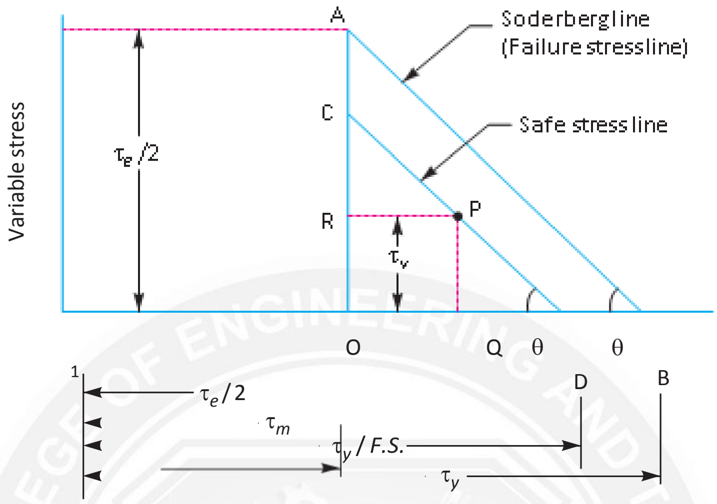

Fig 2.2 Modified Soderberg method for helical springs.

[Source: "A Textbook of Machine Design by R.S. Khurmi J.K. Gupta, Page: 854]

The endurance limit for reversed loading is shown at point A where the mean shear stress is equal to τ_e / 2 and the variable shear stress is also equal to τ_e / 2. A line drawn from A to B (the yield point in shear, τ_y) gives the Soderberg's failure stress line. If a suitable factor of safety (F.S.) is applied to the yield strength (τ_y), a safe stress line CD may be drawn parallel to the line AB, as shown in Fig. 2.1. Consider a design point P on the line CD. Now the value of factor of safety may be obtained as discussed below

From similar triangles PQD and AOB, we have

$$\frac{PQ}{QD} = \frac{OA}{OB}$$

$$\frac{PQ}{O_1D - O_1Q} = \frac{OA}{O_1B - O_1O}$$

$$\frac{\tau_v}{\frac{\tau_y}{F.S} - \tau_m} = \frac{\frac{\tau_e}{2}}{\tau_y - \frac{\tau_e}{2}} = \frac{\tau_e}{2\tau_y - \tau_e}$$

$$2\,\tau_v.\,\tau_y - \tau_v.\,\tau_e = \frac{\tau_e\tau_y}{F.S} - \tau_m.\tau_e$$

$$\frac{\tau_e\tau_y}{F.S} = 2\,\tau_v.\tau_y - \tau_v.\tau_e + \tau_m.\tau_e$$

Dividing both sides by $\tau_e.\tau_y$ and rearranging, we have

$$\frac{1}{F.S} = \frac{\tau_m - \tau_v}{\tau_y} + \frac{2\tau_v}{\tau_e}$$

Problem 2.2

A helical compression spring made of oil tempered carbon steel, is subjected to a load which varies from 400 N to 1000 N. The spring index is 6 and the design factor of safety is 1.25. If the yield stress in shear is 770 MPa and endurance stress in shear is 350 MPa, find: 1. Size of the spring wire, 2. Diameters of the spring, 3. Number of turns of the spring, and 4. Free length of the spring.

The compression of the spring at the maximum load is 30 mm. The modulus of rigidity for the spring material may be taken as 80 kN/mm2.

Given Data:

W_{min} = 400 N

W_{max} = 1000 N

C = 6

F.S. = 1.25

τ_y = 770 MPa = 770 N/mm^2

τ_e = 350 MPa = 350 N/mm^2

δ = 30 mm

G = 80 kN/mm^2 = 80 × 10^3 N/mm^2

1. Size of the spring wire

Let d = Diameter of the spring wire, and

D = Mean diameter of the spring = C.d = 6 d ... (D/d = C = 6)

We know that the mean load,

$$W_m = \frac{W_{max} + W_{min}}{2} = \frac{1000+400}{2}$$

$$W_m = 700 \text{ N}$$

and variable load,

$$W_v = \frac{W_{max} - W_{min}}{2} = \frac{1000-400}{2}$$

$$W_v = 300 \text{ N}$$

Shear stress factor,

$$K_S = 1 + \frac{1}{2C} = 1 + \frac{1}{2\times 6}$$

$$K_S = 1.083$$

Wahl's stress factor,

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$

$$K = \frac{4\times 6-1}{4\times 6-4} + \frac{0.615}{6}$$

$$K = 1.2525$$

We know that mean shear stress,

$$\tau_m = \frac{8\, W_m.D}{\pi\, d^3} K_S$$

$$\tau_m = \frac{8 \times 700\times 6d}{\pi\, d^3} 1.083$$

$$\tau_m = \frac{11582}{d^2} \text{ N/mm}^2$$

and variable shear stress,

$$\tau_v = \frac{8\, W_v.C}{\pi\, d^2} K$$

$$\tau_v = \frac{8 \times 300\times 6d}{\pi\, d^2} 1.2525$$

$$\tau_v = \frac{5740}{d^2} \text{ N/mm}^2$$

We know that

$$\frac{1}{F.S} = \frac{\tau_m - \tau_v}{\tau_y} + \frac{2\tau_v}{\tau_e}$$

$$\frac{1}{1.25} = \frac{\frac{11582}{d^2} - \frac{5740}{d^2}}{770} + \frac{2 \times \frac{5740}{d^2}}{350}$$

$$\frac{1}{1.25} = \frac{7.6}{d^2} + \frac{32.8}{d^2} = \frac{40.4}{d^2}$$

$d^2 = 1.25 \times 40.4 = 50.5$ or

$\therefore d = 7.1$ mm.

2. Diameters of the spring

We know that mean diameter of the spring,

$D = C.d = 6 \times 7.1$

$D = 42.6$ mm.

Outer diameter of the spring,

$D_o = D + d = 42.6 + 7.1$

$D_o = 49.7$ mm.

and inner diameter of the spring,

$D_i = D - d = 42.6 - 7.1$

$D_i = 35.5$ mm.

3. Number of turns of the spring

Let n = Number of active turns of the spring.

We know that deflection of the spring (δ),

$$30 = \frac{8W.D^3.n}{G.d^4}$$

$$30 = \frac{8 \times 1000 \times 42.6^3.n}{80 \times 10^3 \times 7.1^4}$$

$n = 30 / 3.04$

$\therefore n = 9.87$ say 10.

Assuming the ends of the spring to be squared and ground, the total number of turns of the spring,

$n' = n + 2 = 10 + 2$

$n' = 12$.

4. Free length of the spring

We know that free length of the spring,

$L_F = n'. d + \delta + 0.15 \delta$

$L_F = 12 \times 7.1 + 30 + 0.15 \times 30$ mm

$L_F = 119.7$ say 120 mm

Springs in Series

Consider two springs connected in series as shown in Fig. 2.3.

Let W = Load carried by the springs,

δ_1 = Deflection of spring 1,

δ_2 = Deflection of spring 2,

k_1 = Stiffness of spring 1 = W / δ_1, and

k_2 = Stiffness of spring 2 = W / δ_2

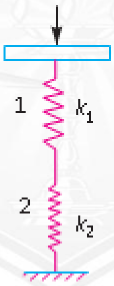

Fig 2.3 Springs in series.

[Source: "A Textbook of Machine Design by R.S. Khurmi J.K. Gupta, Page: 856]

A little consideration will show that when the springs are connected in series, then the total deflection produced by the springs is equal to the sum of the deflections of the individual springs.

∴ Total deflection of the springs,

$$\delta = \delta_1 + \delta_2$$

$$\frac{W}{k} = \frac{W}{k_1} + \frac{W}{k_2}$$

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$

where k = Combined stiffness of the springs.

Springs in Parallel

Consider two springs connected in parallel as shown in Fig 2.4.

Let W = Load carried by the springs,

W_1 = Load shared by spring 1,

W_2 = Load shared by spring 2,

k_1 = Stiffness of spring 1, and

k_2 = Stiffness of spring 2.

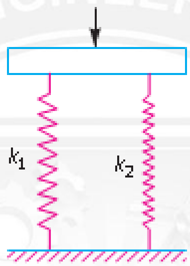

Fig 2.4 Springs in parallel.

[Source: "A Textbook of Machine Design by R.S. Khurmi J.K. Gupta, Page: 856]

A little consideration will show that when the springs are connected in parallel, then the total deflection produced by the springs is same as the deflection of the individual springs.

We know that,

$$W = W_1 + W2$$

$$\text{or } \delta.k = \delta.\ k_1 + \delta.\ k_2$$

$$\therefore k = k_1 + k_2$$

where k = Combined stiffness of the springs, and

δ = Deflection produced.

Concentric or Composite Springs

A concentric or composite spring is used for one of the following purposes:

1. To obtain greater spring force within a given space.

2. To insure the operation of a mechanism in the event of failure of one of the springs.

The concentric springs for the above two purposes may have two or more springs and have the same free lengths as shown in Fig. 1.5 (a) and are compressed equally. Such springs are used in automobile clutches, valve springs in aircraft, heavy duty diesel engines and rail-road car suspension systems. Sometimes concentric springs are used to obtain a spring force which does not increase in a direct relation to the deflection but increases faster. Such springs are made of different lengths as shown in Fig. 1.5 (b). The shorter spring begins to act only after the longer spring is compressed to a certain amount. These springs are used in governors of variable speed engines to take care of the variable centrifugal force.

The adjacent coils of the concentric spring are wound in opposite directions to eliminate any tendency to bind. If the same material is used, the concentric springs are designed for the same stress. In order to get the same stress factor (K), it is desirable to have the same spring index (C).

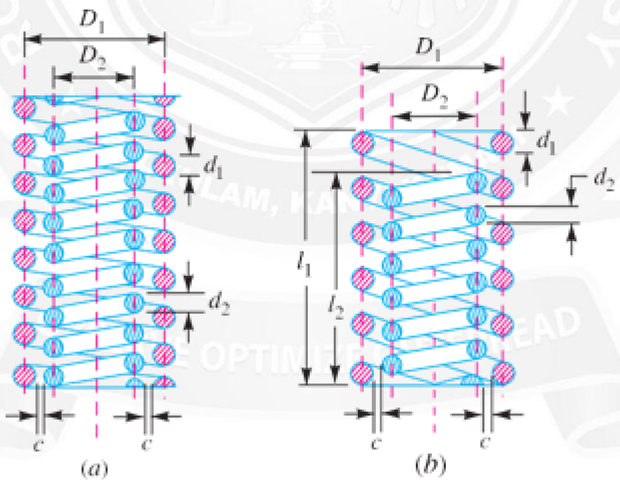

Fig 2.5 Concentric springs.

[Source: "A Textbook of Machine Design by R.S. Khurmi J.K. Gupta, Page: 858]

Consider a concentric spring as shown in Fig. 1.5 (a).

Let W = Axial load,

W_1 = Load shared by outer spring,

W_2 = Load shared by inner spring,

d_1 = Diameter of spring wire of outer spring,

d_2 = Diameter of spring wire of inner spring,

D_1 = Mean diameter of outer spring,

D_2 = Mean diameter of inner spring,

δ_1 = Deflection of outer spring,

δ_2 = Deflection of inner spring,

n_1 = Number of active turns of outer spring, and

n_2 = Number of active turns of inner spring.

Assuming that both the springs are made of same material, then the maximum shear stress induced in both the springs is approximately same, i.e.

$$\tau_1 = \tau_2$$

$$\frac{8\ W_1 D_1 K_1}{\pi\ {d_1}^3} = \frac{8\ W_2 D_2 K_2}{\pi\ {d_2}^3}$$

When stress factor, $K_1 = K_2$, then

$$\frac{W_1 D_1}{{d_1}^3} = \frac{W_2 D_2}{{d_2}^3} \qquad ...(i)$$

If both the springs are effective throughout their working range, then their free length and deflection are equal, i.e.

$$\delta_1 = \delta_2$$

$$\frac{8 W_1 {D_1}^3 n_1}{G.{d_1}^4} = \frac{8 W_2 {D_2}^3 n_2}{G.{d_2}^4}$$

$$\frac{W_1 {D_1}^3 n_1}{{d_1}^4} = \frac{W_2 {D_2}^3 n_2}{{d_2}^4} \qquad ...(ii)$$

When both the springs are compressed until the adjacent coils meet, then the solid length of both the springs is equal, i.e.

$$n_1 . d_1 = n_2 . d_2$$

∴ The equation (ii) may be written as

$$\frac{W_1 {D_1}^3}{{d_1}^5} = \frac{W_2 {D_2}^3}{{d_2}^5} \qquad ...(iii)$$

Now dividing equation (iii) by equation (i), we have

$$\frac{D_1^2}{d_1^2} = \frac{D_2^2}{d_2^2}$$

i.e. the springs should be designed in such a way that the spring index for both the springs is same.

From equations (i) and (iv), we have

$$\frac{W_1}{d_1^2} = \frac{W_2}{d_2^2}$$

$$\frac{W_1}{W_2} = \frac{d_1^2}{d_2^2}$$

From Fig. 1.5 (a), we find that the radial clearance between the two springs,

$$c = \left(\frac{D_1}{2} - \frac{D_2}{2}\right) - \left(\frac{d_1}{2} + \frac{d_2}{2}\right)$$

Usually, the radial clearance between the two springs is taken as $\left(\frac{d_1}{2} - \frac{d_2}{2}\right)$

$$\left(\frac{D_1}{2} - \frac{D_2}{2}\right) - \left(\frac{d_1}{2} + \frac{d_2}{2}\right) = \left(\frac{d_1}{2} - \frac{d_2}{2}\right)$$

$$\frac{D_1 - D_2}{2} = d_1 \qquad \text{...(vi)}$$

From equation (iv), we find that

$D_1 = C.\ d_1$, and

$D_2 = C.\ d_2$

Substituting the values of D_1 and D_2 in equation (vi), we have

$$\frac{Cd_1 - Cd_2}{2} = d_1$$

$$C\ d_1 - 2d_1 = C.\ d_2$$

$$\therefore d_1 (C - 2) = C.\ d_2$$

$$\frac{d_1}{d_2} = \frac{C}{C-2}$$

Problem 2.3

A concentric spring for an aircraft engine valve is to exert a maximum force of 5000 N under an axial deflection of 40 mm. Both the springs have same free length, same solid length and are subjected to equal maximum shear stress of 850 MPa. If the spring index for both the springs is 6, find (a) the load shared by each spring, (b) the main dimensions of both the springs, and (c) the number of active coils in each spring.

Assume G = 80 kN/mm^2 and diametrial clearance to be equal to the difference between the wire diameters.

Given Data:

$W = 5000$ N

$\delta = 40$ mm

$\tau_1 = \tau_2 = 850$ MPa $= 850$ N/mm^2

$C = 6$

$G = 80$ kN/mm^2 $= 80 \times 10^3$ N/mm^2

The concentric spring is shown in Fig. 1.5 (a).

(a) Load shared by each spring

Let W_1 and W_2 = Load shared by outer and inner spring respectively,

d_1 and d_2 = Diameter of spring wires for outer and inner springs respectively, and

D_1 and D_2 = Mean diameter of the outer and inner springs respectively.

Since the diametral clearance is equal to the difference between the wire diameters, therefore

$$(D_1 - D_2) - (d_1 + d_2) = d_1 - d_2$$

$$D_1 - D_2 = 2\, d_1$$

We know that D1 = C. d_1, and D2 = C. d_2

$$\therefore C.\, d_1 - C.\, d_2 = 2\, d_1$$

$$\frac{d_1}{d_2} = \frac{C}{C-2} = \frac{6}{6-2} = 1.5 \qquad ...(i)$$

We also know that

$$\frac{W_1}{W_2} = \frac{{d_1}^2}{{d_2}^2} = 1.5^2 = 2.25 \quad \text{...(ii)}$$

and $$W_1 + W_2 = W = 5000 \text{ N} \quad \text{...(iii)}$$

From equations (ii) and (iii), we find that

$$W_1 = 3462 \text{ N, and } W_2 = 1538 \text{ N}$$

(b) Main dimensions of both the springs

We know that Wahl's stress factor for both the springs,

$$K_1 = K_2 = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$

$$K_1 = K_2 = \frac{4\times6-1}{4\times6-4} + \frac{0.615}{6}$$

$$K_1 = K_2 = 1.2525$$

and maximum shear stress induced in the outer spring (τ_1),

$$850 = \frac{8\, W_1 C K_1}{\pi\, {d_1}^2}$$

$$850 = \frac{8 \times 3462 \times 6 \times 1.2525}{\pi\, {d_1}^2}$$

$$850 = \frac{66243}{(d_1)^2}$$

$$(d_1)^2 = 66\,243 / 850 = 78 \text{ or}$$

$$d_1 = 8.83 \text{ say } 10 \text{ mm.}$$

and $$D_1 = C.\, d1 = 6\, d_1$$

$$D_1 = 6 \times 10 = 60 \text{ mm.}$$

Similarly, maximum shear stress induced in the inner spring (τ_2),

$$850 = \frac{8\, W_2 C K_2}{\pi\, {d_1}^2}$$

$$850 = \frac{8 \times 1538 \times 6 \times 1.2525}{\pi\, {d_2}^2}$$

$$850 = \frac{29428}{(d_2)^2}$$

$$(d_2)^2 = 29428 / 850 = 34.6$$

$$d_2 = 5.88 \text{ say } 6 \text{ mm.}$$

and $D_2 = C.\ d_2 = 6 \times 6 = 36$ mm.

(c) Number of active coils in each spring

Let n_1 and n_2 = Number of active coils of the outer and inner spring respectively.

We know that the axial deflection for the outer spring (δ),

$$30 = \frac{8W_1C^3n_1}{G.d_1}$$

$$30 = \frac{8 \times 3462 \times 6^3 \times n_1}{80 \times 10^3 \times 10}$$

$$30 = 7.48\ n_1$$

$$n_1 = 40 / 7.48$$

$$n_1 = 5.35 \text{ say } 6$$

Assuming square and ground ends for the spring, the total number of turns of the outer spring,

$$n_1' = 6 + 2 = 8$$

∴ Solid length of the outer spring,

$$L_{S1} = n_1'.\ d_1 = 8 \times 10$$

$$L_{S1} = 80 \text{ mm}$$

Let n_2' be the total number of turns of the inner spring. Since both the springs have the same solid length, therefore,

$$n_2'.\ d_2 = n_1'.\ d_1$$

$$n_2' = \frac{n_1'd_1}{d_2}$$

and $n2 = 14 - 2 = 12.$ ... $(n2' = n2 + 2)$

Since both the springs have the same free length, therefore

Free length of outer spring = Free length of inner spring

$= L_{S1} + \delta + 0.15\ \delta$

$= 80 + 40 + 0.15 \times 40$

$= 126$ mm.

Other dimensions of the springs are as follows:

Outer diameter of the outer spring

$= D_1 + d_1 = 60 + 10$

$= 70$ mm.

Inner diameter of the outer spring

$= D_1 - d_1 = 60 - 10$

$= 50$ mm.

Outer diameter of the inner spring

$= D_2 + d_2 = 36 + 6$

$= 42$ mm.

Inner diameter of the inner spring

$= D_2 - d_2 = 36 - 6$

$= 30$ mm

Helical Torsion Springs

The helical torsion springs as shown in Fig. 3.1, may be made from round, rectangular or square wire. These are wound in a similar manner as helical compression or tension springs but the ends are shaped to transmit torque. The primary stress in helical torsion springs is bending stress whereas in compression or tension springs, the stresses are torsional shear stresses. The helical torsion springs are widely used for transmitting small torques as in door hinges, brush holders in electric motors, automobile starters etc. A little consideration will show that the radius of curvature of the coils changes when the twisting moment is applied to the spring. Thus, the wire is under pure bending. According to A.M. Wahl, the bending stress in a helical torsion spring made of round wire is

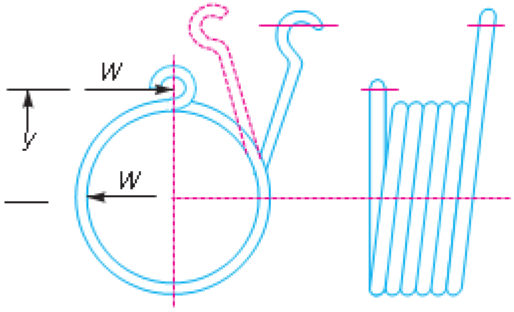

Fig 2.6 Helical torsion spring.

[Source: "A Textbook of Machine Design by R.S. Khurmi J.K. Gupta, Page: 863]

$$\sigma_b = K \times \frac{32\,M}{\pi\,d^3}$$

$$\sigma_b = K \times \frac{32\,W.y}{\pi\,d^3}$$

where K = Wahl's stress factor = $\frac{4C^2 - C - 1}{4C^2 - C}$

C = Spring index,

M = Bending moment = W × y,

W = Load acting on the spring,

y = Distance of load from the spring axis, and

d = Diameter of spring wire.

and total angle of twist or angular deflection,

$$\theta = \frac{M.l}{E.I} = \frac{M \times \pi D n}{E \times \pi d^4/64}$$

where l = Length of the wire = π.D.n,

E = Young's modulus,

I = Moment of inertia = $\frac{\pi}{64} \times d^4$

D = Diameter of the spring, and

n = Number of turns.

and deflection, $\delta = \theta \times y = \frac{64\,M.Dn}{E.d^4} \times y$

When the spring is made of rectangular wire having width b and thickness t, then

$$\sigma_b = K \times \frac{6M}{t.b^2}$$

$$\sigma_b = K \times \frac{6W.y}{t.b^2}$$

where $$K = \frac{3C^2 - C - 0.8}{3C^2 - 3C}$$

Angular deflection, $\theta = \frac{12M\pi D.n}{E.t.b^3}$

$$\delta = \theta \times y$$

$$\delta = \frac{12M\pi D.n}{E.t.b^3} \times y$$

In case the spring is made of square wire with each side equal to b, then substituting t = b, in the above relation, we have

$$\sigma_b = K \times \frac{6M}{t.b^2}$$

$$\sigma_b = K \times \frac{6W.y}{t.b^2}$$

$$\theta = \frac{12M\pi D.n}{E.t.b^3}$$

$$\delta = \frac{12M\pi D.n}{E.t.b^3} \times y$$