

Design Of Shear Key

1.3 Design for Cantilever Retaining Wall For Toe Slab with shear key

Example 3

Design a Toe slab for cantilever retaining wall to retain an earth embankment with a horizontal top 4m above ground level. Density of earth = 18 KN/m^3 . Angle of internal friction $\phi = 30^\circ$. SBC of soil = 200 KN/m^2 . Coefficient of friction between soil and concrete = 0.5. Adopt M20 grade concrete and Fe 415 HYSD bars.

Given data:

Density of earth	$\gamma = 18 \text{ KN/m}^3$
Angle of internal friction	$\phi = 30$
SBC of soil	$q = 200 \text{ KN/m}^2$

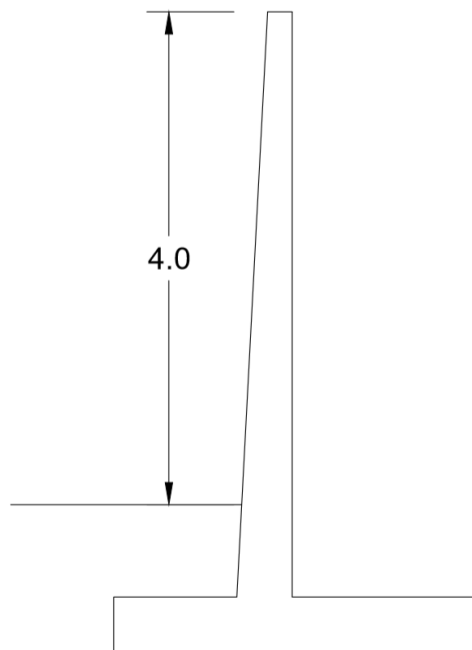


Fig.1.1 Cantilever retaining wall

Step 1: Dimensions of retaining wall

$$\begin{aligned}
 \text{(a) Depth of foundation} &= q / \gamma (1 - \sin \phi / 1 + \sin \phi)^2 \\
 &= 200 / 18 (1 - \sin 30 / 1 + \sin 30)^2
 \end{aligned}$$

$$= 1.2\text{m}$$

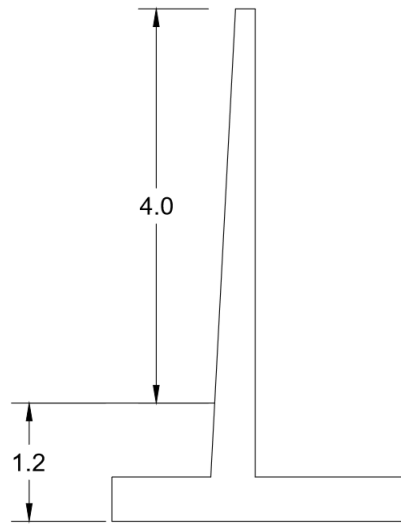


Fig.1.2 Cantilever retaining wall (Depth of foundation)

(b) Overall depth of wall

$$= 4 + 1.2$$

'H

$$= 5.2\text{m}$$

$$= 5200\text{mm}$$

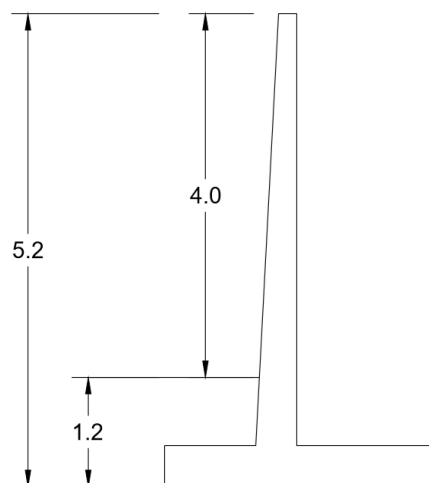


Fig.1.3 Cantilever retaining wall (Overall depth of wall)

(c) Thickness of base slab $= H / 12$

$$= 5200 / 12$$

$$= 433\text{mm} \sim 450\text{mm}$$

(d) Height of stem 'h' $= 5200 - 450$

$$= 4750\text{mm}$$

$$= 4.75\text{m}$$

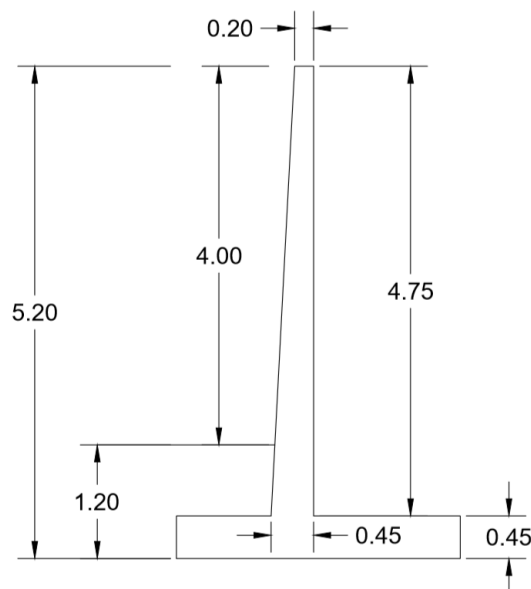


Fig.1.4 Cantilever retaining wall (Thickness of base slab)

(e) Width of base slab 'b' $= 0.5H \text{ to } 0.6H$

$$= 2600 \text{ to } 3120$$

$$= 3000\text{mm}$$

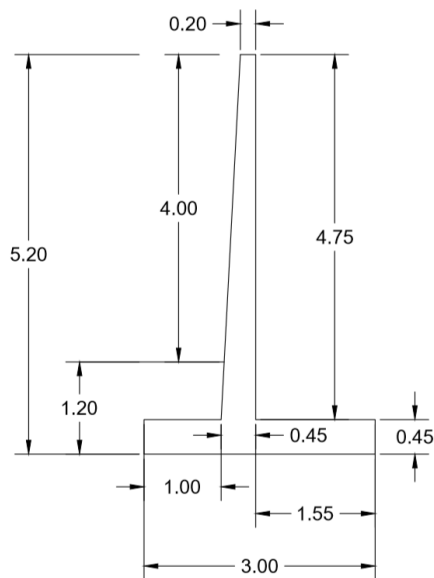


Fig.1.5 Cantilever retaining wall (Width of base slab)

Step 2: Stability calculation

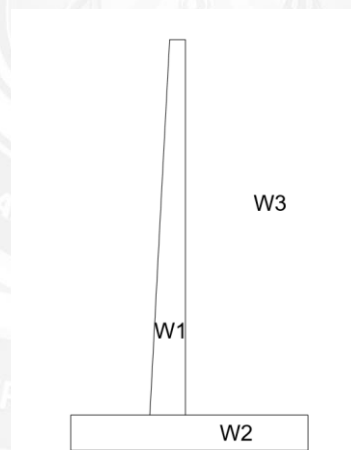


Fig.1.6 Cantilever retaining wall (Stability calculation)

(a) Find load

$$\begin{aligned}
 w_1 &= (b \times d \times \gamma_c) + \left(\frac{1}{2} \times b h \times \gamma_c\right) \\
 &= (0.2 \times 4.75 \times 24) + \left(\frac{1}{2} \times 0.25 \times 4.75 \times 24\right) \\
 &= 22.80 + 14.25 \\
 &= 37.05 \text{ KN}
 \end{aligned}$$

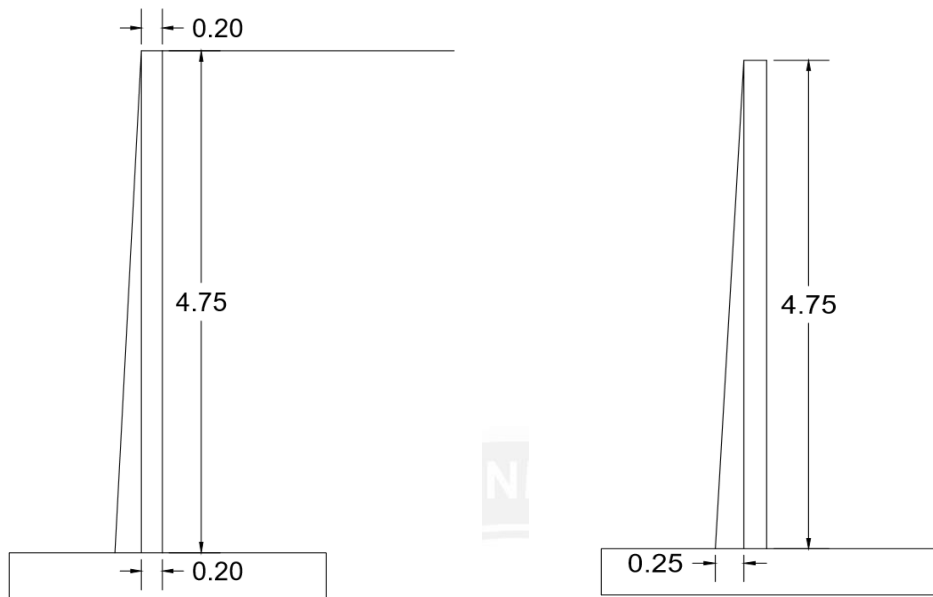


Fig.1.7 Cantilever retaining wall (Stability calculation)

$$\begin{aligned}
 w_2 &= b \times d \times \gamma_c \\
 &= 3 \times 0.45 \times 24 \\
 &= 32.40 \text{ KN}
 \end{aligned}$$

$$\begin{aligned}
 w_3 &= b \times d \times \gamma_s \\
 &= 1.55 \times 4.75 \times 18 \\
 &= 132.50 \text{ KN}
 \end{aligned}$$

$$\begin{aligned}
 \text{Total load} &= w_1 + w_2 + w_3 \\
 &= 201.95 \text{ KN}
 \end{aligned}$$

(b) Find moment @ a

$$\begin{aligned}
 M_1 &= W_1 \times \text{Length} \\
 &= (22.80 \times 1.65) + (14.25 \times 1.83) \\
 &= 37.62 + 26.07 \\
 &= 63.69 \text{ KNm}
 \end{aligned}$$

$$M_2 = W_2 \times \text{Length}$$

$$= 32.40 \times 1.5$$

$$= 48.60 \text{ KNm}$$

$$M_3 = W_3 \times \text{Length}$$

$$= 132.50 \times 0.78$$

$$= 103.35 \text{ KNm}$$

$$M_4 = 107.2 \text{ KNm (Moment at base)}$$

$$\text{Total moment } M = M_1 + M_2 + M_3 + M_4$$

$$= 322.81 \text{ KNm}$$

Point of application

$$Z = \Sigma M / \Sigma W$$

$$= 322.81 / 201.95$$

$$= 1.6\text{m}$$

Eccentricity

$$e = Z - b/2$$

$$= 1.6 - (3/2)$$

$$= 0.1\text{m}$$

$$\text{i.e } b = 3 \text{ (width of base slab)}$$

$$e < b/6$$

$$b/6 = 3/6$$

$$= 0.5$$

$$0.1 < 0.5$$

Hence safe

Max and Min pressure at base

$$\begin{aligned}\sigma &= \Sigma W / b [1 \pm (6e / b)] \\ &= 201.95/3 [1 \pm (6 \times 0.1 / 3)]\end{aligned}$$

$$\begin{aligned}\sigma_{\max} &= 67.32 [1 + 0.2] \\ &= 80.78 \text{ KN/m}^2\end{aligned}$$

$$\begin{aligned}\sigma_{\min} &= 67.32 [1 - 0.2] \\ &= 53.85 \text{ KN/m}^2\end{aligned}$$

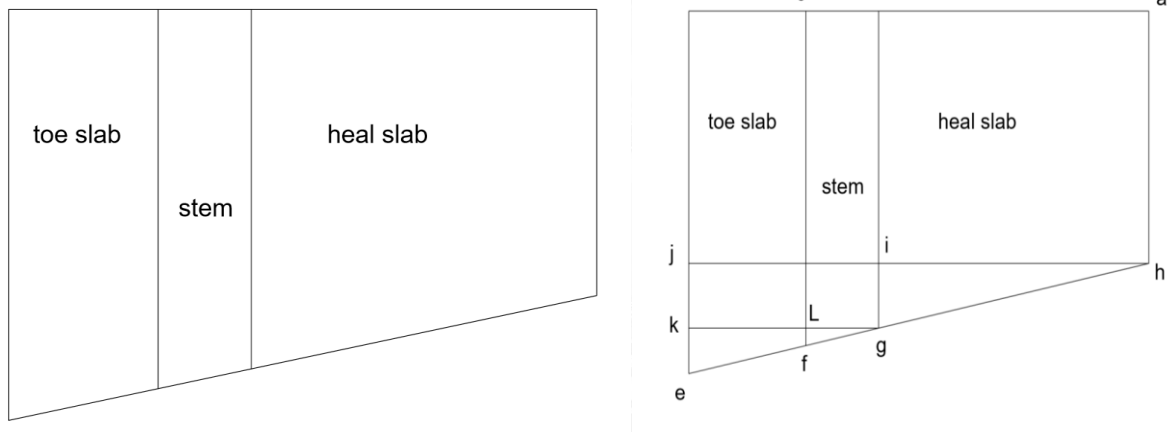


Fig.1.8 Cantilever retaining wall (Stability calculation Top view)

Step 3 : Design of Toe slab

(a) Find load (deductions)

Self weight of toe slab

$$\begin{aligned}\text{'Wd1'} &= B \times D \times \gamma_c \\ &= 1 \times 0.45 \times 24 \\ &= 10.8 \text{ KN}\end{aligned}$$

Self weight of soil over toe slab

$$\begin{aligned}
 'Wd2' &= b \times d \times \gamma_s \\
 &= 1 \times 0.75 \times 18 \\
 &= 13.50 \text{ KN}
 \end{aligned}$$

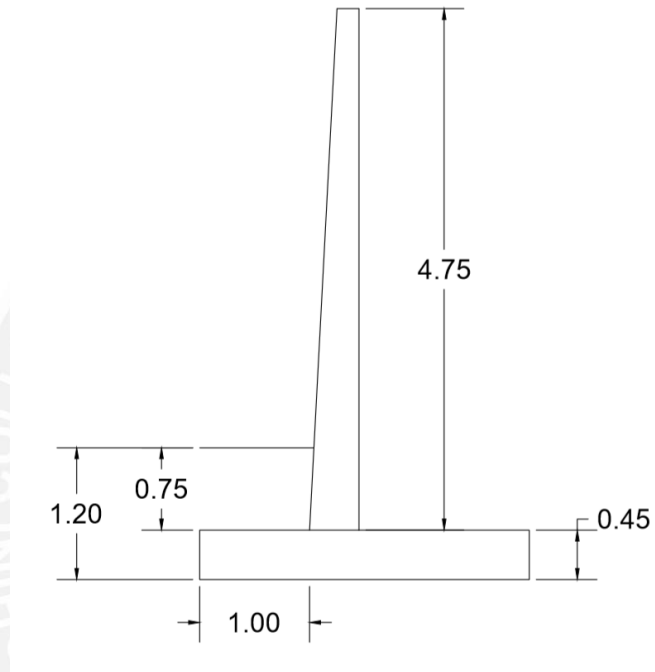


Fig.1.9 Cantilever retaining wall (Toe slab)

Moment deduction

$$\begin{aligned}
 Md1 &= W1 \times \text{length} \\
 &= 10.8 \times 0.5 \\
 &= 5.40 \text{ KNm}
 \end{aligned}$$

$$\begin{aligned}
 Md2 &= 13.50 \times 0.5 \\
 &= 6.75 \text{ KNm}
 \end{aligned}$$

$$\begin{aligned}
 Md &= M1 + M2 \\
 &= 5.40 + 6.75 \\
 &= 12.15 \text{ KNm}
 \end{aligned}$$

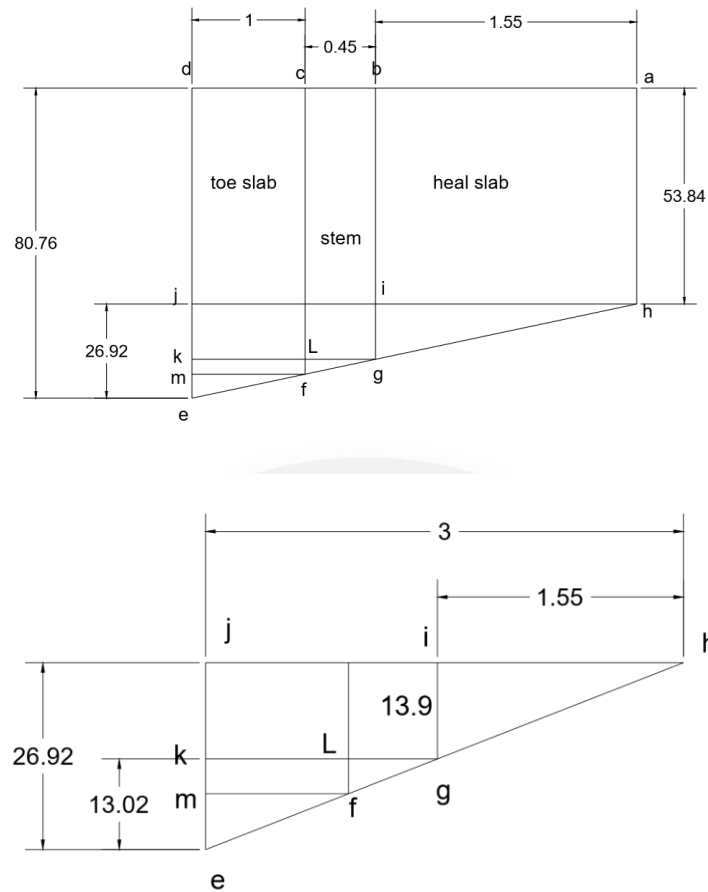


Fig.1.10 Cantilever retaining wall (Heal slab Top view)

(b) Upward pressure

$$\begin{aligned} \text{(cdfm) 'W 1'} &= \sigma_{\max} (\text{breadth}) \times d \\ &= 1 \times 71.78 \\ &= 71.78 \text{ KN} \end{aligned}$$

$$\begin{aligned} \text{(mfe) 'W 2'} &= \frac{1}{2} b \times h \\ &= \frac{1}{2} \times 1 \times 8.98 \\ &= 4.49 \text{ KN} \end{aligned}$$

Moment

$$\begin{aligned} \text{(cdfm) 'M 1'} &= W_1 \times \text{length} \\ &= 71.78 \times 0.5 \\ &= 35.89 \text{ KNm} \end{aligned}$$

$$\begin{aligned}
 (mfe) \text{ 'Md 2' } &= W 2 \times \text{length (triangular)} \\
 &= 4.49 \times 0.33 \\
 &= 1.496 \text{ KNm}
 \end{aligned}$$

$$\begin{aligned}
 M &= M1 + M2 \\
 &= 35.89 + 1.496 \\
 &= 37.38 \text{ KNm}
 \end{aligned}$$

$$\begin{aligned}
 \text{Bending moment 'M'} &= M - Md \\
 &= 37.38 - 12.15 \\
 &= 25.23 \text{ KNm}
 \end{aligned}$$

$$\begin{aligned}
 \text{Factored moment 'Mu'} &= 25.23 \times 1.5 \\
 &= 37.85 \text{ KNm}
 \end{aligned}$$

(c) Find Ast

$$Mu = (0.87 f_y Ast d) [(1 - Ast f_y) / (b d f_{ck})]$$

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$$37.85 \times 10^6 = (0.87 \times 415 \times Ast \times 400) [(1 - 415 \times Ast) / (1000 \times 400 \times 20)]$$

$$37.85 \times 10^6 = (144.42 \times 10^3 Ast) [(1 - 5.187 \times 10^{-5} Ast)]$$

$$37.85 \times 10^6 = (144.42 \times 10^3 Ast) - (7.49 Ast^2)$$

$$37.85 \times 10^6 - (144.42 \times 10^3 Ast) + (7.49 Ast^2) = 0$$

(using calculator) mode > Eqn > degree > 2

$$a = 7.49$$

$$b = -144.42 \times 10^3$$

$$c = 37.85 \times 10^6$$

$$x1 = 19015.96 \text{ mm}^2$$

$$x_2 = 265.7 \text{ mm}^2$$

$$A_{st} = 265.7 \text{ mm}^2$$

Find spacing

Provide 12mm dia bars

$$\begin{aligned} \text{Spacing} &= 1000 \times [(\pi d^2 / 4) / A_{st}] \\ &= 1000 \times [(\pi \times 12^2 / 4) / 265.7] \\ &= 425.65 \sim 300 \text{ mm (max)} \end{aligned}$$

Provide 12mm dia bars at 240mm c/c

Find distribution reinforcement

$$\begin{aligned} A_{st} (\text{dist}) &= (0.12 / 100) \times bD \\ &= (0.12 / 100) \times 1000 \times 450 \\ &= 540 \text{ mm}^2 \end{aligned}$$

Provide 12mm dia bars

$$\begin{aligned} \text{Spacing} &= 1000 \times (\pi d^2 / 4) / A_{st} \\ &= 1000 \times [(\pi \times 12^2 / 4) / 540] \\ &= 209 \text{ mm} \sim 210 \text{ mm} \end{aligned}$$

Provide 12mm dia bars at 210mm c/c

Step 4 : Check for safety against sliding

$$\begin{aligned} P &= K_a \times \gamma (H^2 / 2) \\ &= (1/3) \times 18 \times (5.2^2 / 2) \\ &= 81.12 \text{ KN} \end{aligned}$$

$$\text{i.e } K_a = (1 - \sin \phi / 1 + \sin \phi)$$

$$\text{F.O.S against sliding} = (\mu W / P)$$

$$= (0.5 \times 201.95 / 81.12)$$

$$= 1.24 < 1.5$$

$$\mu = 0.5 \text{ (given)}$$

Since the wall is unsafe, so a shear key is to be designed below the stem

Step 5 : Design of shear key

Intensity of passive pressure in shear key front

$$P_p = K_p \times (\sigma_{\max}) \text{ pressure in shear key front}$$

$$K_p = (1 + \sin \phi / 1 - \sin \phi)$$

$$= (1 + \sin 30 / 1 - \sin 30)$$

$$= 3$$

$$P_p = K_p \times (\sigma_{\max}) \text{ pressure in shear key front}$$

$$= 3 \times 71.78$$

$$= 215.34 \text{ KN/m}^2$$

$$\text{Passive force PF} = P_p \times a$$

$$= 215.34 \times 0.45$$

$$= 97 \text{ KN}$$

$$\text{F.O.S against sliding} = [(\mu W + PF) / P]$$

$$= \{[(0.5 \times 201.95) + 97] / 81.12\}$$

$$= 2.4 > 1.5$$

Hence safe

Minimum % of reinforcement in shear key

$$A_{st} = (0.3/100) \times bD$$

$$= 0.003 \times 1000 \times 450$$

$$= 1350 \text{ mm}^2$$

Provide 16mm dia bars

$$\text{Spacing} = 1000 \times (\pi d^2 / 4) / A_{st}$$

$$= 1000 \times [(\pi \times 16^2 / 4) / 1350]$$

$$= 148.9 \text{ mm} \sim 150 \text{ mm}$$

Provide 16mm dia bars at 150mm c/c

Step 6 : Find shear stress

$$\begin{aligned} \text{Shear force 'V'} &= 1.5P - \mu W \\ &= (1.5 \times 81.12) - (0.5 \times 201.95) \\ &= 20.7 \text{ KN} \end{aligned}$$

Factored Shear force

$$\begin{aligned} \text{'V}_u\text{' } &= 20.7 \times 1.5 \\ &= 31.05 \text{ KN} \end{aligned}$$

$$\begin{aligned} \text{Shear stress '}\tau_v\text{' } &= V_u / bd \\ &= 31.05 \times 10^3 / (1000 \times 400) \\ &= 0.077 \text{ N/mm}^2 \end{aligned}$$

Find τ_c

$$\begin{aligned} 100A_{st} / bd &= 100 \times 1350 / (1000 \times 400) \\ &= 0.335 \text{ N/mm}^2 \end{aligned}$$

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$$0.25 \text{ --- } 0.36$$

$$0.50 \text{ --- } 0.48$$

$$(0.36+0.48) / 2 = 0.42$$

$$\tau_c = 0.42 \text{ N/mm}^2$$

$$\tau_c > \tau_v$$

Hence safe



Reinforcement detail

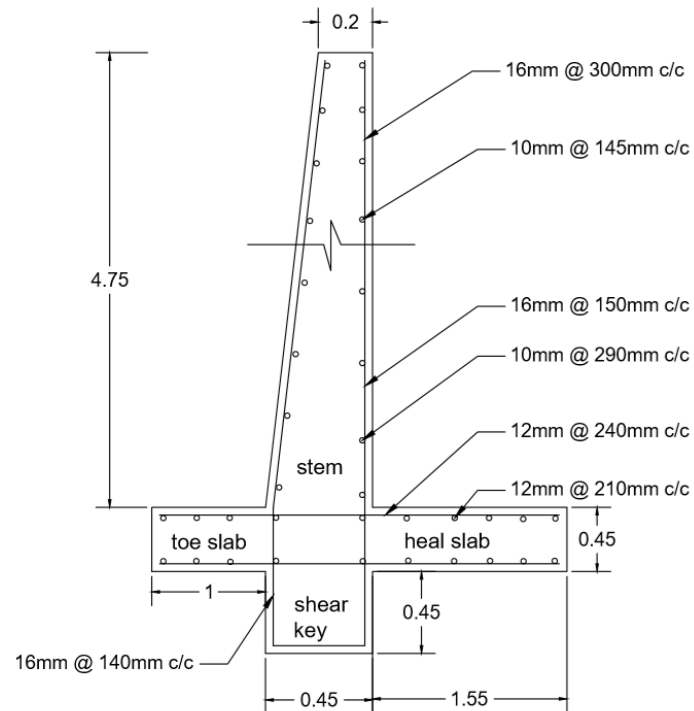


Fig.1.11 Cantilever retaining wall (Reinforcement details cross section)

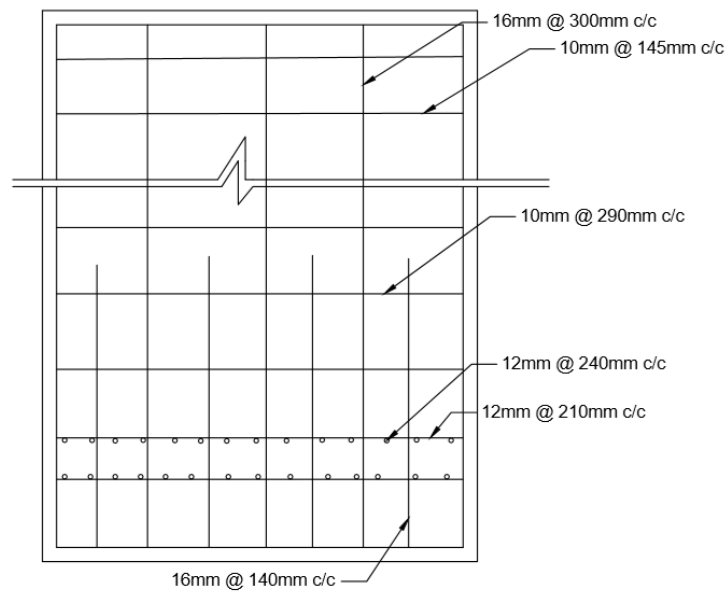


Fig.1.12 Cantilever retaining wall (Reinforcement details Longitudinal cross section)