

1.8. UNIFORM BENDING

ELEVATION AT THE CENTRE OF THE BEAM LOADED AT BOTH ENDS

DEFINITION

The beam is loaded uniformly on its both ends; the bent beam forms an arc of a circle. The elevation is produced in the beam. This type of bending is known as uniform bending.

THEORY

Let us consider a beam AB of negligible mass, supported symmetrically on two knife edges C and D. It is loaded with equal weight W at each end. Let ' l ' be the length between the two knife edges and ' a ' be the length between the knife edge and the load. Due to the applied load the beam bends into an arc of circle and produces an elevation ' y '. Let P be any section of the beam.

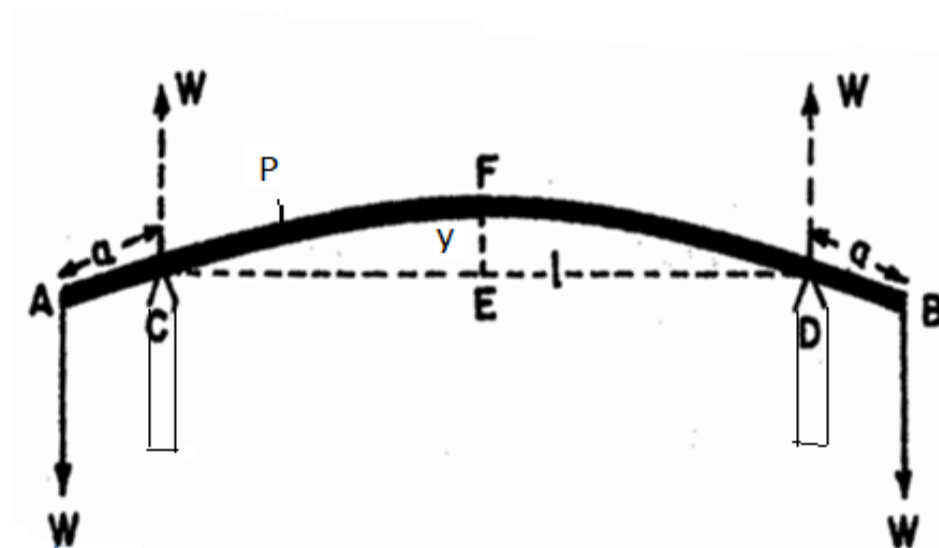


Fig 1.8.1 Young's modulus of the beam by uniform bending.

At the equilibrium position of the section PA of the beam two equal forces, the applied load W at A (downward) and the normal reaction W at C (upward) are acting in the opposite direction constitute a couple.

$$\begin{aligned}\text{The External bending moment} &= (W \times AE) - (W \times CE) \\ &= W(AE - CE) \\ &= W \times AC = Wa \quad \dots\dots\dots (1)\end{aligned}$$

Where, R is the radius of curvature.

$$\text{Internal bending moment} = \frac{Y}{R} I_g \quad \dots\dots\dots (2)$$

Where Y —Young's modulus of the cantilever.

I_g - Geometrical moment of inertia of its cross-section.

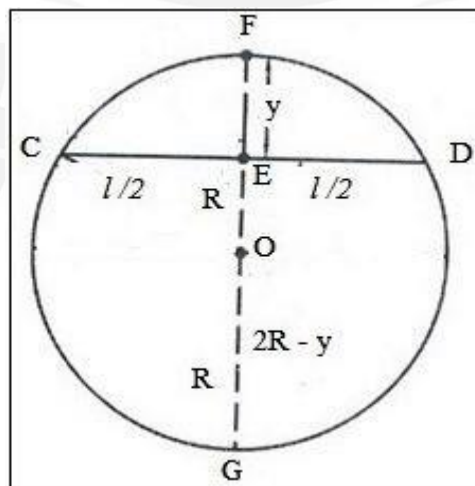
R - Radius of the curvature of the neutral axis at P.

At Equilibrium,

External bending moment = Internal bending moment

$$Wa = \frac{Y}{R} I_g \quad \dots\dots\dots (3)$$

Since, Wa is a constant, R is also constant. Therefore the beam bends into an arc of a circle of radius R . Hence the bending in this case is said to be uniform.



From the property of circle, $CE \times ED = EF \times EG$

$$\frac{l}{2} \times \frac{l}{2} = y(2R - y)$$

$$\frac{l^2}{4} = 2Ry - y^2 \quad \dots\dots\dots (4)$$

Here y is the elevation produced in the beam. Since the elevation is very small compared to the radius of curvature of the circle, y^2 can be neglected.

$$\frac{l^2}{4} = 2Ry$$

$$R = \frac{l^2}{8y} \quad \dots\dots\dots (5)$$

Substituting equation (5) in equation (3)

$$Wa = \frac{YI_g}{\left[\frac{l^2}{8y} \right]}$$

Young's modulus by uniform bending $Y = \frac{Wal^2}{8yI_g} \quad \dots\dots\dots (6)$
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SPECIAL CASES

RECTANGULAR CROSS SECTION

If b and d are the breadth and thickness of the beam, then $A = bd$ and $K^2 = \frac{d^2}{12}$

Substituting the values of $I_g = AK^2 = I_g = \frac{bd^3}{12}$ and $W = Mg$ in equation (6)

$Y = \frac{3Mgal^2}{2bd^3y} \quad \dots\dots\dots (7)$
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Young's modulus of the material of the beam by uniform bending can be calculated as using the relation (7).

CIRCULAR CROSS SECTION

If r be the radius of the beam, then $A = \pi r^2$ and $K^2 = \frac{r^2}{4}$

Substituting the values of $I_g = AK^2 = \frac{\pi r^4}{4}$ and $W = Mg$ in equation (6)

$Y = \frac{Wal^2}{8yI_g} = \frac{Mgal^2}{2y\pi r^4} \quad \dots\dots\dots (8)$
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EXPERIMENTAL DETERMINATION

CONSTRUCTION

A rectangular beam of uniform –section is supported horizontally on two knife –edges A and B as shown in Figure. Two weight hangers of equal masses are suspended from the ends of the beam. A pin is arranged vertically at the mid-point of the beam. A microscope is focused on the tip of the pin.

PROCEDURE

Dead loads are attached to the hangers. The microscope is adjusted such that the horizontal cross-wire coincides with the tip of the image of the pin and the readings on the vertical scale are taken. Equal weights in steps of 50g are added to both hangers simultaneously and the reading of the microscope in the vertical scale is noted. The experiment is repeated for decreasing order of magnitude of the equal masses. The observations are then tabulated and the mean elevation(y) at the mid-point of the bar is determined. The length of the bar between the knife edges ' l ' is measured. The distance of one of the weight hangers from the nearest knife edge ' a ' is measured. The breadth (b) and thickness (d) of the bar are measured by using vernier calipers and screw gauge. The young's modulus of the material of the beam is determined by the relation.

$$Y = \frac{3M g a l^2}{2b d^3 y}$$

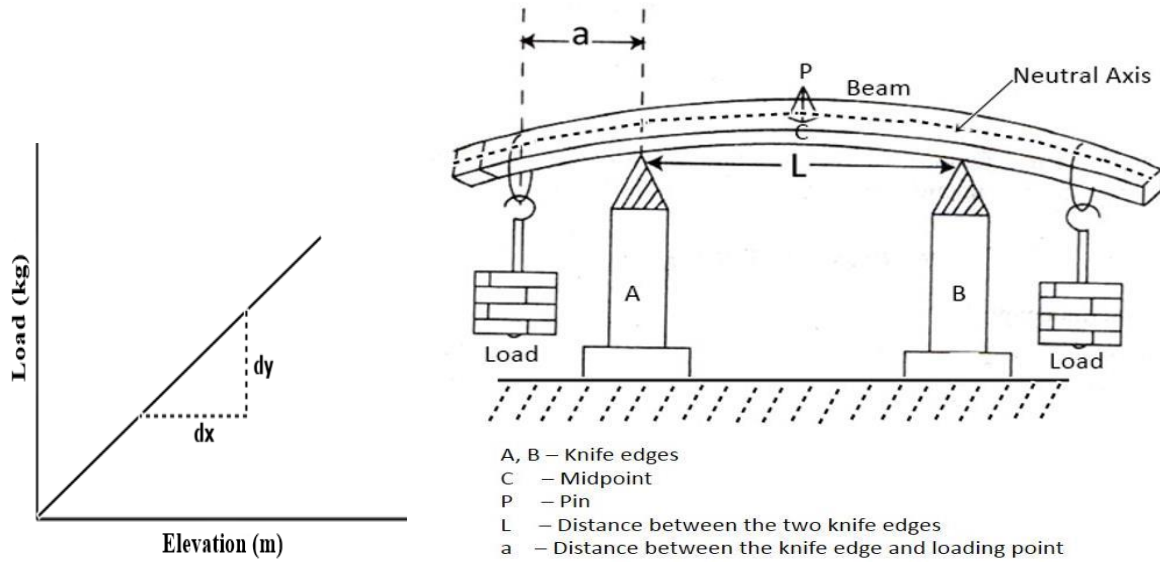


Fig 1.8.3(a) Graphical representation of Young's modulus of the beam by uniform bending.

Fig1.8.3 (b) Experimental arrangement.

Table: Experimental verification of Young's modulus by uniform bending

Load $10^{-3} kg$	Microscope readings							elevation (y) for100 g $10^{-2} m$	Mean (M/y) Kgm ⁻¹
	Loading			Unloading			Mean $10^{-2} m$		
	MSR cm	VSR cm	TR cm	MSR cm	VSR cm	TR Cm			
W									
W+50									
W+100									
W+150									
W+200									

1.9. NON-UNIFORM BENDING

DEFINITION

If the beam is loaded at its midpoint, the depression is produced will not form an arc of a circle. This type of bending is called non uniform bending.

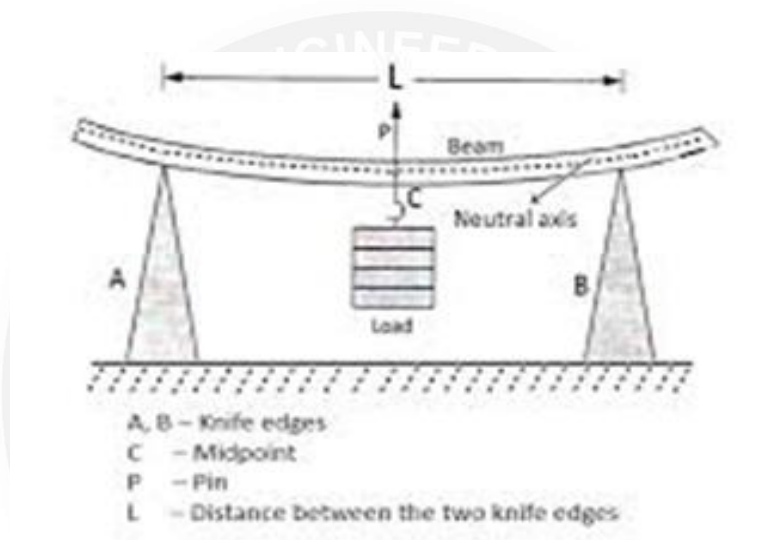


Fig1.9.1.Young's modulus of the beam by non-uniform bending.

THEORY

Consider a uniform beam AB of length L arranged horizontally on two knife edges at its ends A and B. Let it be loaded in the midpoint of the beam with a weight W. The reaction at each Knife edge is equal to $\left(\frac{W}{2}\right)$ in the upward direction. Let 'y' be the depression at the midpoint C.

The bent beam is considered as equivalent to two inverted cantilevers fixed at C each of length $\left(\frac{l}{2}\right)$ and each loaded at A and B with a weight $\left(\frac{W}{2}\right)$ acting vertically upwards. In the case of a

cantilever of length (l) and load (w), the depression = $\frac{Wl^3}{3YI_g}$

Hence for cantilever of length $\left(\frac{l}{2}\right)$ and load $\left(\frac{W}{2}\right)$ depression is $y = \frac{\left(\frac{W}{2}\right)\left(\frac{l}{2}\right)^3}{3YI_g}$

$$y = \frac{Wl^3}{48YI_g} \dots\dots\dots (1)$$

Young's modulus by non-uniform bending $Y = \frac{Wl^3}{48yI_g} \dots\dots\dots (2)$
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SPECIAL CASES

RECTANGULAR CROSS SECTION

If b and d are the breadth and thickness of the beam, then $A = bd$ and $K^2 = \frac{d^2}{12}$

Substituting the values of $I_g = AK^2 = I_g = \frac{bd^3}{12}$ and $W = Mg$ in equation (2)

$$\text{Equation (2)} \rightarrow Y = \frac{Wl^3}{48yI_g} = \frac{Mgl^3}{48y\left(\frac{bd^3}{12}\right)}$$

$\therefore Y = \frac{Mgl^3}{4bd^3y} \text{ Nm}^{-2} \dots\dots\dots (3)$

Young's modulus of the material of the rectangular beam by non-uniform bending can be calculated as using the relation (3).

CIRCULAR CROSS SECTION

If r be the radius of the beam, then $A = \pi r^2$ and $K^2 = \frac{r^2}{4}$

Substituting the values of $I_g = AK^2 = \frac{\pi r^4}{4}$ and $W = Mg$ in equation (2)

$Y = \frac{Wl^3}{48yI_g} \Rightarrow Y = \frac{Mgl^3}{12y\pi r^4} \dots\dots\dots (4)$
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Young's modulus of the material of the circular beam by non-uniform bending can be calculated as using the above relation (4).

EXPERIMENT ARRANGEMENT

The given beam AB of rectangular cross section is arranged horizontally on two knife edges k_1 and k_2 near the ends A and B. A weight hanger is suspended and a pin is fixed vertically at midpoint O. A microscope is focused on the tip of the pin. The initial reading on the vertical scale of the microscope is taken. A suitable mass M is added to the hanger. The beam is depressed. The cross wire is adjusted to coincide with the tip of the pin. The reading of the microscope is noted. The depression corresponding to the mass M is found.

The experiment is repeated by increasing and decreasing the mass step by step. The corresponding readings are tabulated. The average value of depression y is found from the observation.

Table:1.6 Experimental verification of Young's modulus by non-uniform bending

Load $10^{-3} kg$	Microscope readings						Mean depression (y) for 100 g $10^{-2} m$	Mean (M/y) Kgm^{-1}	
	Loading			Unloading					Mean $10^{-2} m$
	MSR cm	VSR cm	TR cm	MSR cm	VS R cm	TR cm			
W									
W+50									
W+100									
W+150									
W+200									

The breadth (b) and thickness (d) of the bar are measured by using Vernier calipers and screw gauge. The young's modulus of the material of the beam is determined by the relation,

$$Y = \frac{Mgl^3}{4bd^3y} \text{ Nm}^{-2}$$

1.10. I-SHAPE GIRDERS

The elastic behavior of materials plays an important role in everyday life. For example while designing a building, the structural design of the columns, beams and supports require knowledge of strength of materials used. Have you ever thought why the beams used in construction of bridges; supports etc. have a cross-section of the type I?

A girder is a support beam used in construction. Girder is the term used to denote the main horizontal support of a structure, which supports smaller beams. A girder is commonly used more in the building of bridges and planes.

The girders with upper and lower section broadened and the middle section tapered, so that it can withstand heavy loads over it. The cross section of the girder takes the shape of the capital letter I and hence it is called as I-shape girders.

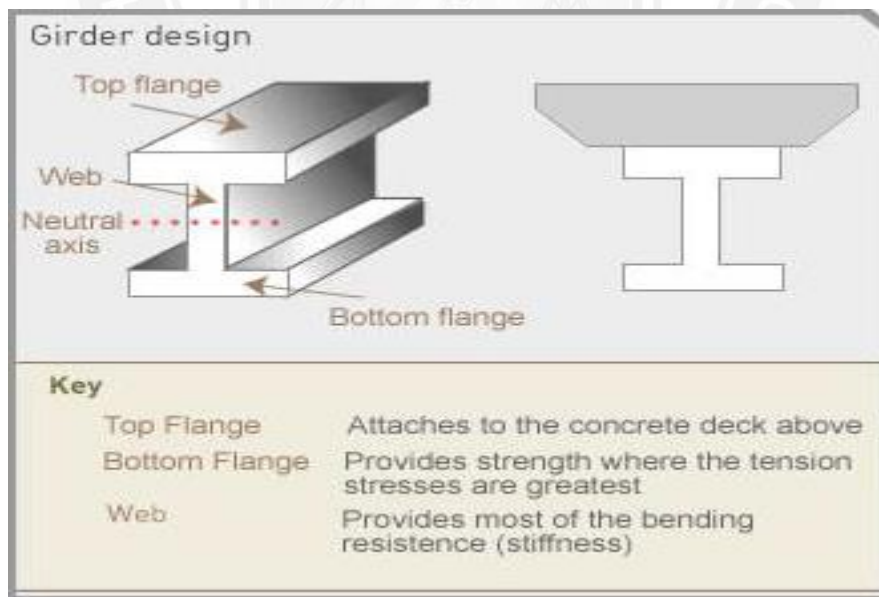


Figure. 1.10 I-shape girders.

The vertical plate in the middle is known as the web and the top and bottom plates are referred to as flanges, steel is one of the most common materials used to make I-beams, since it can withstand very heavy loads, although other materials, such as aluminum are sometimes used. When a beam is used as a girder for a given load, depression must be minimum.

- We know that the depression at the mid-point of a beam loaded at that point is given by

$$Y = \frac{Mgl^3}{4bd^3y} \text{ Nm}^{-2}$$

Here, the depression can be minimized by,

- Decreasing the load (Mg)
- Decreasing the length (l)
- Increasing the Young's modulus (Y)
- Increasing the breadth (b)
- Increasing the thickness of the girder (d)

Since the length (l) and Young's modulus (Y) of the beam are the fixed quantity, it cannot be altered.

- Therefore the breadth and thickness are adjusted to minimize the depression.
- Thus the girders are made of I shape and are called I shape girders.

APPLICATIONS OF I-SHAPE GIRDERS

- Support beam for commercial and residential construction.
- Support frames and columns for trolley ways, lifts and hoists.
- Construction of platforms.
- Trailer and truck bed framing.
- Construction of bridges.
- Machine bases.
- Iron rails include in railway tracks.