5.2 CONVERSION OF STATE VARIABLE MODELS TO TRANSFER FUNCTIONS

The state model of a system consists of state equation and output equation. (or) the state equation and output equation together called as state model of the system.

$$\dot{X}(t) = A\,X(t) + B\,U(t) ------- \boldsymbol{state\ equation}$$

$$Y(t) = C\,X(t) + D\,U(t) ------- \boldsymbol{output\ equation}$$

Where

X (t) = state vector of order (n x 1)

U (t) = Input vector of order (m x 1)

A = System matrix of order (n x n)

B = Input matrix of order (n x m)

Y (t) = Output vector of order (p x 1)

C = Output matrix of order (p x n)

D = Transmission matrix of order (p x m)

Taking Laplace transform (with zero initial condition) in state equation and output equation

$$sX(s) = AX(s) + BU(s)$$

$$Y(s) = CX(s) + DU(s)$$

The state equation can be placed in the form

$$(sI - A)\,X(s) = BU(s)$$

Premultiply both sides by $(sI-A)^{-1}$

$$X(s) = (sI - A)^{-1}B\,U(s)$$

Subtitling X(s) in the output equation

$$Y(S) = [C(sI - A)^{-1}B + D]\,U(s)$$

Hence transfer function Matrix $T(s) = [C(sI - A)^{-1}B + D]$

EXAMPLE:

State space model is given by

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$Y = [1 \quad 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [0]\,u$ .Find the transfer functions of the system.

Let compare given state space model equation with standard state space model equation,

$$\dot{X}(t) = A\,X(t) + B\,U(t) ------\boldsymbol{state\ equation}$$

$$Y(t) = C\,X(t) + D\,U(t) ------\boldsymbol{output\ equation}$$

Hence

$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$; $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$; $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$; $D = [0]$;

W.K.T,

$$T(s) = [C(sI - A)^{-1}B + D]$$

$$T(s) = \begin{bmatrix} 1 & 0 \end{bmatrix} \left(\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \right)^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + [0]$$

$$T(s) = \frac{1}{s^2 + 3s + 2}$$