UNIT III KINEMATICS, DYNAMICS AND DESIGN OF ROBOTS & END-EFFECTORS

3.4 KINEMATICS EQUATIONS:

A fundamental tool in robot kinematics is the kinematics equations of the kinematic chains that form the robot. These non-linear equations are used to map the joint parameters to the configuration of the robot system. Kinematics equations are also used in biomechanics of the skeleton and computer animation of articulated characters.

Forward kinematics uses the kinematic equations of a robot to compute the position of the end-effector from specified values for the joint parameters. The reverse process that computes the joint parameters that achieve a specified position of the end-effector is known as inverse kinematics. The dimensions of the robot and its kinematics equations define the volume of space reachable by the robot, known as its workspace.

There are two broad classes of robots and associated kinematics equations serial manipulators and parallel manipulators. Other types of systems with specialized kinematics equations are air, land, and submersible mobile robots, hyper-redundant, or snake, robots and humanoid robots.

3.5 DENAVIT-HARTENBERG PARAMETERS:

The Denavit–Hartenberg parameters (also called DH parameters) are the four parameters associated with a particular convention for attaching reference frames to the links of a spatial kinematic chain, or robot manipulator.

Denavit-Hartenberg convention:

A commonly used convention for selecting frames of reference in robotics applications is the **Denavit and Hartenberg (D–H) convention**. In this convention, coordinate frames are attached to the joints between two links such that one transformation is associated with the joint, [Z], and the second is associated with the link [X]. The coordinate transformations along a serial robot consisting of *n* links form the kinematics equations of the robot,

$$[T] = [Z_1][X_1][Z_2][X_2]\dots[X_{n-1}][Z_n],$$

Where, [T] is the transformation locating the end-link.

In order to determine the coordinate transformations [Z] and [X], the joints connecting the links are modeled as either hinged or sliding joints, each of which have a unique line S in space that forms the joint axis and define the relative movement of the two links. A typical serial robot is characterized by a sequence of six lines S_i, *i=1,...,6,* one for each joint in the robot. For each sequence of lines S_i and S_{i+1}, there is a common normal line $A_{i,i+1}$. The system of six joint axes S_i and five common normal lines $A_{i,i+1}$ form the kinematic skeleton of the typical six degree of freedom serial robot. Denavit and Hartenberg introduced the convention that Z coordinate axes are assigned to the joint axes S_i and X coordinate axes are assigned to the common normal's $A_{i,i+1}$.

This convention allows the definition of the movement of links around a common joint axis S_i by the screw displacement,

$$[Z_i] = \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & 0 \\ \sin\theta_i & \cos\theta_i & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

Where ϑ_i is the rotation around and d_i is the slide along the Z axis---either of the parameters can be constants depending on the structure of the robot. Under this convention the dimensions of each link in the serial chain are defined by the screw displacement around the common normal $A_{i,i+1}$ from the joint S_i to S_{i+1}, which is given by

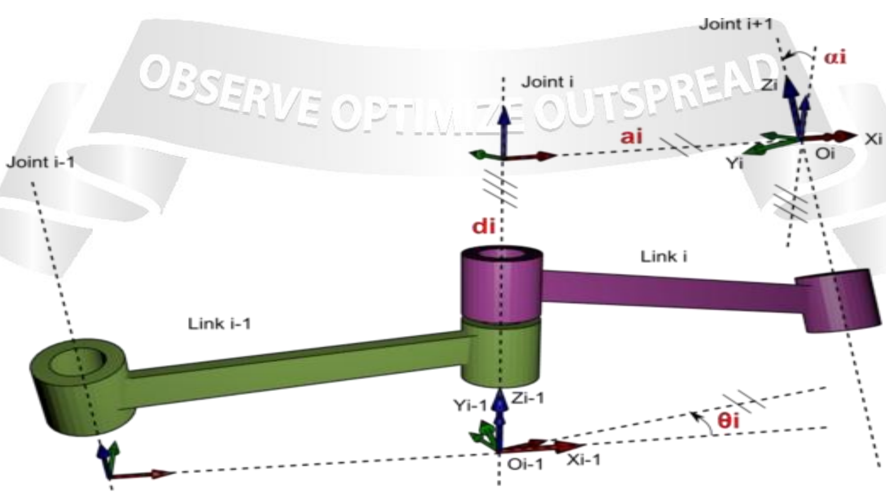
$$[X_i] = \begin{bmatrix} 1 & 0 & 0 & r_{i,i+1} \\ 0 & \cos\alpha_{i,i+1} & -\sin\alpha_{i,i+1} & 0 \\ 0 & \sin\alpha_{i,i+1} & \cos\alpha_{i,i+1} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

Where $\alpha_{i,i+1}$ and $r_{i,i+1}$ define the physical dimensions of the link in terms of the angle measured around and distance measured along the X axis.

In summary, the reference frames are laid out as follows:

- the -axis is in the direction of the joint axis
- the -axis is parallel to the common normal: $x_n = z_{n-1} \times z_n$ If there is no unique common normal (parallel axes), then d (below) is a free parameter. The direction of is from to z_n, as shown in the video below.
- the -axis follows from the - and -axis by choosing it to be a right-handed coordinate system.

Four parameters



The four parameters of classic DH convention are θ_i, d_i, a_i, α_i. With those four parameters, we can translate the coordinates from $O_{i-1}X_{i-1}Y_{i-1}Z_{i-1}$ to $O_iX_iY_iZ_i$.

The transformation the following four parameters known as D–H parameters:

d: offset along previous ***z*** to the common normal

θ: angle about previous ***z***, from old **x** to new **x**

r: length of the common normal. Assuming a revolute joint, this is the radius about previous z.

α: angle about common normal, from old ***z*** axis to new ***z*** axis

There is some choice in frame layout as to whether the previous ***x*** axis or the next **x** points along the common normal. The latter system allows branching chains more efficiently, as multiple frames can all point away from their common ancestor, but in the alternative layout the ancestor can only point toward one successor. Thus the commonly used notation places each down-chain ***x*** axis collinear with the common normal, yielding the transformation calculations shown below.

We can note constraints on the relationships between the axes:

- x_n-axis is perpendicular to both the z_{n-1} and z_n axes
- x_n-axis intersects both z_{n-1} and z_n axes
- Origin of joint   is at the intersection of x_n and z_n
- y_n completes a right-handed reference frame based on x_n and z_n

Denavit-Hartenberg Matrix:

It is common to separate a screw displacement into the product of a pure translation along a line and a pure rotation about the line,[5][6] so that

$$[Z_i] = \mathrm{Trans}_{Z_i}(d_i)\,\mathrm{Rot}_{Z_i}(\theta_i),$$

And,

$$[X_i] = \mathrm{Trans}_{X_i}(r_{i,i+1})\,\mathrm{Rot}_{X_i}(\alpha_{i,i+1}).$$

Using this notation, each link can be described by a coordinate transformation from the previous coordinate system to the next coordinate system.

$${}^{n-1}T_n = \mathrm{Trans}_{z_{n-1}}(d_n)\cdot\mathrm{Rot}_{z_{n-1}}(\theta_n)\cdot\mathrm{Trans}_{x_n}(r_n)\cdot\mathrm{Rot}_{x_n}(\alpha_n)$$

Note that this is the product of two **screw** displacements, the matrices associated with these operations are:

$$\mathrm{Trans}_{z_{n-1}}(d_n) = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_n \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$\mathrm{Rot}_{z_{n-1}}(\theta_n) = \left[\begin{array}{ccc|c} \cos\theta_n & -\sin\theta_n & 0 & 0 \\ \sin\theta_n & \cos\theta_n & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$\mathrm{Trans}_{x_n}(r_n) = \left[\begin{array}{ccc|c} 1 & 0 & 0 & r_n \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$\mathrm{Rot}_{x_n}(\alpha_n) = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha_n & -\sin\alpha_n & 0 \\ 0 & \sin\alpha_n & \cos\alpha_n & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

This gives:

$${}^{n-1}T_n = \left[\begin{array}{ccc|c} \cos\theta_n & -\sin\theta_n\cos\alpha_n & \sin\theta_n\sin\alpha_n & r_n\cos\theta_n \\ \sin\theta_n & \cos\theta_n\cos\alpha_n & -\cos\theta_n\sin\alpha_n & r_n\sin\theta_n \\ 0 & \sin\alpha_n & \cos\alpha_n & d_n \\ \hline 0 & 0 & 0 & 1 \end{array}\right] = \left[\begin{array}{ccc|c} & R & & T \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

Where *R* is the 3×3 sub matrix describing rotation and *T* is the 3×1 sub matrix describing translation.

DENAVIT-HARTENBERG REPRESENTATION OF FORWARD KINEMATIC EQUATIONS OF ROBOT:

Denavit-Hartenberg Representation:

1. Simple way of modeling robot links and joints for any robot configuration, regardless of its sequence or complexity.
2. Transformations in any coordinates are possible.
3. Any possible combinations of joints and links and all-revolute articulated robots can be represented

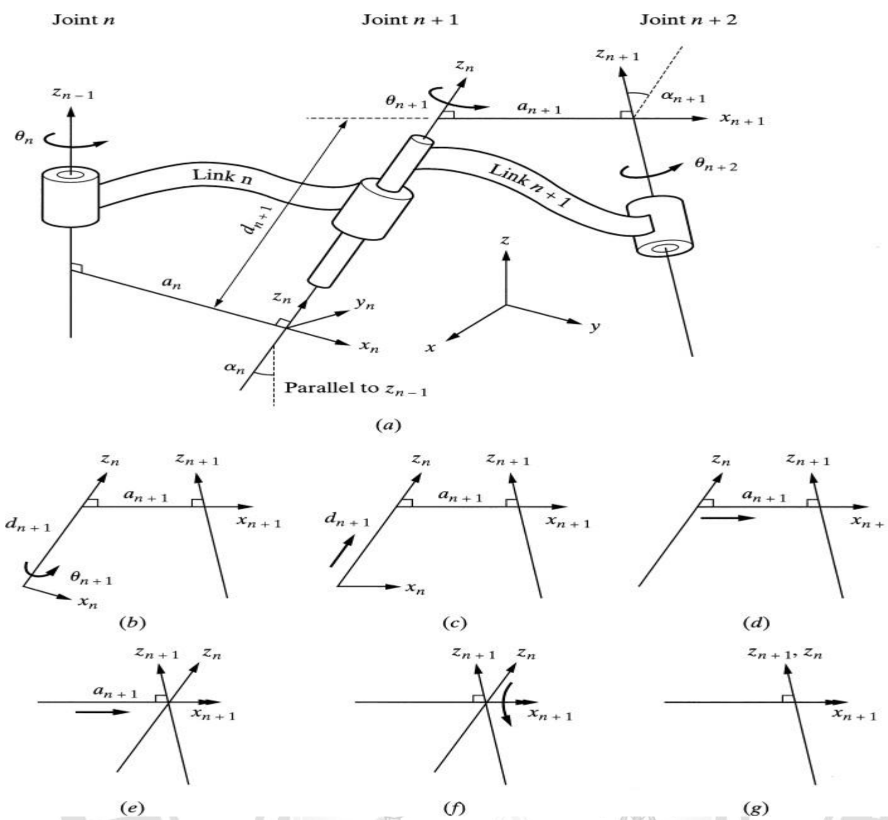


Fig 3.16 a D-H representation of a general-purpose joint-link combination

DENAVIT-HARTENBERG REPRESENTATION PROCEDURES:

Start point:

- Assign joint number *n* to the first shown joint.
- Assign a local reference frame for each and every joint before or after these joints.
- *Y*-axis does not used in D-H representation.

Procedures for assigning a local reference frame to each joint:

All joints are represented by a *z*-axis. (Right-hand rule for rotational joint, linear movement for prismatic joint)

- The common normal is one line mutually perpendicular to any two skew lines.
- Parallel *z*-axes joints make a infinite number of common normal.
- Intersecting *z*-axes of two successive joints make no common normal between them(Length is 0.).

Symbol Terminologies:

- θ : A rotation about the *z*-axis.
- *d* : The distance on the *z*-axis.
- *a* : The length of each common normal (Joint offset).
- α : The angle between two successive *z*-axes (Joint twist)

Only θ and *d* are joint variables

The necessary motions to transform from one reVference frame to the next.

I) Rotate about the z_n-axis an able of θ_{n+1}. (Coplanar)

II) Translate along z_n-axis a distance of d_{n+1} to make x_n and x_{n+1} colinear.

III) Translate along the x_n-axis a distance of a_{n+1} to bring the origins of x_{n+1} together.

IV) Rotate z_n-axis about x_{n+1} axis an angle of α_{n+1} to align z_n-axis with z_{n+1}-axis.

Determine the value of each joint to place the arm at a desired position and orientation.

$$^{R}T_{H} = A_1 A_2 A_3 A_4 A_5 A_6$$