5.4 EFFECTS OF P, PI, PID MODES OF FEEDBACK CONTROL

PROPORTIONAL CONTROLLER (P-Controller)

The proportional controller is a device that produces a control signal, u(t) proportional to the input error signal, e(t)

$$u(t) \propto e(t)$$

$$u(t) = K_p e(t)$$

where, K_p = Proportional gain or constant

On taking Laplace transform of equation, we get,

$$U(s) = K_p E(s)$$

Transfer function,

$$\frac{U(s)}{E(s)} = K_p$$

The equation gives the output of the P-controller for the input E(s) and it is the transfer function of P-controller. The block diagram of the P-controller is shown in the figure 5.4.1.

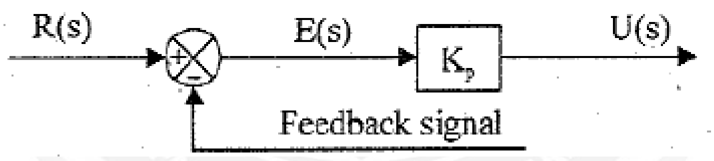

Figure 5.4.1 Block diagram of proportional controller

[Source: "Control Systems" by Nagoor Kani, Page: 2.79]

From the equation, we can conclude that the proportional controller amplifies the error signal by an amount K_p. Also the introduction of the controller on the system increases the loop gain by an amount K_p. The increase in loop gain improves the steady state tracking accuracy, disturbance signal rejection and the relative stability and also makes the system less sensitive to parameter variations. But increasing the gain to very large values mau lead to instability of the system. The drawback in P-controller is that it leads to a constant steady state error.

Example of Electronic P-controller

The proportional controller can be realized by an amplifier with adjustable gain. Either the non- inverting operational amplifier or the inverting operational amplifier followed by sign changer will work as a proportional controller. The op-amp proportional controller is shown in the figures 5.4.2.

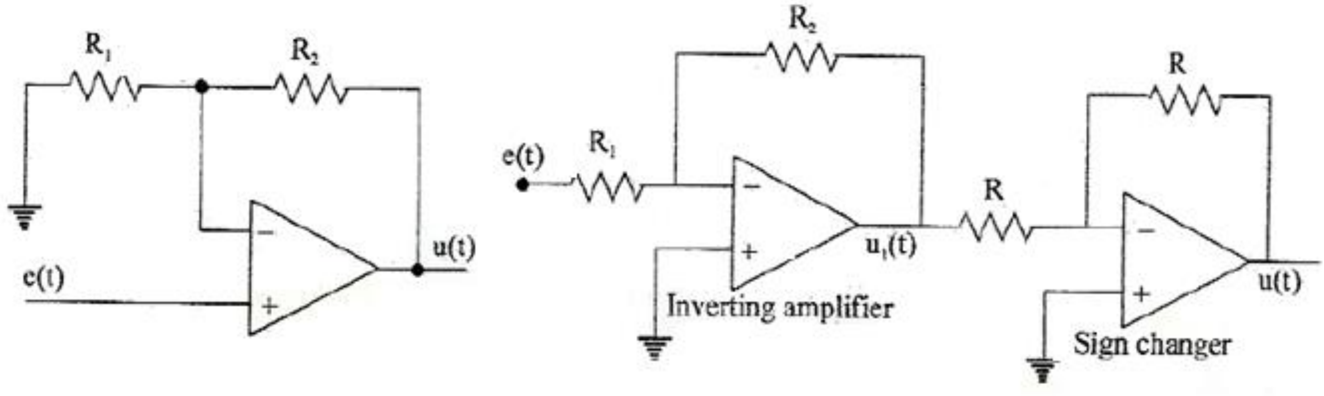

Figure 5.4

5.4.2 P-controller using non-inverting and inverting amplifier

[Source: "Control Systems" by Nagoor Kani, Page: 2.80]

By deriving the transfer function of the controller shown in figures and comparing with the transfer function of P-controller defined by equation, it can be shown that they work as P-controllers.

Analysis of P-controller

In figure 2.8.2, the input e(t) is applied to positive input. By symmetry of op-amp the voltage of negative input is also e(t). Also, we assume an ideal op-amp so that input current is zero. Based on the above assumptions the equivalent circuit of the controller is shown in figure 5.4.3.

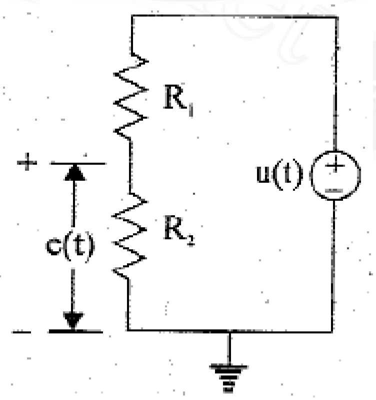

Figure 5.4.3 Equivalent circuit of P-controller

[Source: "Control Systems" by Nagoor Kani, Page: 2.80]

By voltage division rule,

$$e(t) = \frac{R_1}{R_1 + R_2} u(t)$$

On taking Laplace transform of equation we get,

$$\frac{U(s)}{E(s)} = \frac{R_1 + R_2}{R_1}$$

The equation is the transfer function of op-amp P-controller. On comparing, we get,

$$K_p = \frac{R_1 + R_2}{R_1}$$

Therefore, by adjusting the values of R_1 and R_2 the value of gain, K_p can be varied.

Analysis of P-controller

The assumption made in op-amp circuit analysis are,

1. The voltages at both inputs are equal
2. The input current is zero

Based on the above assumptions, the equivalent circuit of op-amp amplifier and sign changer are shown in figure 5.4.4.

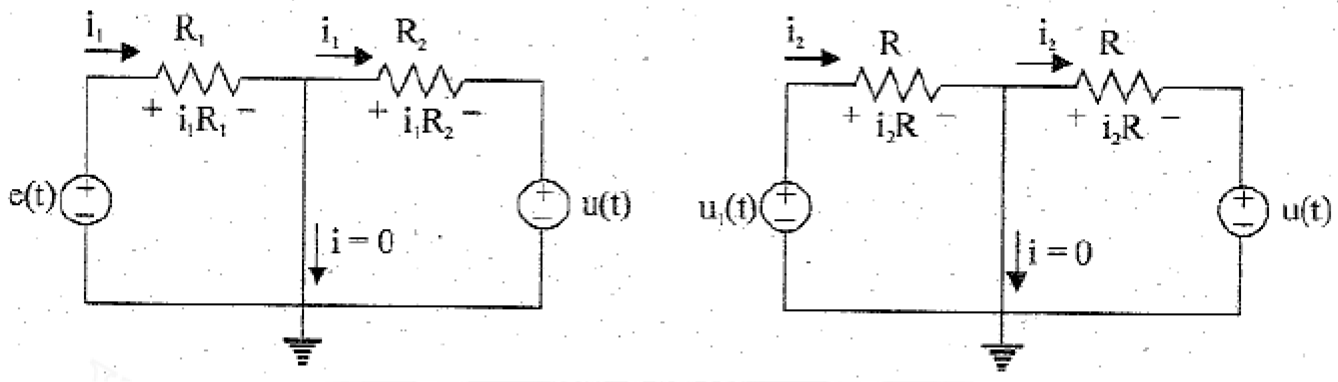

Fig 5.4.4 Equivalent circuit of amplifier and sign changer

[Source: "Control Systems" by Nagoor Kani, Page: 2.81]

From the circuit,

$$e(t) = i_1 R_1$$

$$u_1(t) = -i_1 R_2$$

Substitute for i_1,

$$u_1(t) = -\frac{e(t)}{R_1} R_2$$

Also, from the circuit,

$$u(t) = -i_2 R$$

$$u_1(t) = i_2 R$$

Substitute for i_2,

$$u_1(t) = -u(t)$$

On equating the equations we get,

$$u(t) = \frac{e(t)}{R_1} R_2$$

On taking Laplace transform of equation we get,

$$\frac{U(s)}{E(s)} = \frac{R_2}{R_1}$$

The equation is the transfer function of op-amp P-controller. On the comparing equations, Proportional gain,

$$K_p = \frac{R_2}{R_1}$$

Therefore, by adjusting the values of R_1 and R_2 the value of gain K_p can be varied.

INTEGRAL CONTROLLER (I-Controller)

The integral controller is a device that produces a control signal u(t) which is proportional to integral of the input error signal, e(t).

In I-controller

$$u(t) \propto \int e(t)dt$$

$$u(t) = K_i \int e(t)dt$$

where K_i = integral gain or constant

On taking Laplace transform of equation with zero initial conditions we get,

$$\frac{U(s)}{E(s)} = \frac{K_i}{s}$$

The equation gives the output of the I-controller for the input E(s) and equation is the transfer function of the I-controller, the block diagram of I-controller is shown in the figure 5.4.5.

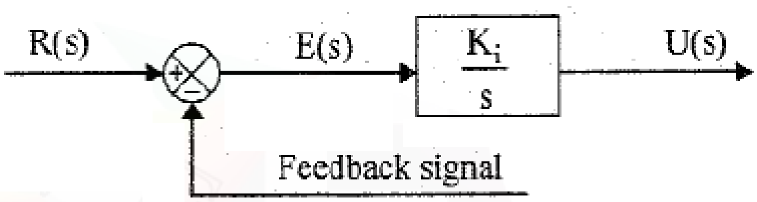

Figure 5.4.5 Block diagram of integral controller

[Source: "Control Systems" by Nagoor Kani, Page: 2.82]

The integral controller removes or reduces the steady error without the need for manual reset. Hence the I-controller is sometimes called automatic reset. The drawback in integral controller is that it may lead to oscillatory response of increasing or decreasing amplitude which is undesirable and the system may become unstable.

Example of electronic I-controller

The integral controller can be realized by an integrator using op-amp followed by a sign changer as shown in figure 2.8.6.

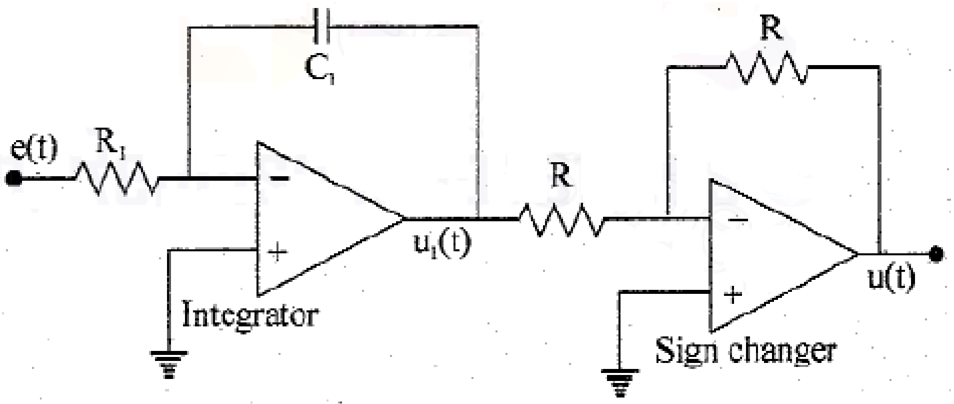

Figure 5.4.6 I-controller using inverting amplifier

[Source: "Control Systems" by Nagoor Kani, Page: 2.82]

By deriving the transfer function of the controller shown in figure and comparing with the transfer function of I-controller defined by equation.

Analysis of I-controller

The assumptions made in op-amp circuit analysis are,

1. The voltages of both inputs are equal
2. The input current is zero.

Based on the above assumptions, the equivalent circuit of op-amp amplifier and sign changer are shown in figure 5.4.7.

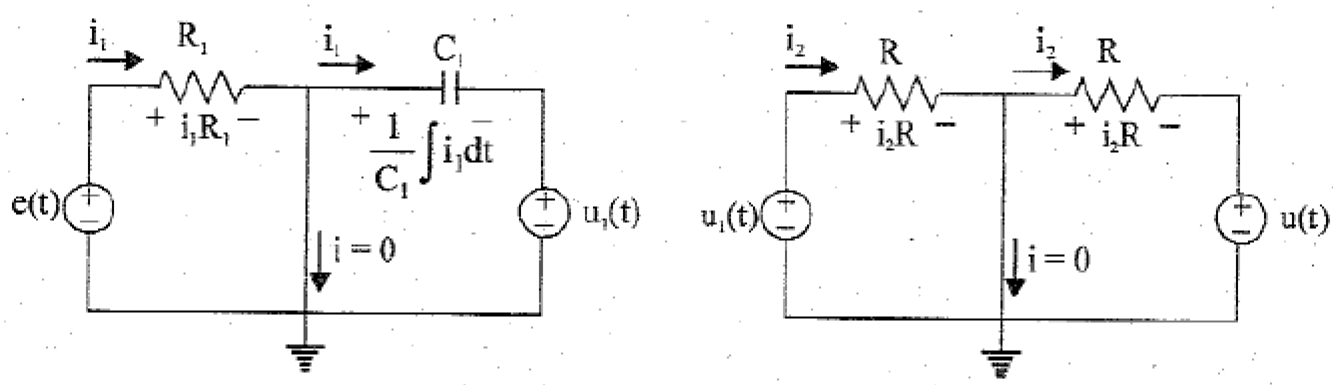

Figure 5.4.7 Equivalent circuit of amplifier and sign changer

[Source: "Control Systems" by Nagoor Kani, Page: 2.83]

From the circuit,

$$e(t) = i_1 R_1$$

$$u_1(t) = -\frac{1}{C_1}\int i_1\, dt$$

Substitute for i_1,

$$u_1(t) = -\frac{1}{R_1C_1}\int e(t)dt$$

Also, from the circuit,

$$u(t) = -i_2R$$

$$u_1(t) = i_2R$$

Substitute for i_2,

$$u_1(t) = -u(t)$$

On equating equations we get

$$u(t) = \frac{1}{R_1C_1}\int e(t)dt$$

On taking Laplace transform of equation we get,

$$\frac{U(s)}{E(s)} = \frac{1}{sR_1C_1}$$

The equation is the transfer function of op-amp P-controller. On the comparing equations, Integral gain,

$$K_i = \frac{1}{R_1C_1}$$

Therefore, by adjusting the values of R_1 and C_1 the value of gain K_i can be varied.

PROPORTIONAL PLUS INTEGRAL CONTROLLER (PI-CONTROLLER)

The proportional plus integral controller (PI controller) produces and output signal consisting of two terms: *one proportional to error signal and the other proportional to the integral of error signal.*

In PI controller,

$$u(t) \propto \left[e(t) + \int e(t)dt\right]$$

$$u(t) = K_p e(t) + \frac{K_p}{T_i}\int e(t)dt$$

On taking Laplace transform of equation with zero initial conditions, we get,

$$\frac{U(s)}{E(s)} = K_p\left(1 + \frac{1}{T_i s}\right)$$

The equation gives the output of the PI-controller for the input E(s) and it is the transfer function of PI-controller. The block diagram of PI-controller is shown in figure 5.4.8.

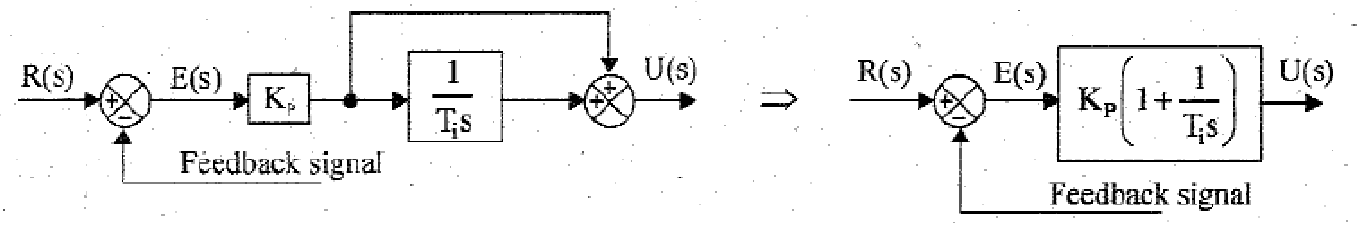

Figure 5.4.8 Block diagram of PI controller

[Source: "Control Systems" by Nagoor Kani, Page: 2.84]

The advantages of both P-controller and I –controller is combined in PI-controller. The proportional action increases the loop gain and makes the system less sensitive to variations of system parameters. The integral action eliminates or reduces the steady state error. The integral control action is adjusted by varying the integral time. The change in value of K_p affects both the proportional and integral parts of control action. The inverse of the integral time T_i is called the reset rate.

Example of Electronic PI-controller

The PI controller can be realized by an op-amp differentiator with gain followed by a sign changer as shown in figure 5.4.9.

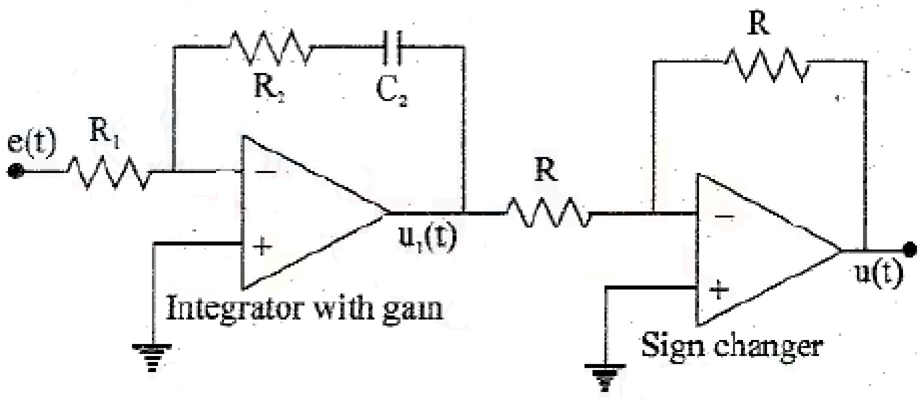

Figure 5.4.9 PI-controller using inverting amplifier

[Source: "Control Systems" by Nagoor Kani, Page: 2.84]

By deriving the transfer function of the controller shown in figure and comparing with the transfer function of PI-controller defined by equation, it can be proved that the circuit shown in figure will work as PI-controller.

Analysis of PI-controller

The assumptions made in op-amp circuit analysis are,

1. The voltages of both inputs are equal
2. The input current is zero.

Based on the above assumptions, the equivalent circuit of op-amp amplifier and sign changer are shown in figure 5.4.10.

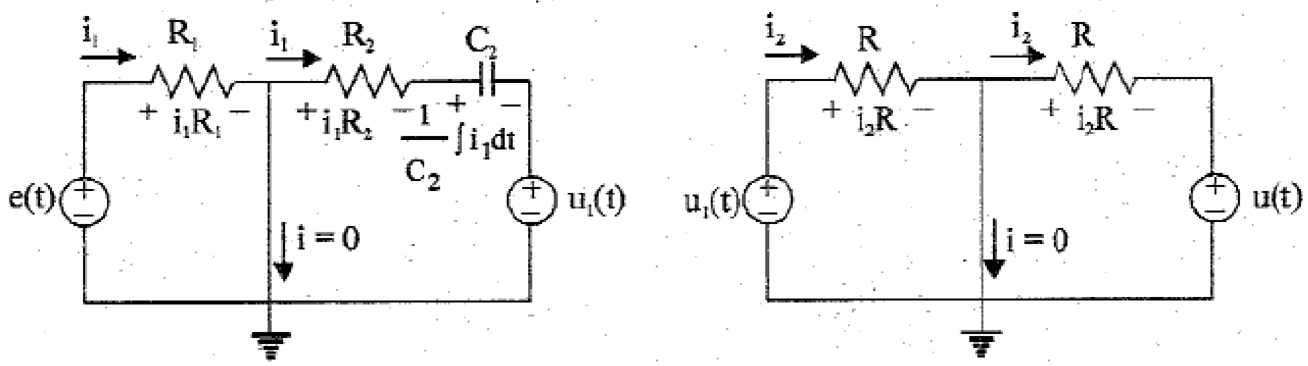

Figure 5.4.10 Equivalent circuit of amplifier and sign changer

[Source: "Control Systems" by Nagoor Kani, Page: 2.85]

From the circuit,

$$e(t) = i_1 R_1$$

$$u_1(t) = -i_1 R_2 - \frac{1}{C_2}\int i_1\, dt$$

Substitute for i_1,

$$u_1(t) = -\frac{e(t)}{R_1}R_2 - \frac{1}{R_1C_2}\int e(t)dt$$

Also, from the circuit,

$$u(t) = -i_2R$$

$$u_1(t) = i_2R$$

Substitute for i_2,

$$u_1(t) = -u(t)$$

On equating equations we get

$$u(t) = \frac{e(t)}{R_1}R_2 + \frac{1}{R_1C_2}\int e(t)dt$$

On taking Laplace transform of equation we get,

$$\frac{U(s)}{E(s)} = \frac{R_2}{R_1}\left(1 + \frac{1}{sR_2C_2}\right)$$

The equation is the transfer function of op-amp P-controller. On the comparing equations,

Proportional gain,

$$K_p = \frac{R_2}{R_1}$$

Integral time,

$$T_i = R_2C_2$$

By varying the values of R_1 and R_2, the value of gain K_p and T_i can be adjusted.

PROPORTIONAL PLUS DERIVATIVE CONTROLLER (PD-CONTROLLER)

The PD controller produces and output signal consisting of two terms: *one proportional to error signal, the other one proportional to derivatives of error signal.*

In PD controller,

$$u(t) \propto \left[e(t) + \frac{d}{dt}e(t)\right]$$

$$u(t) = K_p e(t) + K_p T_d \frac{d}{dt}e(t)$$

On taking Laplace transform of equation with zero initial conditions, we get,

$$\frac{U(s)}{E(s)} = K_p(1 + T_d s)$$

The equation gives the output of the PD-controller for the input E(s) and it is the transfer function of PD-controller. The block diagram of PD-controller is shown in figure 5.4.11.

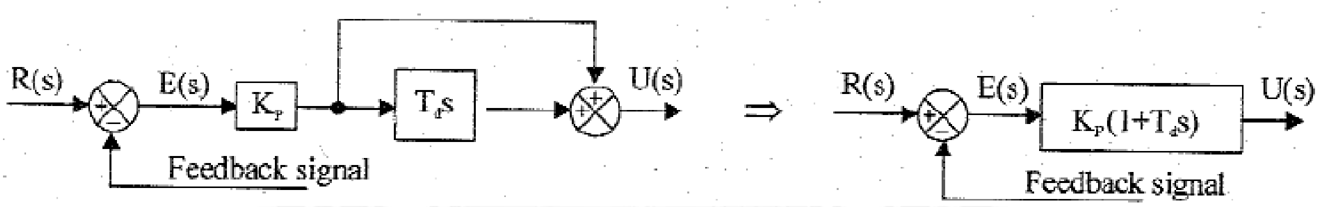

Figure 5.4.11 Block diagram of PD controller

[Source: "Control Systems" by Nagoor Kani, Page: 2.86]

The derivative control acts on a rate of change of error and not on the actual error signal. The derivative control action is effective only during transient periods and so it does not produce corrective measures for any constant error. Hence the derivative controller is never used alone, but it is employed in association with proportional and integral controllers. The derivative controller does not affect the steady-state error directly but anticipates the error, initiates an early corrective action and tends to increase the stability of the system. While derivative control action has an advantage of being anticipatory it has the disadvantage that it amplifies noise signals and may cause a saturation effect in the actuator. The derivative control action is adjusting by varying the derivative time. The change in the value of K_p affects both the proportional and derivative parts of control action. The derivative control is also called rate control.

Example of Electronic PD-controller

The PD controller can be realized by an op-amp amplifier with integral and derivative action followed by a sign changer as shown in figure 5.4.12.

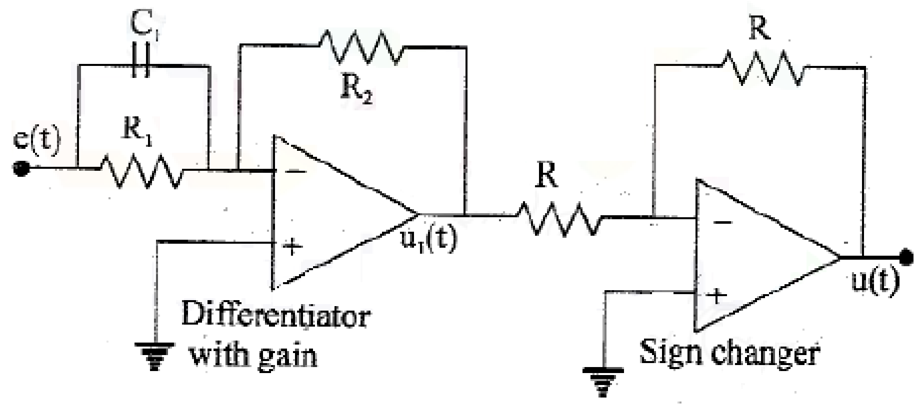

Figure 5.4.12 PD-controller using inverting amplifier

[Source: "Control Systems" by Nagoor Kani, Page: 2.86]

By deriving the transfer function of the controller shown in figure and comparing with the transfer function of PD-controller defined by equation, it can be proved that the circuit shown in figure will work as PD-controller.

Analysis of PD-controller

The assumptions made in op-amp circuit analysis are,

1. The voltages of both inputs are equal
2. The input current is zero.

Based on the above assumptions, the equivalent circuit of op-amp amplifier and sign changer are shown in figure 5.4.13.

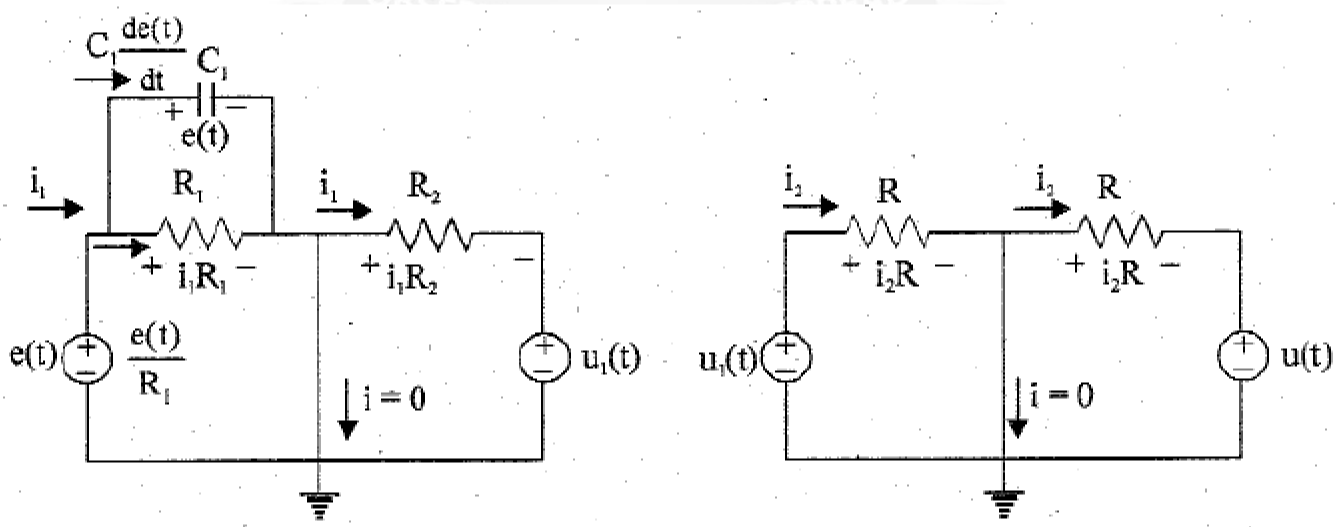

Figure 5.4.13 Equivalent circuit of amplifier and sign changer

[Source: "Control Systems" by Nagoor Kani, Page: 2.87]

From the circuit,

$$i_1 = \frac{e(t)}{R_1} + C_1 \frac{de(t)}{dt}$$

$$u_1(t) = -i_1 R_2$$

Substitute for i_1,

$$u_1(t) = -\frac{e(t)}{R_1} R_2 - R_2 C_1 \frac{d}{dt} e(t)$$

Also, from the circuit,

$$u(t) = -i_2 R$$

$$u_1(t) = i_2 R$$

Substitute for i_2,

$$u_1(t) = -u(t)$$

On equating the equations, we get,

$$u(t) = \frac{e(t)}{R_1} R_2 + R_2 C_1 \frac{d}{dt} e(t)$$

On taking Laplace transform of equation we get,

$$\frac{U(s)}{E(s)} = \frac{R_2}{R_1}(1 + sR_1C_1)$$

The equation is the transfer function of op-amp P-controller. On the comparing equations, Proportional gain,

$$K_p = \frac{R_2}{R_1}$$

Derivative time,

$$T_d = R_1 C_1$$

By varying the values of R_1 and R_2, the value of K_p and T_d are adjusted.

PROPORTIONAL PLUS INTEGRAL PLUS DERIVATIVE (PID) CONTROLLER

The PID controller produces and output signal consisting of two terms: *one proportional to error signal, another one proportional to the integral of error signal and the third one proportional to derivatives of error signal.*

$$u(t) \propto \left[e(t) + \int e(t)dt + \frac{d}{dt}e(t)\right]$$

$$u(t) = K_p e(t) + \frac{K_p}{T_i}\int e(t)dt + K_p T_d \frac{d}{dt}e(t)$$

On taking Laplace transform of equation with zero initial conditions, we get,

$$\frac{U(s)}{E(s)} = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$

The equation gives the output of the PID-controller for the input E(s) and it is the transfer function of PID-controller. The block diagram of PID-controller is shown in figure 5.4.14.

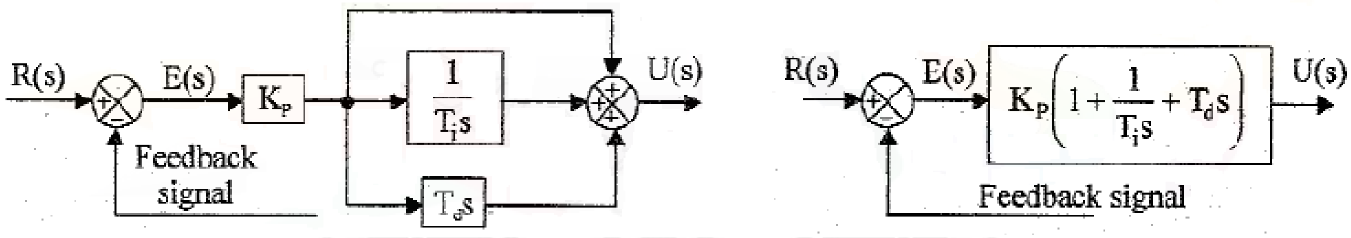

Figure 5.4.14 Block diagram of PID controller

[Source: "Control Systems" by Nagoor Kani, Page: 2.88]

The combination of proportional control action, integral control action and derivative control action is called PID-control action. This combined action has the advantages of the each of the three individual control actions. The proportional controller stabilizes the gain but produces a steady state error. The integral controller reduces or eliminates the steady state error. The derivative controller reduces the rate of change of error.

Example of Electronic PID-controller

The PID controller can be realized by an op-amp amplifier with integral and derivative action followed by a sign changer as shown in figure 5.4.15.

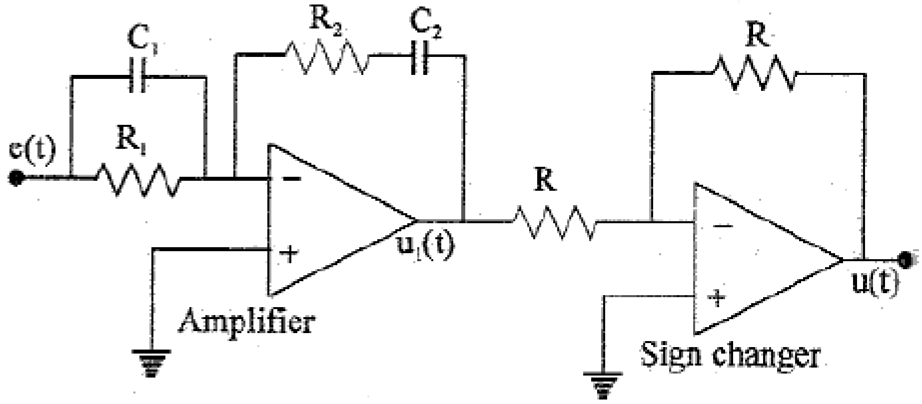

Figure 5.4.15 PID-controller using inverting amplifier

[Source: "Control Systems" by Nagoor Kani, Page: 2.88]

By deriving the transfer function of the controller shown in figure and comparing with the transfer function of PID-controller defined by equation, it can be proved that the circuit shown in figure will work as PID-controller.

Analysis of PID-controller

The assumptions made in op-amp circuit analysis are,

1. The voltages of both inputs are equal
2. The input current is zero.

Based on the above assumptions, the equivalent circuit of op-amp amplifier and sign changer are shown in figure 5.4.16.

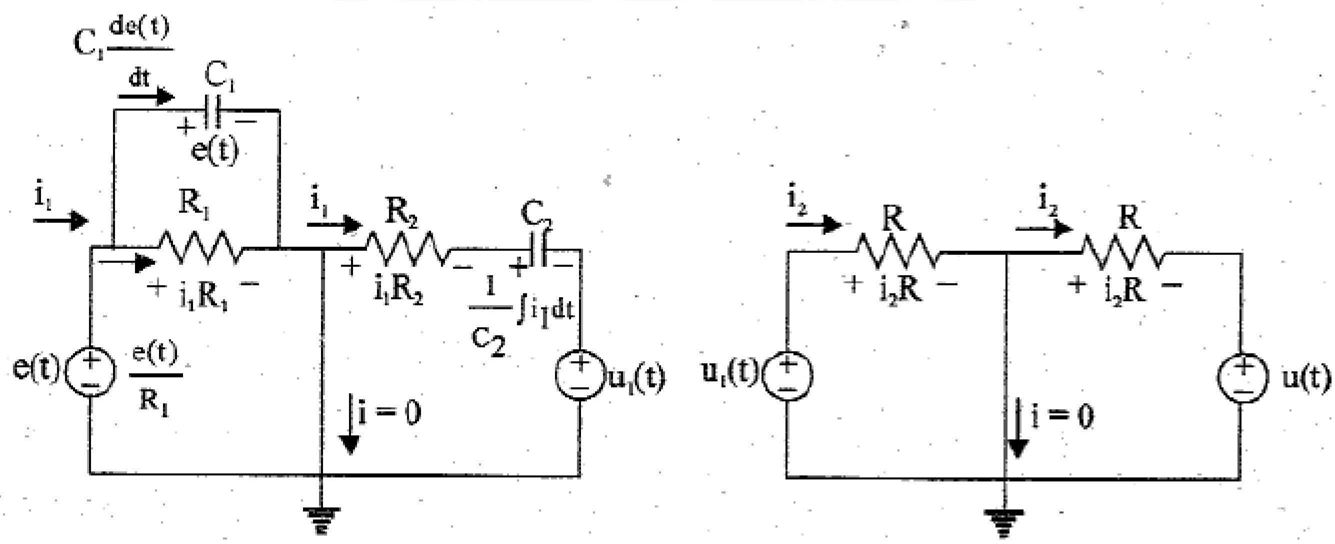

Figure 5.4.16 Equivalent circuit of amplifier and sign changer

[Source: "Control Systems" by Nagoor Kani, Page: 2.89]

From the circuit,

$$i_1 = \frac{e(t)}{R_1} + C_1 \frac{de(t)}{dt}$$

On taking Laplace transform of equation with zero initial conditions, we get,

$$I_1(s) = \left(\frac{1}{R_1} + C_1 s\right) E(s)$$

Also, from the circuit,

$$u_1(t) = -i_1 R_2 - \frac{1}{C_2} \int i_1 \, dt$$

On taking Laplace transform of equation with zero initial conditions, we get,

$$U_1(s) = -I_1(s) R_2 - \frac{1}{sC_2} I_1(s)$$

Substitute for i_1, from equations

$$U_1(s) = -\left(\frac{R_2}{R_1} + \frac{C_1}{C_2} + \frac{1}{R_1 C_2 s} + R_2 C_1 s\right) E(s)$$

Also, from the circuit,

$$u(t) = -i_2 R$$

$$u_1(t) = i_2 R$$

Substitute for i_2,

$$u_1(t) = -u(t)$$

On equating the equations, we get,

$$\frac{U(s)}{E(s)} = \frac{R_2}{R_1}\left(1 + \frac{R_1 C_1 + R_2 C_2}{R_2 C_2} + \frac{1}{R_2 C_2 s} + R_1 C_1 s\right)$$

The equation is the transfer function of op-amp PID-controller. On the comparing, we get,

Proportional gain, $K_p = \frac{R_2}{R_1}$

Derivative time, $T_d = R_1 C_1$

Integral time, $T_i = R_2 C_2$

Also, $\frac{R_1 C_1 + R_2 C_2}{R_2 C_2} = 1$

By varying the values of R_1 and R_2, the value of K_p, T_d and T_i are adjusted.