Frequency Sampling Method:

Discuss the design procedure of FIR filters using frequency sampling method.

Generally, FIR filter can be specified by giving impulse response coefficients *h(n)* (or) DFT coefficients H(k).

$$h(n) = \frac{1}{N}\sum_{k=0}^{N-1} H(k) e^{\frac{j2\pi kn}{N}} \text{ ------------------------------> (1)}$$

$$H(k) = \sum_{n=0}^{N-1} h(n) e^{-\frac{j2\pi kn}{N}} \text{ ---------------------------------> (2)}$$

$$H(k) = DFT\ samples$$

$$H(k) = H(z)\Big|_{z=e^{\frac{j2\pi kn}{N}}} \text{ -------------------------------------> (3)}$$

and $$H(z) = \sum_{n=0}^{N-1} h(n) z^{-n} \text{ -------------------------------------> (4)}$$

Put (1) in (4) $$H(z) = \sum_{n=0}^{N-1}\left[\frac{1}{N}\sum_{k=0}^{N-1} H(k) e^{\frac{j2\pi kn}{N}}\right] z^{-n}$$

If $z = e^{j\omega}$ $$H(e^{j\omega}) = H(z)$$

$$\omega_k = \left(\frac{2\pi k}{N}\right) \qquad where\ \omega_k \ --> sampling\ \ frequency.$$

General steps to design FIR filter using frequency sampling method [type-I design]:

Step 1: Draw the filter graph, as in FIR design using window function.

Step 2: Draw the unit circle and mark the points, if k-0,1…N-1.

$$\theta_k = \frac{360^o}{N} * k$$

if $k = 1;$ $$\theta_1 = \frac{360^0}{N} * 1$$

Step 3: To find H(k), replace ω by $\frac{2\pi k}{N}$ is the given equation.

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\alpha\omega} & , \ 0 \le \omega \le \omega_C \\ 0 & for\ otherwise \end{cases}$$

(Or)

$$H(k) = \begin{cases} e^{-\frac{j(N-1)\pi k}{N}} & , for\ k = 0\ pass\ band\ values \\ 0 & for\ k = stop\ band\ values. \end{cases}$$

Note: In type – II design replace $\omega = \frac{2\pi}{N}\left(k + \frac{1}{2}\right)$

Step 4: Find h(n)

If 'N' = odd

$$h(n) = \frac{1}{N}\left\{H(0) + \sum_{k=1}^{\frac{N-1}{2}} 2\,\mathrm{Re}\left(H(k) e^{\frac{j2\pi kn}{N}}\right)\right\} \qquad \therefore H(0) = 1$$

If 'N' = Even:

$$h(n) = \frac{1}{N}\left\{H(0) + \sum_{k=1}^{\frac{N}{2}-1} 2\,\mathrm{Re}\left(H(k) e^{\frac{j2\pi kn}{N}}\right)\right\}$$

Step 5: Find H(z):

$$H(z)=\sum_{n=0}^{N-1}h(n)z^{-n}$$

Design LPF which has the following specifications, N=7 using frequency sampling Technique. [Nov/Dec-2016][Nov/Dec-15]

$$H_d(e^{j\omega})=\begin{cases}e^{-j3\omega} & ,0\le\omega\le\frac{\pi}{2}\\ 0 & ,\frac{\pi}{2}\le\omega\le\pi.\end{cases}$$

Given: $$H_d(e^{j\omega})=\begin{cases}e^{-j3\omega} & ,0\le\omega\le\frac{\pi}{2}\\ 0 & ,\frac{\pi}{2}\le\omega\le\pi.\end{cases}$$

Step 1: From a unit circle, mark points from

$$\theta_k=\frac{360^o}{N}*k$$
$$\theta_1=51.42^0$$
$$\theta_2=102.8^0$$
$$\theta_3=154.26^0$$
$$\theta 4=205.68^0$$
$$\theta_5=257.1^0$$
$$\theta_6=308.52^0$$

Step 2:

$$H(k)=\begin{cases}e^{-j\left(\frac{N-1}{N}\right)\pi k} & for\ k=0,1,6\\ 0 & for\ otherwise\end{cases}$$

N=7.

$$H(k)=\begin{cases}e^{-j\frac{6\pi k}{7}} & for\ k=0,1,6\\ 0 & for\ k=2,3,4,5\end{cases}$$

$$h(n)=\frac{1}{N}\left\{H(0)+\sum_{k=1}^{\frac{N-1}{2}}2\operatorname{Re}\left(H(k)e^{\frac{j2\pi kn}{N}}\right)\right\}\qquad \therefore H(0)=1$$

$$=\frac{1}{7}\left\{1+\sum_{k=1}^{3}2\operatorname{Re}\left(e^{-j\frac{6\pi k}{7}}.e^{\frac{j2\pi kn}{7}}\right)\right\}$$

$$=\frac{1}{7}\left\{1+2\sum_{k=1}^{3}\operatorname{Re}\ .e^{\frac{j\pi k(2n-6)}{7}}\right\}$$

$$h(n)=\frac{1}{7}\left\{1+2\sum_{k=1}^{3}\cos\left(\frac{\pi k(2n-6)}{7}\right)\right\}$$

$$h(n)=\frac{1}{7}\left\{1+2\left(\cos\left(\frac{\pi(2n-6)}{7}\right)\right)\right\}\quad for\ k=1.$$

$$h(0)=-0.114;h(1)=0.079;h(2)=0.321;h(3)=0.4286;h(4)=h(2);h(5)=h(1);h(6)=h(0)$$

Find coefficient of LP FIR with N=15 and it has symmetric unit sample response. It satisfies the following condition.

Solution:

Step 1:

$$H(k)=\begin{cases}1.e^{-j\left(\frac{N-1}{N}\right)\pi k} & ,k=0,1,2,3\\ 0.4e^{-j\left(\frac{N-1}{N}\right)\pi k} & ,k=4\\ 0 & ,k=5,6,7\end{cases}$$

$$H(k)=\begin{cases}1.e^{-j\frac{14\pi k}{15}} & ,k=0,1,2,3\\ 0.4e^{-j\frac{14\pi k}{15}} & ,k=4\\ 0 & ,k=5,6,7\end{cases}$$

Step 2:

$$h(n)=\frac{1}{N}\left\{H(0)+\sum_{k=1}^{\frac{N-1}{2}}2\operatorname{Re}\left(H(k)e^{\frac{j2\pi kn}{N}}\right)\right\} \qquad \therefore H(0)=1$$

$$h(n)=\frac{1}{15}\left\{1+\sum_{k=1}^{7}2\operatorname{Re}\left(\left(e^{-\frac{j14\pi k}{15}}+0.4e^{-\frac{j14\pi k}{15}}\right)e^{\frac{j2\pi kn}{15}}\right)\right\}$$

$$=\frac{1}{15}\left\{1+2\operatorname{Re}\left[\sum_{k=1}^{3}e^{\frac{j2\pi k(n-7)}{15}}\right]+2(0.4)\sum_{k=4}e^{\frac{j2\pi k}{15}(n-7)}\right\}$$

$$=\frac{1}{15}\left\{1+2\left[\cos\frac{2\pi(n-7)}{15}+\cos\frac{4\pi(n-7)}{15}+\cos\frac{6\pi(n-7)}{15}\right]+0.8\cos\left(\frac{8\pi(n-7)}{15}\right)\right\}$$

$h(0)=-0.0141; h(1)=-0.0195; h(2)=0.04; h(3)=0.0122$

$h(4)=-0.0913; h(5)=-0.01809; h(6)=0.313; h(7)=0.52.$

Determine the coefficients {h (n)} of a linear phase FIR filter of length M =15 has a symmetric unit sample response and a frequency response that satisfies the condition $H_r\left(\frac{2\pi k}{15}\right)=\begin{cases}1 \; for\; k=0,1,2,3\\ 0 \; for\; k=4,5,6,7\end{cases}$ **(May/June-13) (April/May-11)(Nov/Dec-09)**

Solution:

$|\overline{H}(k)|=1 \qquad$ for $0\le k\le 3$ and $12\le k\le 14$

$=0 \qquad$ for $4\le k\le 11$

$$\theta(k) = -\left(\frac{N-1}{N}\right)\pi k$$

$$= -14/15\ \pi k \quad 0 \le k \le 7$$

and

$$\theta(k) = 14\pi - \frac{14\pi k}{15} \quad \text{for } 8 \le k \le 14$$

$$H(k) = e^{-j14\pi k/15} \quad \text{for } k=0,1,2,3$$

$$= 0 \quad \text{for } 4 \le k \le 11$$

$$= e^{-j14\pi(k-15)/15} \quad \text{for } 12 \le k \le 14$$

$$h(n) = \frac{1}{n}\left[H(0) + 2\sum_{k=1}^{\frac{N-1}{2}} \mathrm{Re}(H(k)e^{j2\pi nk/15})\right]$$

$$= \frac{1}{5}\left[1 + 2\sum_{k=1}^{7} \mathrm{Re}(e^{-j14\pi k/15} e^{j2\pi nk/15}\right]$$

$$= \frac{1}{15}\left[1 + 2\sum_{k=1}^{3} \cos\frac{2\pi k(7-n)}{15}\right]$$

$$= \frac{1}{15}\left[1 + 2\cos\frac{2\pi(7-n)}{15} + 2\cos\frac{4\pi(7-n)}{15} + 2\cos\frac{6\pi(7-n)}{15}\right]$$

` $h(0) = h(14) = -0.05;\ h(1) = h(3) = 0.041;\ h(4) = h(10) = -0.1078$

$h(2) = h(12) = 0.0666;\ \ h(3) = h(11) = -0.0365;\ h(5) = h(9) = 0.034$

$h(6) = h(8) = 0.3188;\ \ h(7) = 0.466$

Using frequency sampling method, design BPF with the following specifications. [May/June-2016]
Sampling frequency F=8000Hz
Cut off frequencies f_{c1}=1000Hz
Cut off frequencies f_{c2}=3000Hz Determine the filter coefficients for N=7.

Solution:

$$\omega_{c_1} = 2\pi f_{c_1} T = \frac{2\pi f_{c_1}}{F} = \frac{2\pi(1000)}{8000} = \frac{\pi}{4}$$

$$\omega_{c_2} = 2\pi f_{c_2} T = \frac{2\pi f_{c_2}}{F} = \frac{2\pi(3000)}{8000} = \frac{3\pi}{4}$$

$$H(K) = H_d(e^{j\omega})\Big|_{\omega = \frac{2\pi}{7}k} \quad k=0,1,\ldots 6$$

$$|H(k)| = \begin{cases} 0 & for\, k = 0,3 \\ 1 & for\, k = 1,2 \end{cases}$$

$$\theta(k) = -\left(\frac{N-1}{N}\right)\pi \quad for \quad 0 \le k \le \frac{N-1}{2}$$

$$= -\frac{6}{7}\pi k \quad for \quad 0 \le k \le 3$$

$$|H(k)| = \begin{cases} 0 & for\, k = 0,3 \\ e^{\frac{-j6\pi k}{7}} & for\, k = 1,2 \end{cases}$$

The filter coefficients are given by

$$h(n) = \frac{1}{N}\left[H(0) + \sum_{k=1}^{\frac{N-1}{2}} 2.\mathrm{Re}\left[H(k).e^{\frac{j2\pi kn}{N}}\right]\right]$$

$$= \frac{1}{7}\left[2\sum_{k=1}^{\frac{N-1}{2}} \mathrm{Re}\left[e^{\frac{j6\pi k}{7}}\, e^{\frac{j2\pi kn}{7}}\right]\right]$$

$$= \frac{1}{7}\left[2\sum_{k=1}^{\frac{N-1}{2}} \cos\frac{2\pi k}{7}((3-n))\right]$$

$$= \frac{2}{7}\left[\cos\frac{2\pi}{7}(3-n) + \cos\frac{4\pi}{7}(3-n)\right]$$

$h(0) = h(6) = -0.07928.$

$h(1) = h(5) = -0.321$

$h(2) = h(4) = -0.11456$

$h(3) = 0.57$

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