

Addition is one of the most basic operations performed by different electronic devices like computers, calculators, etc. The electronic circuit that performs the addition of two or more numbers, more specifically binary numbers, is called as **adder**. Since, the logic circuits use binary number system to perform the operations, hence the adder is referred to as **binary adder**

Depending on the number of bits that the circuit can add, adders (or binary adders) are of two types –

- Half Adder
- Full Adder

In this article, we will discuss the **half adder**, its definition, circuit diagram, truth table, kmap, characteristic equations, and applications.

What is a Half-Adder?

A combinational logic circuit which is designed to add two binary digits is called as a **half adder**. The half adder provides the output along with a carry value (if any). The half adder circuit is designed by connecting an EX-OR gate and one AND gate. It has two input terminals and two output terminals for sum and carry. The block diagram and circuit diagram of a half adder are shown in Figure-1.

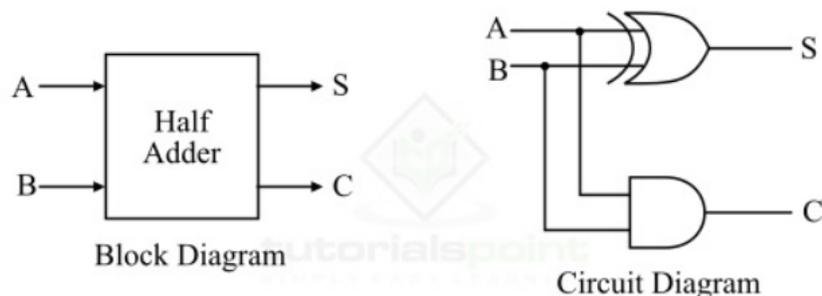


Figure 1 - Half Adder

From the logic circuit diagram of half adder, it is clear that A and B are the two input bits, S is the output sum, and C is the output carry bit.

In the case of a half adder, the output of the EX-OR gate is the sum of two bits and the output of the AND gate is the carry. Although, the carry obtained in one addition will not be forwarded in the next addition because of this it is known as half adder.

Operation of Half Adder

Half adder adds two binary digits according to the rules of binary addition. These rules are as follows –

$$0 + 0 = 0$$

$$0 + 1 = 1$$

$$1 + 0 = 1$$

$$1 + 1 = 10 \text{ (Sum} = 0 \text{ \& Carry} = 1)$$

According to these rules of binary addition, we can see that the first three operations produce a sum whose length is one digit, whereas in the case of last operation (1 and 1), the sum consists of two digits. Here, the MSB (most significant bit) of this result is called a carry (which is 1) and the LSB (least significant bit) is called the sum (which is 0).

Explore our **latest online courses** and learn new skills at your own pace. Enroll and become a certified expert to boost your career.

Truth Table of Half Adder

Truth table is one that gives the relationship between inputs and outputs of a logic circuit and explains the operation of the circuit. The following is the truth table of the half-adder –

Inputs		Outputs	
A	B	S (Sum)	C (Carry)
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

K-Map for Half Adder

We can use the K-Map (Karnaugh Map), a method for simplifying Boolean algebra, to determine equations of the sum bit (S) and the output carry bit (C) of the half adder circuit.

The k-map for half adder circuit is shown in Figure-2.

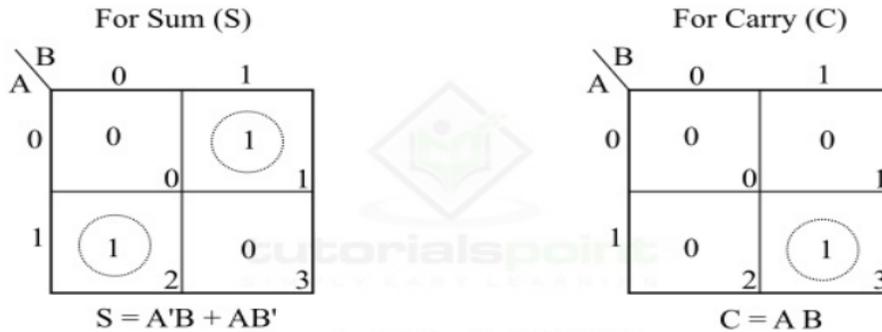


Figure 2 - K-Map for Half Adder

Characteristic Equations of Half-Adder

The characteristic equations of half adder, i.e., equations of sum (S) and carry (C) are obtained according to the rules of binary addition. These equations are given below –

The sum (S) of the half-adder is the XOR of A and B. Thus,

$$\text{Sum, } S = A \oplus B = AB' + A'B$$

The carry (C) of the half-adder is the AND of A and B. Therefore,

$$\text{Carry, } C = A \cdot B$$

Applications of Half Adder

The following are some important applications of half adder –

- Half adder is used in ALU (Arithmetic Logic Unit) of computer processors to add binary bits.
- Half adder is used to realize full adder circuit.
- Half adder is used in calculators.
- Half adder is used to calculate addresses and tables.

What is a Full Adder?

A combinational logic circuit that can add two binary digits (bits) and a carry bit, and produces a sum bit and a carry bit as output is known as a **full-adder**.

In other words, a combinational circuit which is designed to add three binary digits and produces two outputs (sum and carry) is known as a full adder. Thus, a full adder circuit adds three binary digits, where two are the inputs and one is the carry forwarded from the previous addition. The block diagram and circuit diagram of the full adder are shown in Figure-1.

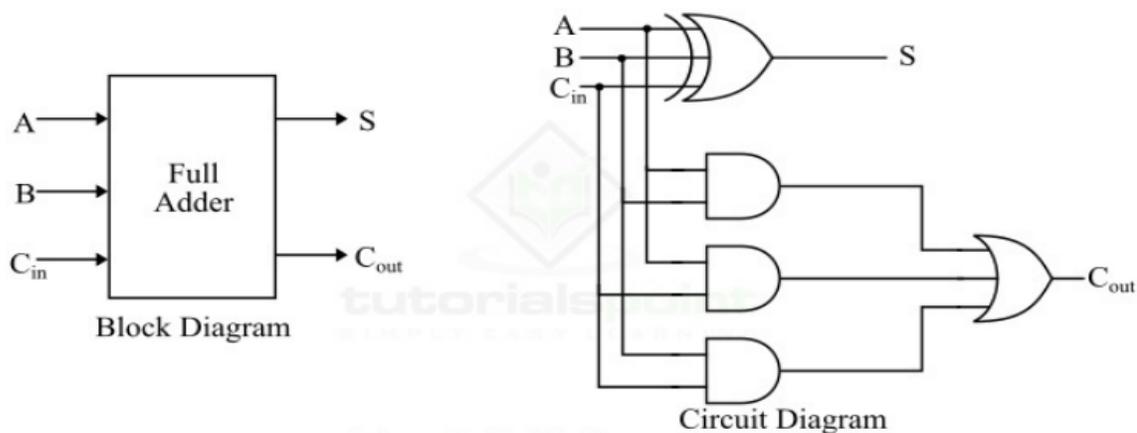


Figure 1 - Full Adder

Hence, the circuit of the full adder consists of one EX-OR gate, three AND gates and one OR gate, which are connected together as shown in the full adder circuit in Figure-1.

Operation of Full Adder

Full adder takes three inputs namely A , B , and C_{in} . Where, A and B are the two binary digits, and C_{in} is the carry bit from the previous stage of binary addition. The sum output of the full adder is obtained by XORing the bits A , B , and C_{in} . While the carry output bit (C_{out}) is obtained using AND and OR operations.

Explore our **latest online courses** and learn new skills at your own pace. Enroll and become a certified expert to boost your career.

Truth Table of Full Adder

Truth table is one that indicates the relationship between input and output variables of a logic circuit and explains the operation of the logic circuit. The following is the truth table of the full-adder circuit –

Inputs			Outputs	
A	B	C _{in}	S (Sum)	C _{out} (Carry)
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Hence, from the truth table, it is clear that the sum output of the full adder is equal to 1 when only 1 input is equal to 1 or when all the inputs are equal to 1. While the carry output has a carry of 1 if two or three inputs are equal to 1.

K-Map for Full Adder

K-Map (Karnaugh Map) is a tool for simplifying binary complex Boolean algebraic expressions. The K-Map for full adder is shown in Figure-2.

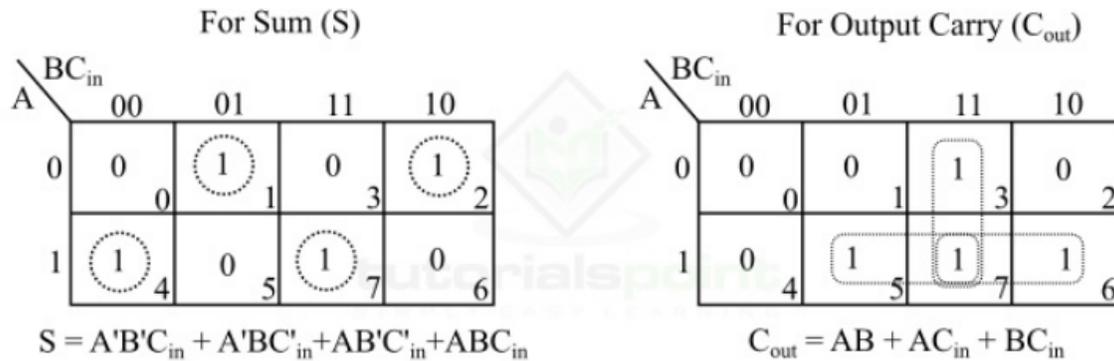


Figure 2 - K Map for Full Adder

Characteristic Equations of Full Adder

The characteristic equations of the full adder, i.e. equations of sum (S) and carry output (C_{out}) are obtained according to the rules of binary addition.

These equations are given below –

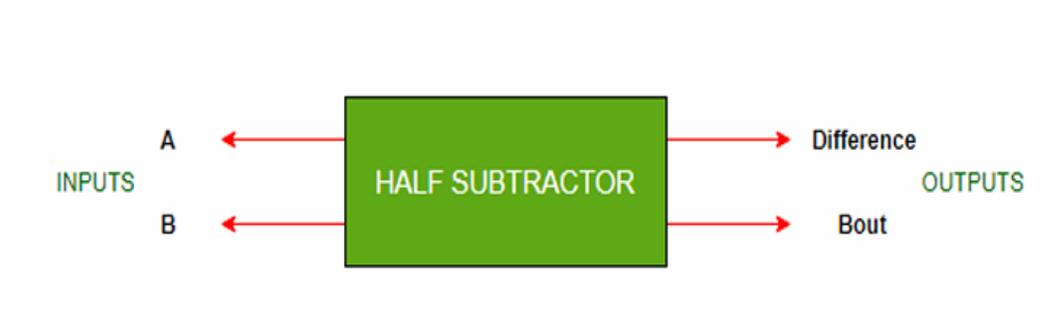
The sum (S) of the full-adder is the XOR of A, B, and C_{in}. Therefore,

$$\text{Sum, } S = A \oplus B \oplus C_{in} = A'B'C_{in} + A'BC'_{in} + AB'C'_{in} + ABC_{in}$$

The carry (C) of the half-adder is the AND of A and B. Therefore,

$$\text{Carry, } C = AB + AC_{in} + BC_{in}$$

3. Half Subtractor:

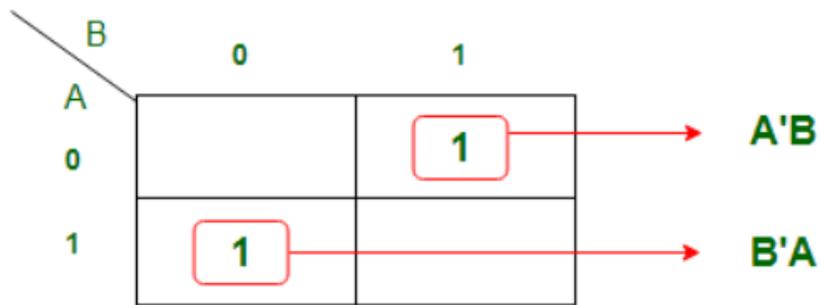


- It is a combinational logic circuit designed to perform the subtraction of two single bits.
- It contains two inputs (A and B) and produces two outputs (Difference and Borrow-output).

Truth Table of Half Subtractor:

Inputs		Outputs	
A	B	D	B ₀
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

K-map Simplification for output variable 'D':



$$A'B + B'A = A \text{ xor } B$$

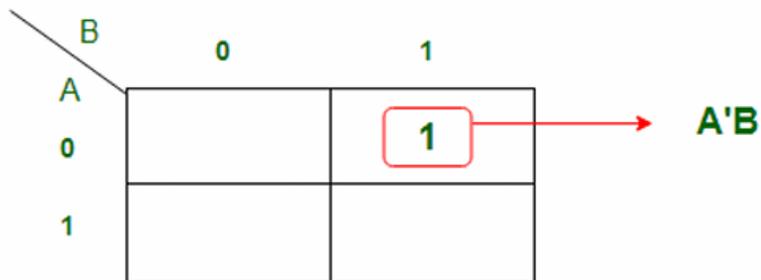
The equation obtained is,

$$D = A'B + AB'$$

which can be logically written as,

$$D = A \text{ xor } B$$

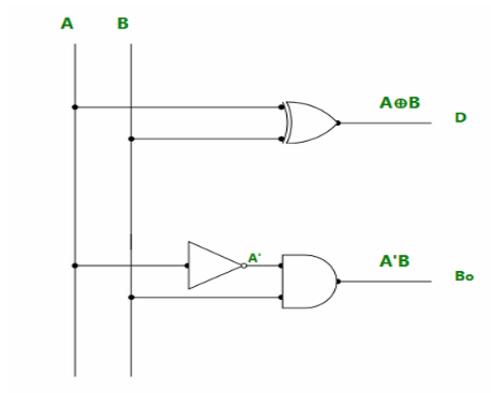
K-map Simplification for output variable 'B_{out}' :



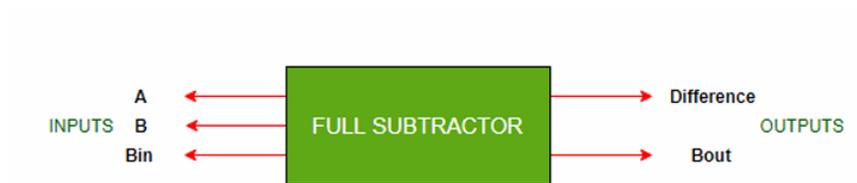
The equation obtained from above K-map is,

$$B_{out} = A'B$$

Logic Diagram of Half Subtractor:



4. Full Subtractor:

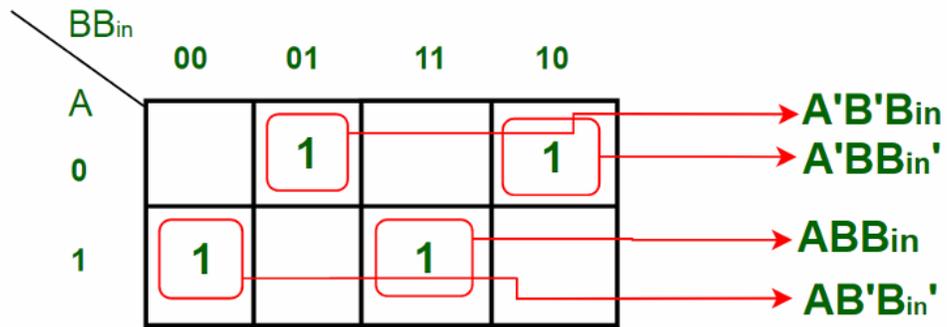


- It is a Combinational logic circuit designed to perform subtraction of three single bits.
- It contains three inputs(A, B, B_{in}) and produces two outputs (D, B_{out}).
- Where, A and B are called **Minuend** and **Subtrahend** bits.
- And, B_{in} -> Borrow-In and B_{out} -> Borrow-Out

Truth Table of Full Subtractor:

Inputs			Outputs	
A	B	B _{in}	D	B _{out}
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

K-map Simplification for output variable 'D' :



The equation obtained from above K-map is,

$$D = A'B'B_{in} + AB'B_{in}' + ABB_{in} + A'BB_{in}'$$

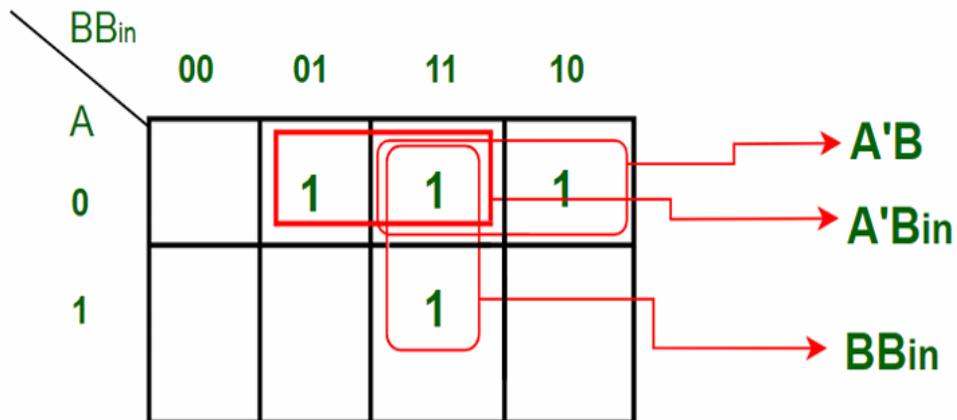
which can be simplified as,

$$D = B'(A'B_{in} + AB_{in}') + B(AB_{in} + A'B_{in}')$$

$$D = B'(A \text{ xor } B_{in}) + B(A \text{ xor } B_{in})'$$

$$D = A \text{ xor } B \text{ xor } B_{in}$$

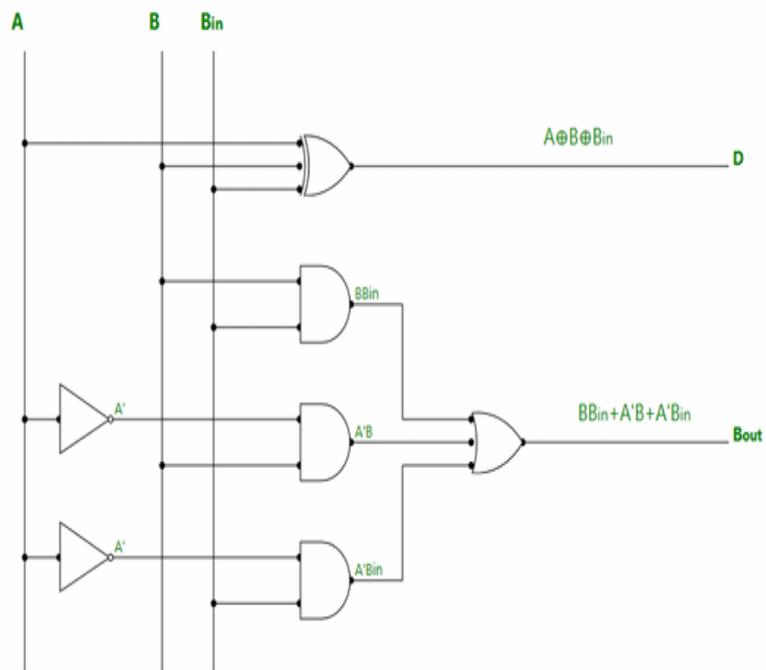
K-map Simplification for output variable 'B_{out}' :



The equation obtained is,

$$B_{out} = BB_{in} + A'B + A'B_{in}$$

Logic Diagram of Full Subtractor:



Applications:

1. For performing arithmetic calculations in electronic calculators and other digital devices.
2. In Timers and Program Counters.