SAMPLING RATE CONVERSION BY A RATIONAL FACTOR I/D:

Let as consider the general case of sampling rate conversion by a rational factor I/D .We can achieve this sampling rate conversion by first performing interpolation by the factor I and then decimating the output of the interpolator by the factor. Inother words, the sampling rate conversion by the rational factor $I/D$is accomplished by cascading an interpolator with a decimator.

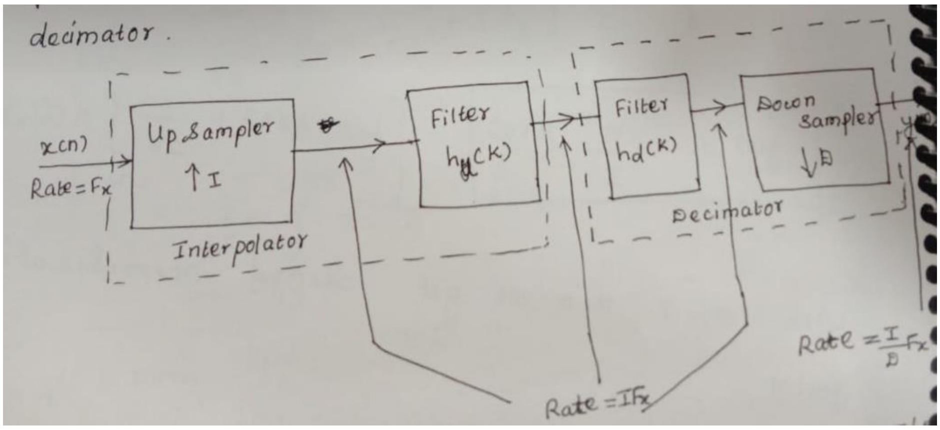


Fig: method for sampling rate conversion by factor I/D

The interpolation by a factor I is obtained first to increase the sampling rate to IF_x.

The output of the interpolator is then decimated by a factor D, so that the final output rate is

$$F_y = \frac{IF_x}{D}$$

In the above figure, observe that there is a cascade of two low pass filters. The overall cut off frequency will be minimum of the two cut off frequencies.

The frequency response of the anti-imaging filter is given as,

$$H_u(\omega) = \begin{cases} C & , \frac{-\pi}{I} \leq \omega \leq \frac{\pi}{I} \\ & 0, else \end{cases}$$

The scaling factor $C = I$ for a desired normalization.

$$H_u(\omega) = \begin{cases} I & , \frac{-\pi}{I} \leq \omega \leq \frac{\pi}{I} \\ & 0, else \end{cases}$$

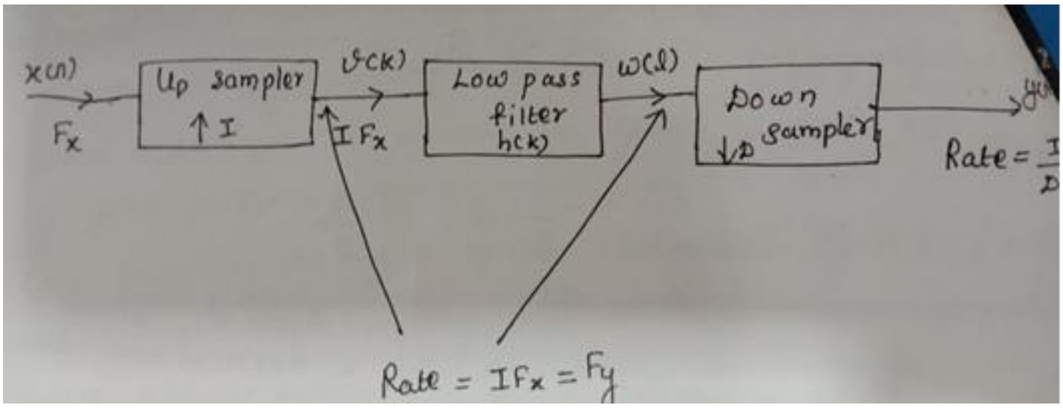
The frequency response of anti-imaging filter is given as,

$$H_d(\omega) = \begin{cases} 1 & , \frac{-\pi}{D} \leq \omega \leq \frac{\pi}{D} \\ & 0, else \end{cases}$$

The overall cascading effect of low pass filter will have a cutoff frequency which is minimum of $\frac{\pi}{I}$ and $\frac{\pi}{D}$. Hence we can write the frequency response of the combined filter as

$$H(\omega) = \begin{cases} I, |\omega| \leq min\left(\frac{-\pi}{D}, \frac{\pi}{I}\right) \\ \quad 0, other\ wise \end{cases}$$

Thus single filter can be used ,as designed by above equation. Then the block diagram can be modified as follows.



<u>Derivation for output $y(m)$.</u>

The output of low pass filter is given by

$$w(l) = v(k)h(k)$$

$$w(l) = \sum_{k=-\infty}^{\infty} v(k)\, h(l-k)$$

$k = Ik$

$$w(l) = \sum_{k=-\infty}^{\infty} v(kI)\, h(l-kI)$$

Let $V(kI) = x(k)$

$$w(l) = \sum_{k=-\infty}^{\infty} x(k)\, h(l-kI)$$

The output of the down sampler

$$y(m) = w(mD)$$

Sub $l = mD$ in the above equ,

$$w(mD) = \sum_{k=-\infty}^{\infty} x(k)h(mD - kI)$$

$$y(m) = \sum_{k=-\infty}^{\infty} x(k)h(mD - kI)$$

This is the equation for output sequence.

The frequency domain relationship can be obtained by combining the results of the interpolation and decimation processes. Thus the spectrum of the up sample is

$$Y(\omega_y) = \begin{cases} CX(\omega_y I), 0 \le |\omega_y| \le \frac{\pi}{I} \\ 0, else \end{cases}$$

$where\ C = I$

$$Y(\omega_y) = \begin{cases} IX(\omega_y I), 0 \le |\omega_y| \le \frac{\pi}{I} \\ 0, else \end{cases}$$

Here $w(l)$ is the output of the linear filter

$$W(\omega_v) = \begin{cases} IX(\omega_v I), 0 \le |\omega_v| \le min\left(\frac{\pi}{D}, \frac{\pi}{I}\right) \\ 0, else \end{cases} ----(1)$$

The spectrum of the down sampler is

$$Y(\omega_y) = \frac{1}{D} X\left(\frac{\omega_y}{D}\right) \;\; for\ 0 \le |\omega_y| \le \pi$$

Here $w(l)$ is the input of the down sampler

$$Y(\omega_y) = \frac{1}{D} W\left(\frac{\omega_y}{D}\right) ----(2)$$

The input and output frequency are related by the equ,

$$\omega_x = \frac{\omega_y}{D}$$

Here

$$\omega_x = \omega_v$$

$$\omega_v = \frac{\omega_y}{D} --- (3)$$

(2) implies

$$Y(\omega_y) = \frac{1}{D} W(\omega_v)$$

$$Y(\omega_y) = \frac{1}{D}\begin{cases} IX(\omega_v I), 0 \leq |\omega_v| \leq min\left(\frac{\pi}{D}, \frac{\pi}{I}\right) \\ 0\,, else \end{cases}$$

$$\omega_v = \frac{\omega_y}{D}$$

$$Y(\omega_y) = \begin{cases} \frac{I}{D} X\left(\frac{I}{D}\omega_y\right), 0 \leq \left|\frac{\omega_y}{D}\right| \leq min\left(\frac{\pi}{D}, \frac{\pi}{I}\right) \\ 0\,, else \end{cases}$$

$$Y(\omega_y) = \begin{cases} \frac{I}{D} X\left(\frac{I}{D}\omega_y\right), 0 \leq |\omega_y| \leq min\left(\pi, \frac{\pi D}{I}\right) \\ 0\,, else \end{cases}$$

This gives the spectrum of the output sequence.