ELECTRIC FIELD INSIDE DIELECTRIC POLARIZATION

Dielectric Materials:

The charges in dielectrics are bound by the finite forces and are called bound charges. If an electric field E is applied to a dielectric. The bound charges shift their relative positions against the normal molecular and atomic forces. This shifts allows the dielectric to store energy.

When the dipole results from the displacement of the bound charges the dielectric is said to the polarized.

Separation of bound charges produce electric dipoles under the influence of electric field E is called polarization.

POLARIZATION:

An atom of dielectric consists of a nucleus with positive charge and negative charges in the form of revolving electrons in the orbits.

If no $\bar{E}$ is applied ,the number of positive charge is same as negative charges and hence atom is electrically neutral .The charges are coinciding at the centre. Hence there cannot exists an electric dipole .This is called unpolarized atom.

When electric field $\bar{E}$ is applied the symmetrical distribution of charges gets disturbed. The positive charges experience a force F= QE

While the negative charges experience a force F= -QE in the opposite direction.

An electron cloud has a centre separated from the nucleus . This forms an electric dipole .The dipole gets aligned with applied field is called polarization of dielectrics.

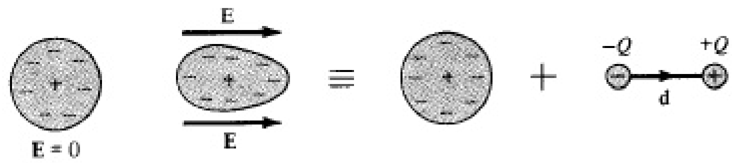

Fig 3: Polarization of nonpolar atom or molecule.

Types of Dielectrics

1.Non Polar molecules

2. Polar molecules.

In nonpolar molecules the dipole arrangement is totally absent in absence of electric field $\bar{E}$. Dipole results only when an external field $\bar{E}$ is applied to it .In polar molecules permanent dipole exists without application of $\bar{E}$. But such dipoles are randomly oriented . Under the application of $\bar{E}$ the dipole align themselves with the direction of the applied field $\bar{E}$. This is called polarization of polar molecules.

Non polar molecules Eg. Hydrogen, oxy gen

Polar molecules Eg. Water ,sulphur dioxide ,hydrochroric acid

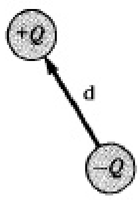

Fig 4 a: Polarization of polar molecule
Permanent dipole (E=0)

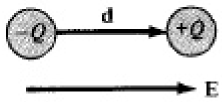

Fig b: Alignment of permanent dipole (E ≠ 0)

Mathematical Expression for Polariztion

When the dipole is formed due to polarization there exists an electric dipole moment $\overline{P}$.

$$\overline{P} = Q\overline{d}$$

Q= Magnitude of one of the two charges

$\overline{d}$=Distance vector from negative to positive charges

n = Number of dipoles per unit volume

Δv= Total volume of the dielectric

N=Total dipoles= $n\Delta v$

The total dipole moment is ,

$\overline{P}_{total} = Q1\overline{d1} + Q2\overline{d2} + \ldots\ldots\ldots + Qn\overline{dn}$

$\overline{P}_{total} = \sum_{i=1}^{n} Qi\,\overline{di}$

If the dipoles are randomly oriented $\overline{P}_{total}$ is zero but if dipoles are aligned in the direction of E then P_{total} has a significant value.

The polarization $\overline{P}$ is defined as the total dipole moment per unit volume

$$\overline{P} = \lim_{\Delta v \to 0} \sum_{i=1}^{N} \frac{Qi\overline{di}}{\Delta v}$$

Its unit is coulombs per square meter $[C/m^2]$

Field due to polarized dielectric:

Consider the dielectric material consisting of dipoles with dipole moment P per unit volume. The potential dv at an exterior point O due to dipole moment is

$$dv = \frac{P \cdot a_R \, dv'}{4\pi\varepsilon_0 R^2}$$

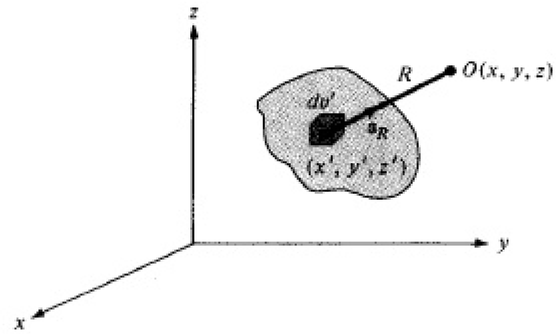

Fig 5: A block of dielectric material with dipole moment P per unit Volume

$$R^2 = (x - x')^2 + (y - y')^2 + (z - z')^2$$

R is the distance between volume element dv' at (x',y',z') and field point O at(x,y,z),. The gradient of 1/R with respect to primed coordinate is

$$\nabla'\left(\frac{1}{R}\right) = \frac{a_R}{R^2}$$

Thus $\frac{P \cdot a_R}{R^2} = P . \nabla'\left(\frac{1}{R}\right)$

Apply the vector identity $\nabla' . fA = f\nabla' . A + A . \nabla' f$

$$\frac{P.a_R}{R^2} = \nabla' . \frac{P}{R} - \frac{\nabla' . P}{R}$$

Integrate the above equation over the volume v' of the dielectric ,

$$v = \int_{v'} \frac{1}{4\pi\varepsilon_0} \left[\nabla' . \frac{P}{R} - \frac{1}{R} \nabla' . P\right] dv'$$

Apply divergence theorem to the first term leads to

$$V = \int_{S'} \frac{P . a'_n}{4\pi\varepsilon_0 R} dS' + \int_{v'} \frac{- \nabla' . P}{4\pi\varepsilon_0 R} dv'$$

Where a_n' → outward unit normal to the surface ds' of the dielectric.

The potential due to surface and volume charge distributions with densities is $\rho_{\rho s} = P . a_n$

$$\rho_{\rho v} = -\nabla . P$$

$\rho_{\rho s} \rightarrow$ bound surface charge density , $\rho_{\rho v}$ → bound volume charge density

Bound charges are those that are not free to move within the dielectric material. The total positive bound charge on surface S bounding the dielectric is

$$Q_b = \oint P . ds = \int \rho_{ps} \, ds$$

The charge remains inside surface S is

$$- Q_b = \int_v \rho_{pv} \, dv = - \int_v \nabla . P \, dv$$

Total charge of the dielectric material remains zero

Total charge = $\oint_s \rho_{ps} \, ds + \int_v \rho_{pv} \, dv = Q_b - Q_b = 0$

The dielectric is electrically neutral before polarization.

The dielectric has a free volume charge density ρ_v , total volume charge density in the dielectric is

$$p_t = \rho_v + \rho_{pv} = \nabla . D = \nabla . \varepsilon_0 E$$

$$\rho_v = \nabla . \varepsilon_0 E - \rho_{pv}$$

$$= \nabla . (\varepsilon_0 E + P)$$

$$= \nabla . D$$

The polarization increases the electric flux density in a dielectric medium. Flux density in a dielectric is ,

$\overline{D} = \varepsilon_0\overline{E} + \overline{P}$ ----- (A)

For an isotropic and linear medium ,the $\overline{P}$ and $\overline{E}$ are parallel to each other at every point and related to each other as

$\overline{P}=\psi_e \varepsilon_0 \overline{E}$ ------①

$\psi_e \rightarrow$ Electric susceptibility of the material [$dimension\ less$]

Substitute equation ① in eqn A

$\overline{D}= \varepsilon_0 \bar{E} + \psi_e \varepsilon_0 \overline{E}$

$=(1 + \psi_e)\varepsilon_0 \overline{E}$

D = εE , where $\varepsilon = \varepsilon_0 \varepsilon_r$

1+$\psi_e \rightarrow$ relative permittivity (or) dielectric constant of the dielectric material

In an isotropic material $\overline{D}$,$\overline{E}$ and $\overline{P}$ are not parallel to each other and ε & ψ_e vary in all directions.

DIELECTRIC CONSTANT :

Dielectric constant is the ratio of permittivity of the dielectric to that of free space. $\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$

DIELECTRIC STRENGTH:

The minimum value of the applied electric field at which the dielectric breakdown occurs is called dielectric strength of that dielectric.

OR

The dielectric strength is the maximum electric field that a dielectric can tolerate without breakdown.The dielectric strength is measured in V/m (or) KV/m

CURRENT & CURRENT DENSITY:

The flow of electric charge per unit time is called electric current. It is measured in ampere (or) coulombs /Sec (C/s).

I = $\frac{dQ}{dt}$ C/s (i.e A)

One Ampere : A current of " 1 (one)" ampere is said to be flowing across the surface when a charge of one coulomb is passing across the surface in one second.

The current in the conductPors exists ,due to the drifting of electrons ,under the influence of the applied voltage is called drift current.

While in dielectrics ,there can be flow of charges ,under the influence of the electric field intensity is called displacement (or) convection current.

The current density is defined as the current passing through the unit surface area ,when the surface is held normal to the direction of the current.

It is represented as J & expressed in A/m^2.

RELATION BETWEEN I & J:

Consider a surface 'S' and 'I' is the current passing through the surface 'S'. The current 'I ' is normal to the surface hence 'J' is also normal to the surface.

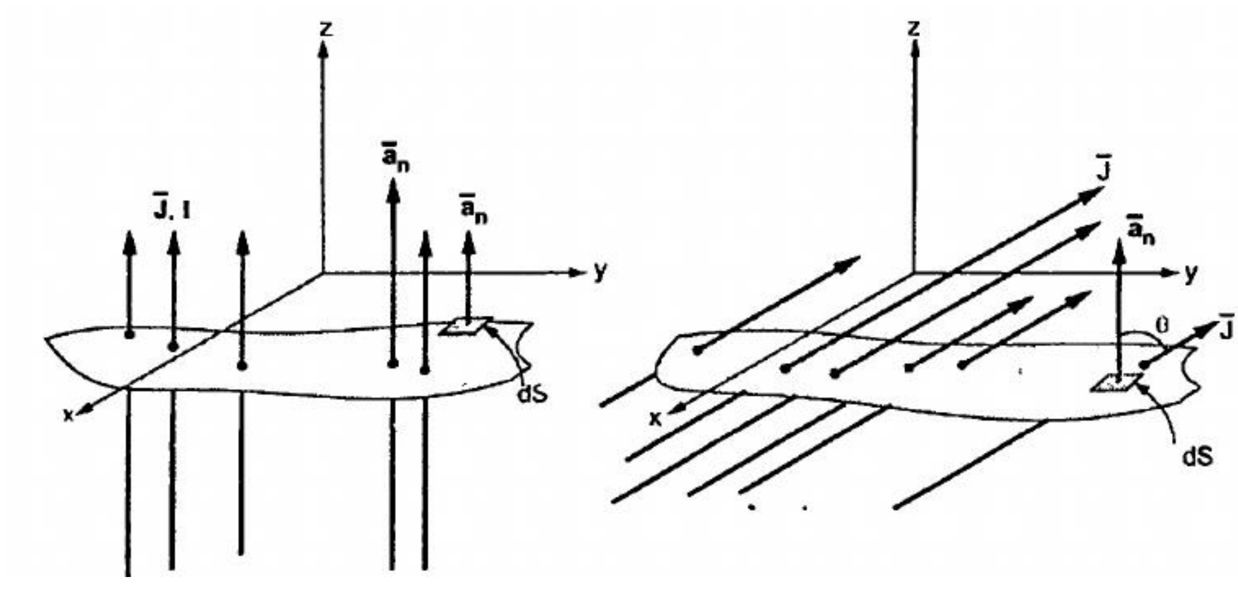

Fig 6 a: J and ds are normal Figb: $\bar{J}$ and $\overline{ds}$ are not at right angles

The current density at a given point is the current through a unit normal area at that point.

ds= ds $\overline{an}$ while J =J $\overline{an}$ -------①

The diffential current dI passing through ds is given by

dI = J.ds (dot product)

when J and ds are at right angles θ=90°

dI=J $\overline{an}$.ds

I =$\int_s J.ds$-------②

J→ current density in A/m^2

If J is not normal to the differential area $\overline{ds}$ then current is given as

I =$\int_s J.ds$

J need not be uniform over S.

RELATION BETWEEN J AND ρV:

The set of charged particles give rise to a charge density ρv in a volume V . The current density is related to the velocity with which the volume charge density crosses the surface 's' at a point.

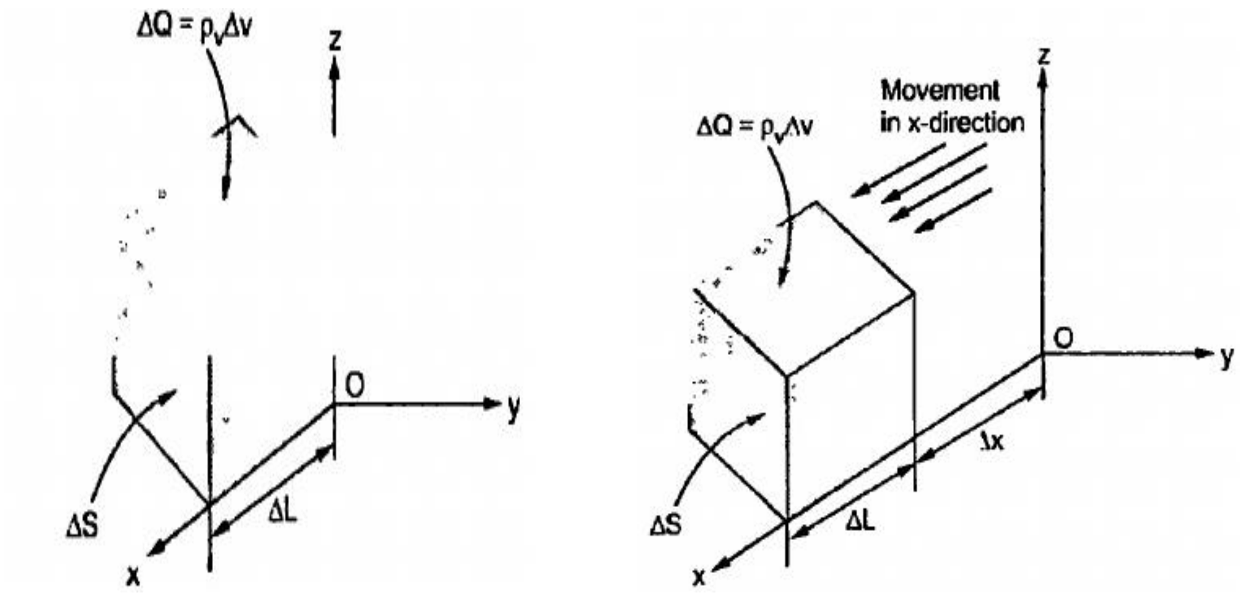

Fig 7: Incremental charge moving in x-direction

Consider differential volume Δv having charge in a volume a volume is ,

$Q = \rho v \Delta v$-------① $\Delta L \rightarrow Incremental\ length$

$\Delta s \rightarrow incremental\ surface\ area$

Hence incremental volume $\Delta v = \Delta s \Delta L$----②

$\Delta Q = \rho v \Delta s \Delta L$------③ $= \rho v \Delta s \Delta x$-------3a

Let the charge moving in x-direction with velocity ϑx

The current can be expressed as ,

$\Delta I = \frac{\Delta Q}{\Delta t}$ ---------④

Substitute eqn 3a in ④

$\Delta I = \rho v \Delta s \frac{\Delta x}{\Delta t}$, $\frac{\Delta x}{\Delta t} = ux$

$\Delta I = \rho v \Delta s u_x$

ux →velocity in x-direction

But ΔI =J.ds

J & ds are normal

Jx = $\rho v u_x$ →Convection current density

In general J=$\rho v \overline{u}$

u → velocity vector

The electron with mass m is moving in an electric field E with an average drift velocity u, according to Newton's law the average change in momentum of the free electron must match the applied force. Thus

$$\frac{mu}{T} = -QE$$

$$u = -\frac{QT}{m} E$$

Where T -> average time between collisions

If there are n charges per unit volume , the charge density is given by

$$\rho_v = -nQ$$

Thus the conduction current density is $J = \rho_v u = \frac{nQ^2T}{m} E = \sigma E$

Or $J = \sigma E$ is known as point form of ohm’s law.

Where $\sigma = \frac{nQ^2T}{m}$ is the conductivity of the conductor