3.4 DETERMINATION OF CLOSED LOOP RESPONSE FROM OPEN LOOP RESPONSE

M and N circles

Peak magnitude

$$M_r = 20\log\left|\frac{C(j\omega)}{R(j\omega)}\right| dB$$

where, 3 dB is considered good.

M-CIRCLES

$$M(j\omega) = \frac{G(j\omega)}{1+G(j\omega)}$$

$$G(j\omega) = X + jY$$

$$M(j\omega) = \frac{X+jY}{1+X+jY} = \frac{\sqrt{X^2+Y^2}\angle\tan^{-1}\left(\frac{Y}{X}\right)}{\sqrt{(1+X)^2+Y^2}\angle\tan^{-1}\left(\frac{Y}{1+X}\right)}$$

$$= \frac{\sqrt{X^2+Y^2}}{\sqrt{(1+X)^2+Y^2}}\angle\tan^{-1}\left(\frac{Y}{X}\right) - \tan^{-1}\left(\frac{Y}{1+X}\right)$$

Let, M = Magnitude of M(jω)

$$|M(j\omega)| = \frac{\sqrt{X^2+Y^2}}{\sqrt{(1+X)^2+Y^2}}$$

$$M^2(1+X)^2 + M^2Y^2 = X^2 + Y^2$$

$$X^2(1-M^2) + (1-M^2)Y^2 - 2M^2X = M^2$$

$$X^2 + Y^2 - 2\frac{M^2}{(1-M^2)}X = \frac{M^2}{(1-M^2)}$$

Adding $\left(\frac{M^2}{(1-M^2)}\right)^2$ on both sides, we get,

$$\left(X - \frac{M^2}{(1-M^2)}\right)^2 + Y^2 = \left(\frac{M}{(1-M^2)}\right)^2$$

The above equation represents a family of circles with its

centre at $\left(\frac{M^2}{(1-M^2)}, 0\right)$ and radius $\frac{M}{(1-M^2)}$

Family of M-circles corresponding to the closed loop magnitudes, M of a unit feedback system is given by the figure 3.4.1.

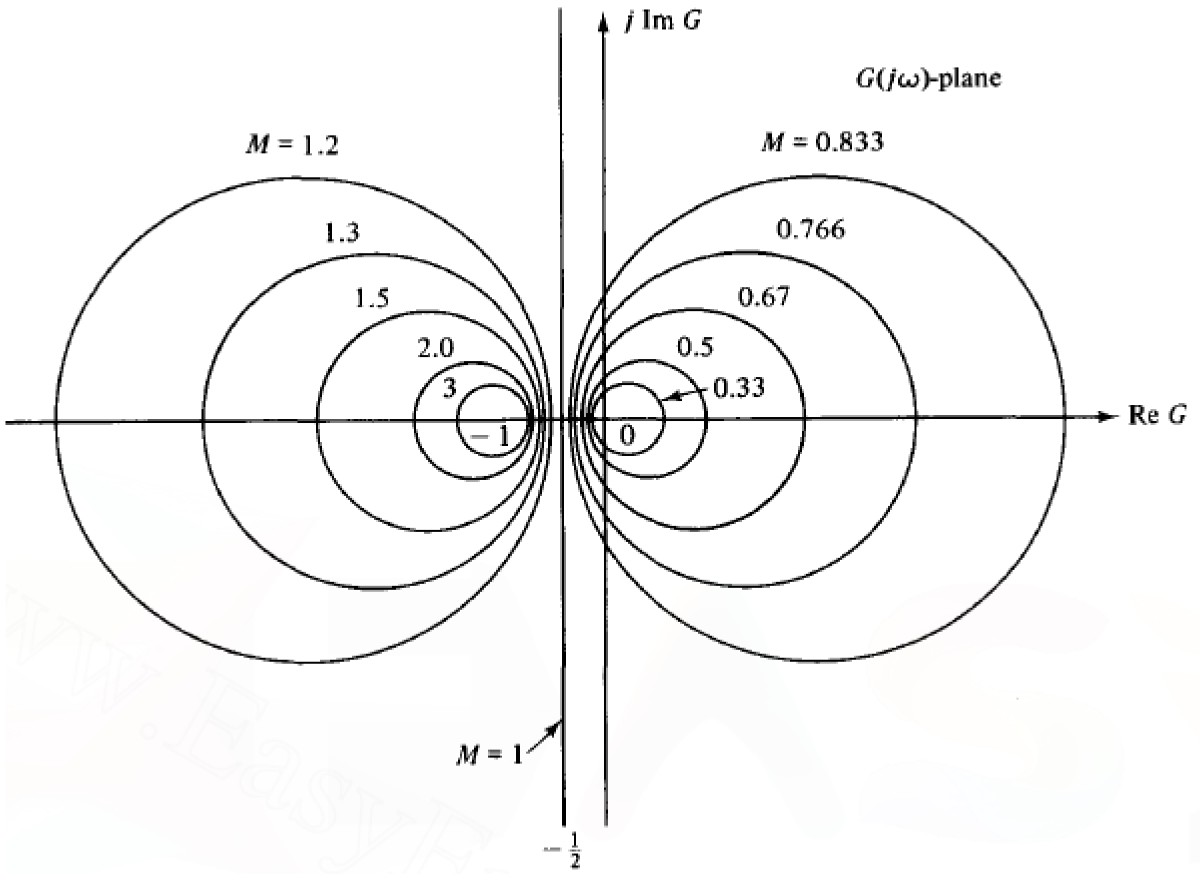

Figure 3.4.1 Constant M-circles in the polar co-ordinates

[Source: "Automatic Control Systems" by Benjamin C. Kuo, Page: 487]

N-CIRCLES

$$\angle M(j\omega) = \alpha = \frac{\angle G(j\omega)}{\angle(1 + G(j\omega))}$$

$$\alpha = \tan^{-1}\frac{Y}{X} - \tan^{-1}\frac{Y}{1+X}$$

$$\tan\alpha = N = \tan\left(\tan^{-1}\frac{Y}{X} - \tan^{-1}\frac{Y}{1+X}\right)$$

We know,

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\mathrm{N} = \left(\frac{Y}{X^2 + X + Y^2}\right)$$

$$\left(X + \frac{1}{2}\right)^2 + \left(Y - \frac{1}{2N}\right)^2 = \frac{1}{4} + \left(\frac{1}{2N}\right)^2$$

The above equation represents the family of circles with its

Centre at $\left(-\frac{1}{2}, \frac{1}{2N}\right)$ and radius $\sqrt{\frac{1}{4} + \left(\frac{1}{2N}\right)^2}$

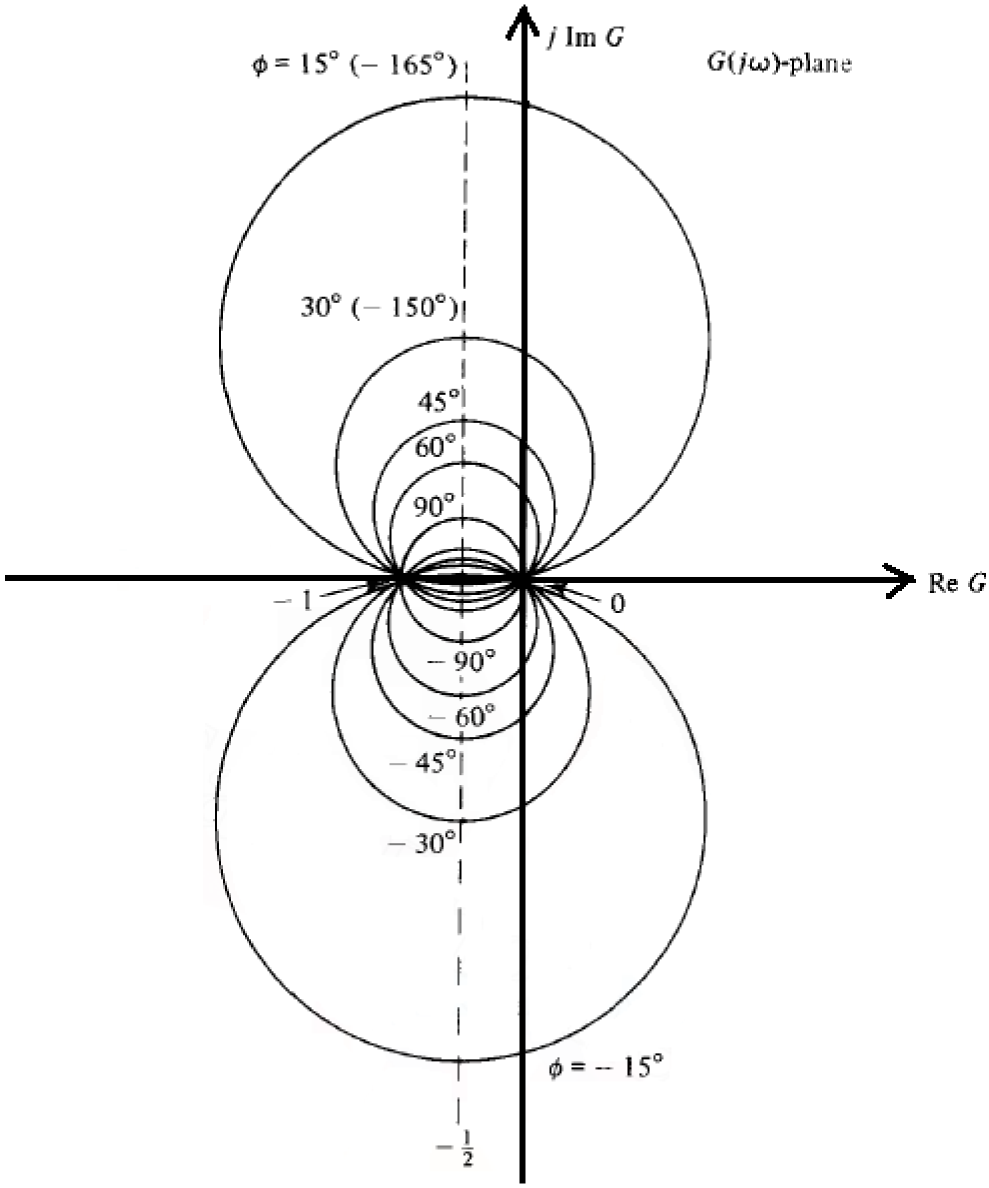

Figure 3.4.2 Constant N-circles in the polar co-ordinates

[Source: "Automatic Control Systems" by Benjamin C. Kuo, Page: 490]