Introduction to digital systems– Number system

Many number systems are in use in digital technology.
The most common are the decimal, binary, octal, and hexadecimal systems.
The decimal system is clearly the most familiar to us because it is tools that we use every day.

Types of Number Systems are

1. Decimal Number system
2. Binary Number system
3. Octal Number system
4. Hexadecimal Number system

DECIMAL	BINARY	OCTAL	HEXADECIMAL
0	0000	0	0
1	0001	1	1
2	0010	2	2
3	0011	3	3
4	0100	4	4
5	0101	5	5
6	0110	6	6
7	0111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

Numbering Systems		
System	**Base**	**Digits**
Binary	2	0 1
Octal	8	0 1 2 3 4 5 6 7
Decimal	10	0 1 2 3 4 5 6 7 8 9
Hexadecimal	16	0 1 2 3 4 5 6 7 8 9 A B C D E F

(a) Decimal Number System

The number system that we use in our day-to-day life is the decimal number system. Decimal number system has base 10 as it uses 10 digits from 0 to 9. In decimal number system, the successive positions to the left of the decimal point represents units, tens, hundreds, thousands and so on.

Each position represents a specific power of the base (10). For example, the decimal number 1234 consists of the digit 4 in the units position, 3 in the tens position, 2 in the hundreds position, and 1 in the thousands position, and its value can be written as

$(1\times1000) + (2\times100) + (3\times10) + (4\times1)$

$(1\times10^3) + (2\times10^2) + (3\times10^1) + (4\times10^0)$

$1000 + 200 + 30 + 1$

1234

S.No	Number System & Description
1	**Binary Number System** Base 2. Digits used: 0, 1
2	**Octal Number System** Base 8. Digits used: 0 to 7
3	**Hexa Decimal Number System** Base 16. Digits used: 0 to 9, Letters used: A- F

(b) Binary Number System

Characteristics

- Uses two digits, 0 and 1.
- Also called base 2 number system
- Each position in a binary number represents a 0 power of the base (2). Example: 2^0
- Last position in a binary number represents an x power of the base (2). Example: 2^x where x represents the last position - 1.

Example

Binary Number: 10101_2

Calculating Decimal Equivalent −

Step	Binary Number	Decimal Number
Step 1	10101_2	$((1 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0))_{10}$
Step 2	10101_2	$(16 + 0 + 4 + 0 + 1)_{10}$
Step 3	10101_2	21_{10}

Note: 10101_2 is normally written as 10101.

(c) Octal Number System

Characteristics

- Uses eight digits, 0,1,2,3,4,5,6,7.
- Also called base 8 number system
- Each position in an octal number represents a 0 power of the base (8). Example: 8^0
- Last position in an octal number represents an x power of the base (8). Example: 8^x where x represents the last position - 1.

Example

Octal Number − 12570_8

Calculating Decimal Equivalent −

Step	Octal Number	Decimal Number
Step 1	12570_8	$((1 \times 8^4) + (2 \times 8^3) + (5 \times 8^2) + (7 \times 8^1) + (0 \times 8^0))_{10}$
Step 2	12570_8	$(4096 + 1024 + 320 + 56 + 0)_{10}$
Step 3	12570_8	5496_{10}

Note: 12570_8 is normally written as 12570.

(d) Hexadecimal Number System

Characteristics

- Uses 10 digits and 6 letters, 0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F.
- Letters represents numbers starting from 10. A = 10, B = 11, C = 12, D = 13, E = 14, F = 15.
- Also called base 16 number system.
- Each position in a hexadecimal number represents a 0 power of the base (16). Example 16^0.
- Last position in a hexadecimal number represents an x power of the base (16). Example 16^x where x represents the last position - 1.

Example −

Hexadecimal Number: $19FDE_{16}$

Calculating Decimal Equivalent −

Step	**Hexadecimal Number**	**Decimal Number**
Step 1	$19FDE_{16}$	$((1 \times 16^4) + (9 \times 16^3) + (F \times 16^2) + (D \times 16^1) + (E \times 16^0))_{10}$
Step 2	$19FDE_{16}$	$((1 \times 16^4) + (9 \times 16^3) + (15 \times 16^2) + (13 \times 16^1) + (14 \times 16^0))_{10}$
Step 3	$19FDE_{16}$	$(65536 + 36864 + 3840 + 208 + 14)_{10}$
Step 4	$19FDE_{16}$	106462_{10}

Note − $19FDE_{16}$ is normally written as 19FDE.

4.1.1 Number System Conversion

There are many methods or techniques which can be used to convert numbers from one base to another.

- Decimal to Other Base System
- Other Base System to Decimal
- Other Base System to Non-Decimal
- Shortcut method − Binary to Octal
- Shortcut method − Octal to Binary
- Shortcut method − Binary to Hexadecimal
- Shortcut method − Hexadecimal to Binary

(i) Decimal to Other Base System

Steps

- **Step 1** − Divide the decimal number to be converted by the value of the new base.
- **Step 2** − Get the remainder from Step 1 as the rightmost digit (least significant digit) of new base number.
- **Step 3** − Divide the quotient of the previous divide by the new base.
- **Step 4** − Record the remainder from Step 3 as the next digit (to the left) of the new base number.

Repeat Steps 3 and 4, getting remainders from right to left, until the quotient becomes zero in Step 3.

The last remainder thus obtained will be the Most Significant Digit (MSD) of the new base number.

Example

Decimal Number: 29_{10}

Calculating Binary Equivalent −

Step	Operation	Result	Remainder
Step 1	29 / 2	14	1
Step 2	14 / 2	7	0
Step 3	7 / 2	3	1
Step 4	3 / 2	1	1
Step 5	1 / 2	0	1

As mentioned in Steps 2 and 4, the remainders have to be arranged in the reverse order so that the first remainder becomes the Least Significant Digit (LSD) and the last remainder becomes the Most Significant Digit (MSD).

Decimal Number − 29_{10} = Binary Number − 11101_2.

(ii) Other Base System to Decimal System

Steps

- **Step 1** − Determine the column (positional) value of each digit (this depends on the position of the digit and the base of the number system).
- **Step 2** − Multiply the obtained column values (in Step 1) by the digits in the corresponding columns.
- **Step 3** − Sum the products calculated in Step 2. The total is the equivalent value in decimal.

Example

Binary Number − 111012

Calculating Decimal Equivalent −

Step	**Binary Number**	**Decimal Number**
Step 1	11101_2	$((1 \times 2^4) + (1 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0))_{10}$
Step 2	11101_2	$(16 + 8 + 4 + 0 + 1)_{10}$
Step 3	11101_2	29_{10}

Binary Number − 11101_2 = Decimal Number − 29_{10}

(iii) Other Base System to Non-Decimal System

Steps

- **Step 1** − Convert the original number to a decimal number (base 10).
- **Step 2** − Convert the decimal number so obtained to the new base number.

Example

Octal Number − 25_8

Calculating Binary Equivalent −

- Step 1 − Convert to Decimal

Step	**Octal Number**	**Decimal Number**
Step 1	25_8	$((2 \times 8^1) + (5 \times 8^0))_{10}$
Step 2	25_8	$(16 + 5\ )_{10}$
Step 3	25_8	21_{10}

Octal Number − 25_8 = Decimal Number − 21_{10}

- Step 2 − Convert Decimal to Binary

Step	**Operation**	**Result**	**Remainder**
Step 1	21 / 2	10	1
Step 2	10 / 2	5	0
Step 3	5 / 2	2	1
Step 4	2 / 2	1	0
Step 5	1 / 2	0	1

Decimal Number − 21_{10} = Binary Number − 10101_2

Octal Number − 25_8 = Binary Number − 10101_2

(iii) Shortcut method - Binary to Octal

Steps

- **Step 1** − Divide the binary digits into groups of three (starting from the right).
- **Step 2** − Convert each group of three binary digits to one octal digit.

Example

Binary Number − 10101_2

Calculating Octal Equivalent −

Step	**Binary Number**	**Octal Number**
Step 1	10101_2	010 101
Step 2	10101_2	2_8 5_8
Step 3	10101_2	25_8

Binary Number − 10101_2 = Octal Number − 25_8

(iv) Shortcut method - Octal to Binary

Steps

- **Step 1** − Convert each octal digit to a 3 digit binary number (the octal digits may be treated as decimal for this conversion).
- **Step 2** − Combine all the resulting binary groups (of 3 digits each) into a single binary number.

Example

Octal Number − 25_8

Calculating Binary Equivalent −

Step	**Octal Number**	**Binary Number**
Step 1	25_8	2_{10} 5_{10}
Step 2	25_8	010_2 101_2
Step 3	25_8	010101_2

Octal Number − 25_8 = Binary Number − 10101_2

(v) Shortcut method - Binary to Hexadecimal

Steps

- **Step 1** − Divide the binary digits into groups of four (starting from the right).
- **Step 2** − Convert each group of four binary digits to one hexadecimal symbol.

Example

Binary Number – 10101_2

Calculating hexadecimal Equivalent –

Step	Binary Number	Hexadecimal Number
Step 1	10101_2	0001 0101
Step 2	10101_2	1_{10} 5_{10}
Step 3	10101_2	15_{16}

Binary Number – 10101_2 = Hexadecimal Number – 15_{16}

(vi) Shortcut method - Hexadecimal to Binary

Steps

- **Step 1** – Convert each hexadecimal digit to a 4 digit binary number (the hexadecimal digits may be treated as decimal for this conversion).
- **Step 2** – Combine all the resulting binary groups (of 4 digits each) into a single binary number.

Example

Hexadecimal Number – 15_{16}

Calculating Binary Equivalent –

Step	Hexadecimal Number	Binary Number
Step 1	15_{16}	1_{10} 5_{10}
Step 2	15_{16}	0001_2 0101_2
Step 3	15_{16}	00010101_2

Hexadecimal Number – 15_{16} = Binary Number – 10101_2

Number System Conversion Table

Decimal Base-10	Binary Base-2	Octal Base-8	Hexadecimal Base-16
0	0	0	0
1	1	1	1
2	10	2	2
3	11	3	3
4	100	4	4
5	101	5	5
6	110	6	6
7	111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F
16	10000	20	10
17	10001	21	11
18	10010	22	12
19	10011	23	13
20	10100	24	14
21	10101	25	15
22	10110	26	16
23	10111	27	17
24	11000	30	18
25	11001	31	19
26	11010	32	1A
27	11011	33	1B
28	11100	34	1C
29	11101	35	1D
30	11110	36	1E
31	11111	37	1F
32	100000	40	20