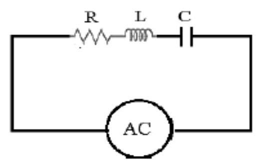
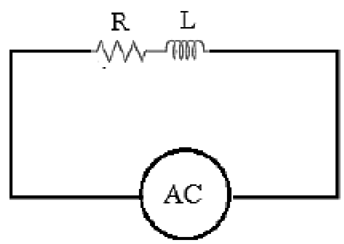
Derive the expression for the current, power, power factor and impedance for series RL and series RC CIRCUITS connected to an AC Voltage source

Solution

RLC Circuit



RL Circuit



R_L circuit

$$V_R = IR$$

$$V_L = I_{XL} \quad (X_L = 2\pi fL)$$

$$V = V_R + jV_L = IR + jI_{XL}$$

$$= I[R + jX_L]$$

$$\boxed{V = IZ}$$

$$Z \text{ is impedance} = R + jX_L$$

$$|Z| = \sqrt{R^2 + X_L^2}$$

$$\angle Z = \theta = \tan^{-1}\frac{X_L}{R}$$

$$\boxed{Z = |Z|\angle\theta}$$

Real power P = VI cos θ

Reactive power θ = VI sin θ

Apparent power S = VI

$$\text{power factor } \cos\phi = \frac{R}{Z}$$

Series R-C circuit:

$$V_R = IR$$

$$V_C = IX_C = I\left(\frac{1}{c\omega}\right)$$

$$X_C = \frac{1}{2\pi fC}$$

$$\begin{aligned} V &= V_R - jV_C \\ &= I_R - jIX_C \\ &= I[R - jX_C] \end{aligned}$$

$$\boxed{V = IZ}$$

Z is impedance = $R - jX_C$

$$|Z| = \sqrt{R^2 + X_C^2}$$

$$\angle Z = \theta = \tan^{-1}\left[\frac{X_C}{R}\right]$$

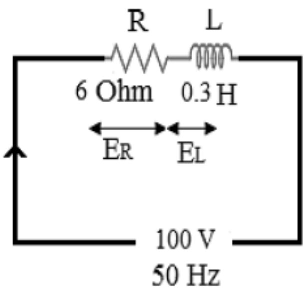
Power factor = $\cos\theta$

Real power $P = VI\cos\theta$

Reactive power $\theta = VI\sin\theta$

Apparent power $S = VI$

RL series circuit having a resistance of 6Ω and an inductance of 0.03 H is connected across a 100 V 50 Hz, ac supply. Calculate the phase angle between the current and voltage and the power factor. **(May/June 2014)**



Solution

$$\text{Inductive Reactance } X_L = 2\pi f L$$
$$= 2\pi \times 50 \times 0.3$$
$$X_L = 94.2$$

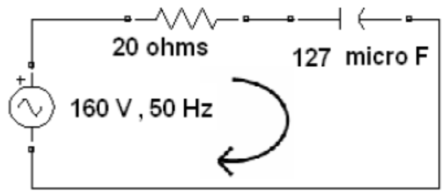
$$\text{Impedance } Z = \sqrt{R^2 + X_L^2} = \sqrt{6^2 + 94.2^2}$$
$$= \sqrt{36 + 8873.64}$$
$$= \sqrt{8909.64}$$
$$Z = 94.390$$

$$\text{Circuit current } I = \frac{E}{Z} = \frac{100}{94.39}$$
$$I = 1.059$$

$$\text{Phase angle } \phi = \tan^{-1}\left(\frac{\omega L}{R}\right) = \tan^{-1}\left(\frac{94.2}{6}\right)$$
$$= \tan^{-1}(15.7)$$
$$\phi = 86.3555$$

$$\text{Power factor} = \cos\phi = \cos(86.555)$$
$$= -\,0.03828$$

A series R-C circuit with R = 20 Ω *and* C = 127 μF has 160 V, 50 Hz supply connected to it. Find the impedance, current, power, power factor, Voltage drop across R, and C.



Solution

Given:

R = 20Ω
C = 127μF

$$X_C = \frac{1}{2\pi fC}$$

$$X_C = \frac{1}{2\pi \times 50 \times 127 \times 10^{-6}}$$

$$X_C = 25.06\Omega$$

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{(20)^2 + (25.06)^2} = 32.06\Omega$$

$$I = \frac{V}{Z} = \frac{160}{32.06} = 4.99A$$

$$\text{Power factor : } \frac{R}{Z} = \frac{20}{25.06} = 0.798$$

$$\text{Power } (P) = VI\cos\phi$$

$$= 160 \times 4.99 \times 0.798$$

$$\boxed{P = 637.12 \text{ w}}$$

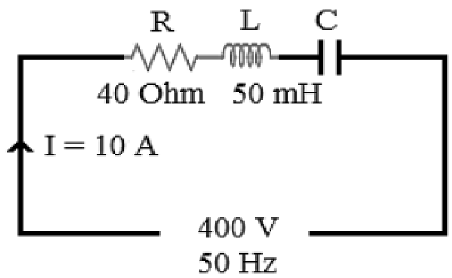
$$V_R = IR = 4.99 \times 20 = 99.8 \text{ V}$$

$$V_C = IX_C = 4.99 \times 25.06 = 125.05 \text{ V}$$

$$\therefore V_R = 99.8V$$

$$\boxed{V_C = 125.05V}$$

A series circuit having pure resistance of 40Ω, pure inductance of 50 mH and a capacitor is connected across a 400 V, 50 Hz ac supply. This RLC circuit draws a current of 10 A. Calculate a) Power factor of the circuit b) Capacitor value.



Solution

$$\text{Current } I = \frac{E}{Z}$$

$$\text{Impedance} = Z = \frac{E}{I} = \frac{400}{10} = 40\ \Omega$$

$$\cos\phi = \frac{R}{Z} = \frac{40}{40} = 1$$

$$\text{Inductive reactance} = X_L = 2\pi f L = 2\pi \times 50 \times 50 \times 10^{-3} = 15.7\Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$40 = \sqrt{40^2 + (X_L - X_C)^2}$$

$$40^2 = 40^2 + (X_L - X_C)^2$$

$$(X_L - X_C)^2 = 0$$

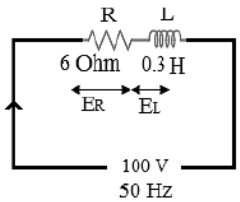
$$X_L = X_C = 15.7\Omega$$

$$X_C = 15.7 = \frac{1}{2\pi f C}$$

$$C = \frac{1}{2\pi \times 50 \times 15.7}$$

$$C = 202.74\mu F$$

RL series circuit having a resistance of 6Ω and an inductance of 0.03 H is connected across a 100 V, 50 Hz, ac supply. Calculate the phase angle between the current and voltage and the power factor **(May/June 2014)**



Solution

$$\text{Inductive Reactance } X_L = 2\pi fL$$
$$= 2\pi \times 50 \times 0.3$$
$$X_L = 94.2$$

$$\text{Impedance } Z = \sqrt{R^2 + X_L^2} = \sqrt{6^2 + 94.2^2}$$
$$= \sqrt{36 + 8873.64}$$
$$= \sqrt{8909.64}$$
$$Z = 94.390$$

$$\text{Circuit current } I = \frac{E}{Z} = \frac{100}{94.39}$$
$$I = 1.059$$

$$\text{Phase angle } \phi = \tan^{-1}\left(\frac{\omega L}{R}\right) = \tan^{-1}\left(\frac{94.2}{6}\right)$$
$$= \tan^{-1}(15.7)$$
$$\phi = 86.3555$$

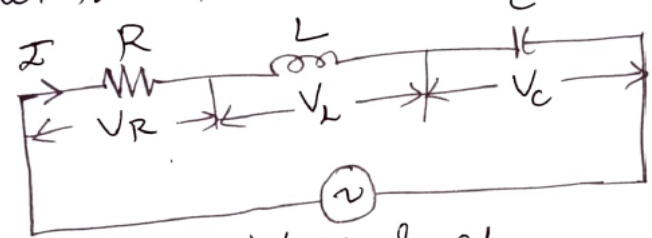
Instantaneous Power:

The Product of instantaneous Voltage and instantaneous Current is Called Instantaneous Power.

$$P = VI \cos\phi \text{ Watts.}$$

Steady State Analysis of RLC Circuits:

Consider the RLC Series Circuits Shown in figure.



$V = V_m \sin\omega t$

Supply Voltage $V = V_m \sin\omega t$ Volts

Supply Current I Amps.

Voltage across resistance, $V_R = IR$ Volts

Voltage across Inductance, $V_L = IX_L$ Volts

Voltage across Capacitance, $V_C = IX_C$ Volts.

Impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2}\ \Omega$.

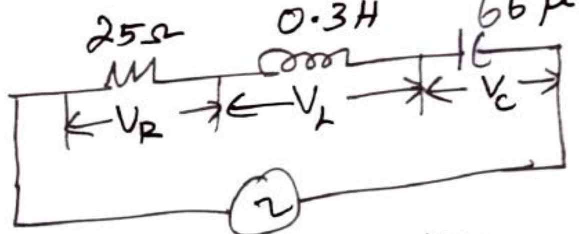
$X_L = 2\pi f L\ \Omega$ & $X_C = \frac{1}{2\pi f C}\ \Omega$

$I = \frac{V}{Z}$ Amps

Power $= VI \cos\phi$ Watts.

A Series RLC Circuits with $R = 25\Omega$, $L = 0.3H$ and $C = 66\mu F$ is Supplied at 220V, 50Hz. Find the impedance of the Circuit, Current and Voltage across each Element Power, Reactive power and power factor.

Solution:



220V, 50HZ

Given Datas:

$R = 25\Omega$
$L = 0.3H$
$C = 66\mu F$
$V = 220V$
$f = 50Hz$

Find i) impedance (Z)
ii) Current (I)
iii) V_R, V_L & V_C
iv) Power
v) Reactive power
vi) Power factor.

i) Impedance (Z)

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Inductive Reactance $\Big\}$ $X_L = 2\pi f L = 2 \times \pi \times 50 \times 0.3$

$$X_L = 94.24\Omega.$$

Capacitive Reactance } $X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi 50 \times 66 \times 10^{-6}}$

$$X_C = 48.22\,\Omega$$

$$\therefore Z = \sqrt{25^2 + (94.24 - 48.22)^2}$$

$$Z = 52.37\,\Omega$$

ii) Current, $I = \frac{V}{Z} = \frac{220}{52.37} = 4.2\,A.$

iii) a) Voltage across the resistor, V_R

$$V_R = I \cdot R = 4.2 \times 25 = 105 \text{ volts}$$

b) Voltage across the Inductor, V_L

$$V_L = I X_L = 4.2 \times 94.24 = 395.8\,V$$

c) Voltage across the Capacitor, V_C

$$V_C = I X_C = 4.2 \times 48.22 = 202.52\,V$$

iv) Power $= I^2 R = 4.2^2 \times 25 = 441$ Watts.

v) Power factor $= \cos\phi = \frac{R}{Z} = \frac{25}{52.37} = 0.477$ (lag)

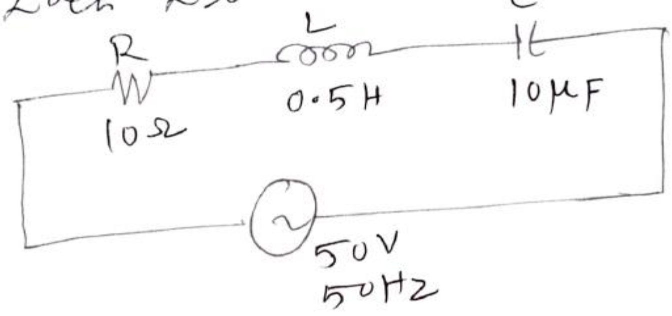
vi) Reactive power, Q.

$$Q = VI\sin\phi = 220 \times 4.2 \sin 61.5 = 812 \text{ VAR}$$

$\cos\phi = 0.477$

$\phi = \cos^{-1} 0.477 = 61.51$

In the Circuit given determine the total impedance Current, Phase angle and the Voltage across Each Elements



Solution:

Given datas: $V = 50V$ $R = 10\Omega$

$f = 50Hz$ $L = 0.5H$

$C = 10\mu F$

Find i) Impedance, Z

ii) Current, I

iii) Phase angle θ (or) ϕ

iv) Voltage across Resistor, V_R

Voltage across Inductor, L_L

Voltage across Capacitor, V_C

i) Impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

Inductive Reactance, $X_L = 2\pi f L$

$= 2 \times \pi \times 50 \times 0.5$

$\boxed{X_L = 157\Omega}$

Capacitive Reactance, $X_C = \frac{1}{2\pi f C} = \frac{1}{2\times\pi\times 50\times 10\times 10^{-6}}$

$$X_C = 318.3\ \Omega$$

$\therefore Z = \sqrt{10^2 + (157 - 318.3)^2} = \sqrt{10^2 + (-161.3)^2}$

$= \sqrt{100 + 26017.69} = 161.6\ \Omega$

$$Z = 161.6\ \Omega$$

ii) Current, $I = \frac{V}{Z} = \frac{50}{161.6} = 0.31$ Amps

$$I = 0.31 \text{ Amps}$$

iii) Phase angle (θ) (or) (ϕ)

$\cos\phi = \frac{R}{Z} = \frac{10}{161.6} = 0.0618$

$\phi = \cos^{-1}(0.0618) = 86.45$

$$\phi = 86.46^\circ$$

iv) Voltage across the resistor, $V_R = I \times R$

$= 0.31 \times 10 = 3.1$ V.

$$V_R = 3.1 \text{ Volts}$$

v) Voltage across the Inductor $= V_L = I X_L = 0.31 \times 157$

$$V_L = 48.67 \text{ Volts}$$

vi) Voltage across the Capacitor $= V_C = I X_C = 0.31 \times 318.3$

$$V_C = 98.67 \text{ Volts}$$