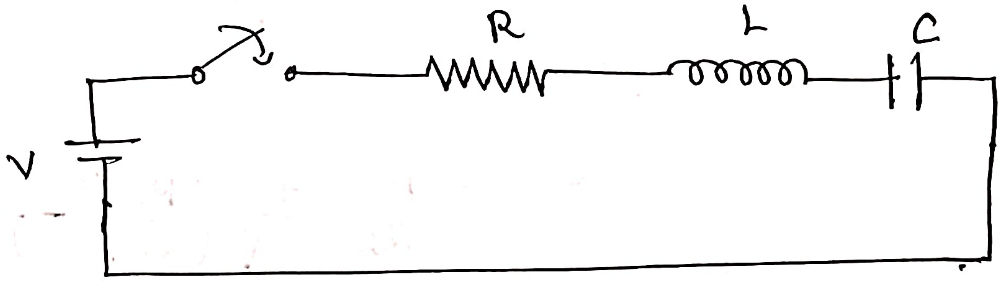
Tranciemt Response of RLC circuit.

Forced Responce of RLC circuit

Driven Responce of RLC circuit

An RLC circuit is connected to a DC battery through a switch. Assume the initial current in the circuit tobe 0 and voltage in capacitor be 0.



Apply KVL

$$V = V_R + V_L + V_C$$

$$= iR + L \cdot \frac{di}{dt} + \frac{1}{C}\int i \cdot dt$$

Taking Laplace transform on bothsides.

$$\frac{V}{S} = R\,I(S) + L\left[S\,I(S) - \cancel{I(0)}^{0}\right] + \frac{1}{C}\frac{I(S)}{S}$$

$$= I(S)\left[R + LS + \frac{1}{CS}\right]$$

$$I(S) = \frac{V}{S\left[R + LS + \frac{1}{CS}\right]}$$

$$= \frac{V}{S^2L + SR + \frac{1}{C}}$$

(24)

$$I(s) = \frac{V/L}{\left(s^2 + s\frac{R}{L} + \frac{1}{LC}\right)}$$

$$s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-R/L \pm \sqrt{(R/L)^2 - \frac{4}{LC}}}{2}$$

$$= \underbrace{-\frac{R}{2L}}_{\Downarrow} \pm \underbrace{\sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}}_{\Downarrow}$$

$$s = \alpha \pm \beta$$

If roots are real & distinct $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$.

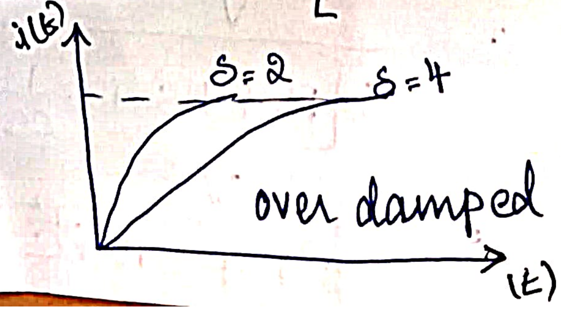
$s_1 = \alpha + \beta \qquad s_2 = \alpha - \beta$.

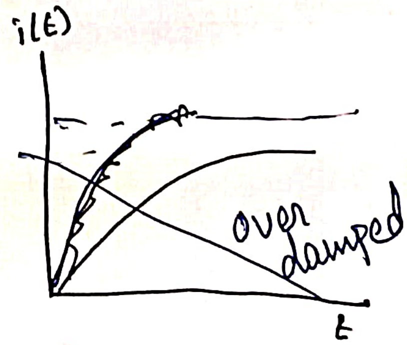
$$I(s) = \frac{A}{s - (\alpha+\beta)} + \frac{B}{s - (\alpha-\beta)}$$

Taking inverse Laplace Transform.

$$i(t) = Ae^{(\alpha+\beta)t} + Be^{(\alpha-\beta)t}.$$

$$= e^{\alpha t}\left[Ae^{\beta t} + Be^{-\beta t}\right]$$





<u>case ii</u> Roots are real. $\left(\frac{R}{2L}\right)^2 - \frac{1}{LC} = 0.$

$$\therefore \left(\frac{R}{2L}\right)^2 = \frac{1}{LC}.$$

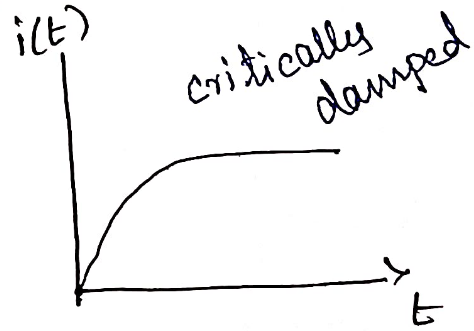
$\therefore$ The roots are $(s-\alpha)^2$ and $(s-\alpha)$

$$I(s) = \frac{A}{(s-\alpha)^2} + \frac{B}{(s-\alpha)}$$

Taking inverse Laplace Transform.

$$i(t) = Ate^{\alpha t} + Be^{\alpha t}.$$

$$i(t) = e^{\alpha t}(At+B)$$



<u>when roots are complex</u>

$$\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$$

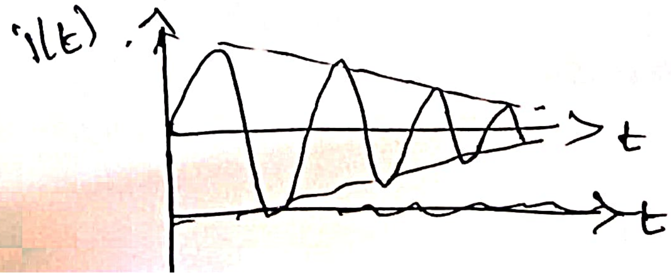
$\therefore$ The roots are $(\alpha+j\beta)$ & $(\alpha-j\beta)$

$$I(s) = \frac{A}{s-(\alpha+j\beta)} + \frac{B}{s-(\alpha-j\beta)}.$$

Taking inverse Laplace transform.

$$i(t) = Ae^{(\alpha+j\beta)t} + Be^{(\alpha-j\beta)}$$

$$i(t) = e^{\alpha t}\left[Ae^{j\beta t} + Be^{-j\beta t}\right]$$



This says the current is oscillating and decaying.