Electric flux density :

The flux due to the electric field 'E' can be calculated by using the definition of the flux.

According to eqn $E = \frac{Q}{4\pi\varepsilon_0 R^2}\ aR$

the electric field intensity is dependent on the medium in which the charge is place.

New vector D is defined by

$$D = \varepsilon_0\, E \qquad \ldots\ldots\ldots\ldots \qquad (1)$$

The electric flux Ψ in terms of D ,

$$\Psi = \int_s\ D.ds \qquad \ldots\ldots\ldots\ldots \qquad (2)$$

$D\ \rightarrow\ electric\ flux\ density\ \rightarrow\ c/m^2$

$x \rightarrow measured\ in\ coloumb's$

Electric flux density is also called electric displacement

Example : For an infinite sheet of charge

$$E = \frac{\rho_s}{2\varepsilon_0}\ an \qquad \ldots\ldots\ldots\ldots \qquad (3)$$

$$D = \varepsilon_0 E = \frac{\varepsilon_0 \rho_s}{2\varepsilon_0} a_n$$

$$D = \frac{\rho_s}{2} a_n \qquad \ldots\ldots\ldots \qquad (4)$$

For a volume charge distribution

$$a = \int_v\ \rho_{v\ dv}$$

$$D = \int_v \frac{\rho_{v\ av}}{4\pi R^2}\ aR \qquad \ldots\ldots\ldots\ldots \qquad (5)$$

From eqn (4) & (5) 'D' is a function of charge & position only it is independent of medium

Gauss's law – Maxwell's equation

Gauss's law states that the total electric flux Ψ through any closed surface is equal to the total charge enclosed by that surface.

$$\Psi = \text{Q enc} \qquad \ldots\ldots\ldots\ldots \quad (1)$$

ie $\Psi = \oint d\Psi = \oint D.ds$

total charge enclosed $Q = \int_v \rho_v \, dv$

$$Q = \oint D.ds = \int_v \rho_v \, dv \quad \ldots\ldots..(2)$$

By applying divergence theorem

$$\oint D.ds = \int_v \nabla.D \, dv \qquad \ldots\ldots. (3)$$

Compare the two volume integrals in eqn (2) & (3)

$$\rho_v = \nabla.D \qquad \ldots\ldots \quad (4)$$

The above equation is a Maxwell first equation. The equation states that volume charge density is same as the divergence of electric flux density

ρ_v at any point – charge per unit volume at that point.

Note :

- Equation (A) & (4) state Gauss's law in different ways eqn (A) is the integral form
  Eqn (4) is the differential (or) point form of Gauss's law
- Gauss's law is an alternative statement of Coulomb's law. Proper application of divergence theorem to Coulomb's law results Gauss's Law.
- By using Gauss's law E (or) D for symmetrical charge distributions such as point charge an infinite line charge an infinite cylindrical surface charge and a spherical distribution of charge.
- Rectangular symmetry depends on x $(or\ y\ or\ z)$
- Cylindrical symmetry depends on ρ
- Spherical symmetry depends on r (independent of θ & φ)

Gauss's law cannot be used to determine E (or) D. When the charge distribution is not symmetric.

Applications of Gauss's law :

- Gauss's law is used to calculate the electric field whether symmetry exists
- Surface of D is chosen such that D is normal or tangential to the Gaussian surface.

D is normal to the surface

$$D.ds = Dds$$

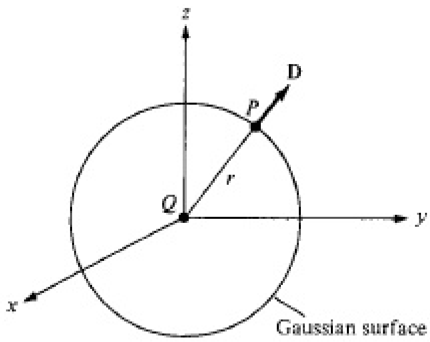
D is tangential to the surface

$$D.ds = 0$$

Point charge :

A point charge Q is located at the origin. To determine D at point p. Here spherical surface containing 'p' satisfy the symmetry conditions is used.

Spherical surface centered at the origin is the Gaussian surface



Gaussian Surface

Fig25. Gaussian surface about a point charge

D is normal to the Gaussian surface

$$D = D_r . a_r \quad \ldots\ldots\ldots\ldots \quad (1)$$

Gauss's law $\quad Q = \oint_s D.\, ds \quad \ldots\ldots\ldots \quad (2)$

$$Q = \oint_s D_r .\ ds = D_r\, 4\pi r^2 \quad \ldots\ldots\ldots \quad (3)$$

$$\oint_s ds = 4\pi r^2 = \int_{\varphi=0}^{2\pi} \int_{\theta=0}^{\pi} r^2\, Sin\theta\, d\theta\, d\varphi$$

$$D_r = D = \frac{Q}{4\pi r^2}\ ar \qquad …….. \qquad (4)$$

Infinite line charge :

Infinite line of uniform charge $\rho_L\ \ c/m$ along z- axis. To determine D at point p

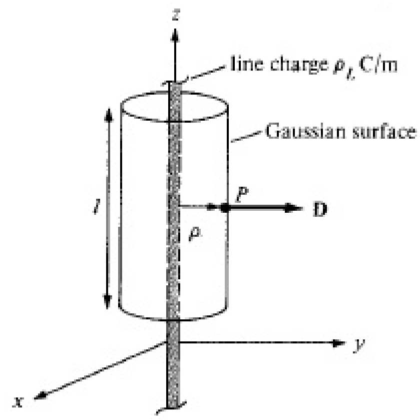


Fig26. Gaussian surface about an infinite line charge

The electric flux density 'D' is constant on and normal to the cylindrical Gaussian surface

$$D = D_\rho a_\rho$$

Apply Gauss's law to an arbitrary length l of the line.

$$\rho_l\, L = Q = \oint_s D.\, ds = D_\rho \oint_s ds$$

$$= D_\rho\, 2\pi\rho l$$

$\oint_s ds = 2\pi\rho l$ → Surface area of the Gaussian surface.

In the top & bottom of the surface 'D' is tangential to the Gaussian surface.

$$\oint_s D\,.ds = 0, \quad D_\rho = D = \frac{Q\,\rho_L\, l}{2\pi\rho l} = \frac{\rho_L}{2\pi\rho} a_\rho$$

Infinite sheet of charge :

Infinite sheet of uniform charge $\rho_s \; c/m^2$ lying on the z=0 plane. To determine 'D' at point. $'P'$

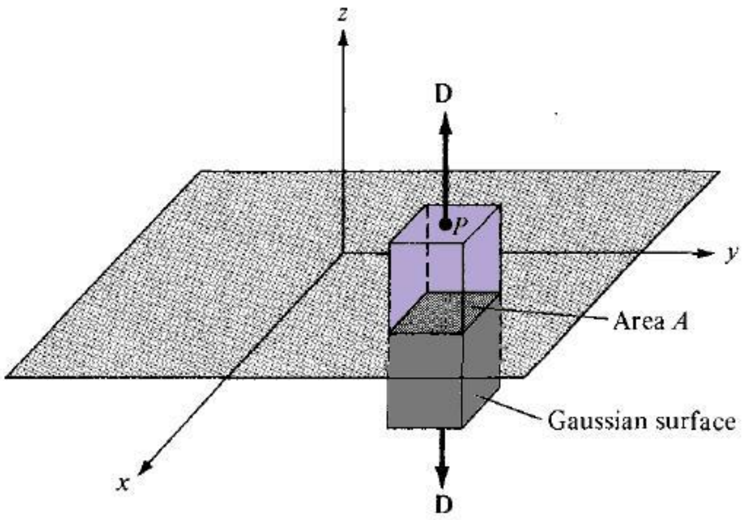


Fig6. Gaussian surface about an infinite sheet of charge

D is normal to the sheet

$$D = D_z . a_z$$

Apply Gauss's law

$$\rho_s \oint_s ds = Q = \oint D. \; ds$$

$$= D_z \left[\int_{top} ds + \int_{bottom} ds \right]$$

'D' has no x & y components. The surface area of the box is 'A'.

$$\rho_s . A = D_z \; [A + A]$$

$$D_z = D = \frac{\rho_s}{2} a_z \text{ (or)}$$

$$E = \frac{D}{\varepsilon_0} = \frac{\rho_s}{2\varepsilon_0} a_z$$

Uniformly charged sphere :

Consider a sphere of radius 'a' with a uniform charge $\rho_o\ c/m^3$. To determine D everywhere construct Gaussian surface for $r \le a$ & $r \ge a$. Separately

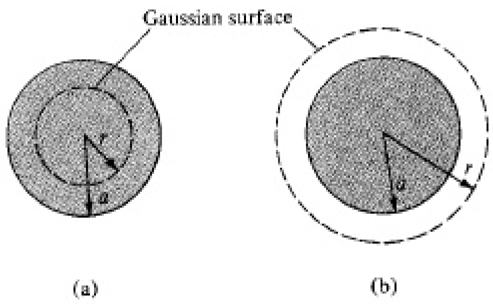


Fig27. Gaussian surface for a uniformly charged sphere $a)\ r \le a \quad b)\ r \ge a$

For $r \le a$. The total charge enclosed by the sphere of radius r is

$$Q_{enc} = \int_v \rho_v\, dv = \rho_v \int_v dv = \rho_v \int_{\varphi=0}^{2\pi} \int_{\theta=0}^{\pi} \int_{r=0}^{r} r^2 \sin\theta\, dr\, d\theta\, d\varphi$$

$$Q_{enc} = \rho_v \frac{4\pi}{3} r^3 \qquad \ldots\ldots\ldots\ldots\ldots (1)$$

and

$$\Psi = \oint_s D.ds = \oint_s D_r.ds$$

$$= D_r \int_{\varphi=0}^{2\pi} \int_{\theta=0}^{\pi} r^2 \sin\theta\, d\theta\, d\varphi$$

$$= D_r\, 4\pi r^2 \qquad \ldots\ldots\ldots \quad (2)$$

Hence $\Psi = Q_{enc}$

Equate (1) & (2)

$$D_r\, 4\pi r^2 = \frac{4\pi}{3} r^3 \rho_v$$

$$D_r = D = \frac{r}{3}\rho_v\, dr \qquad 0 < r \leq a$$

For $r \geq a$ The charge enclosed by the surface is

$$Q_{enc} = \int_v \rho_v\, dv = \rho_v \int_v dv = \rho_v \int_{\varphi=0}^{2\pi}\int_{\theta=0}^{\pi}\int_{r=0}^{r} r^2 \sin\theta\, dr\, d\theta\, d\varphi$$

$$Q_{enc} = \rho_v \frac{4\pi}{3} a^3 \qquad \ldots\ldots\ldots\ldots\ldots (3)$$

while $\Psi = \oint_s D.ds = D_r \oint_s ds$

$$= D_r\, 4\pi r^2 \qquad \ldots\ldots\ldots \quad (4)$$

Equate (3) & (4)

$$D_r\, 4\pi r^2 = \frac{4\pi}{3} a^3 \rho_v$$

$$D_r = \frac{a^3}{3r^2}\rho_v\, dr \quad \ldots\ldots\ldots \quad (5)$$

$$D = \frac{r}{3}\rho_v a_r \qquad 0 \leq r \leq a$$

$$D = \frac{a^3}{3r^2}\rho_v a_r \qquad r \geq a$$

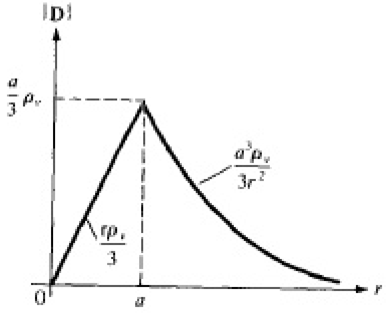


Fig28. Sketch of $|D|$ against r for uniformly charged sphere