unit-2. ①

Network reduction Theorems.

Network reduction - voltage and current division - source transformation - star delta conversion - Theorems - superposition, Thevenin's and Norton's Theorem - maximum power transfer theorem - Reciprocity Theorem - statement, application to DC and AC circuits.

Network reduction :-

Network reduction is effectively a mathematical process that combines and eliminates network nodes and branches, so that the reduced circuits have smaller number of nodes and branches when compared to the original circuit. The essential characteristic of the network is not disturbed.

Advantages :- Faster computations, reduced resource consumption and easier analysis.

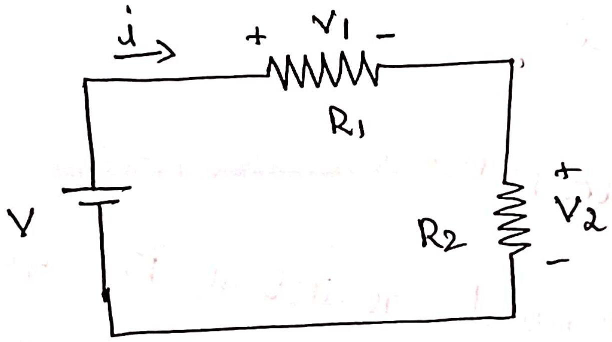
Appln :- power systems, computer networks, enabling efficient evaluation of system

(2) Performance.

Voltage division

The voltage division rule states that in a series circuit, the voltage across each resistor is directly equal proportional to ~~its~~ product of that resistance & total supply voltage divided by the total resistance of the series circuit

Let us consider the circuit below,



Apply KVL

$$V = V_1 + V_2$$
$$= iR_1 + iR_2$$
$$V = i(R_1 + R_2)$$
$$i = \frac{V}{R_1 + R_2}$$

The voltage drop across the resistor R_2 is,

$$V_2 = iR_2 = \frac{V}{R_1+R_2} \cdot R_2$$
$$V_2 = V\left(\frac{R_2}{R_1+R_2}\right)$$

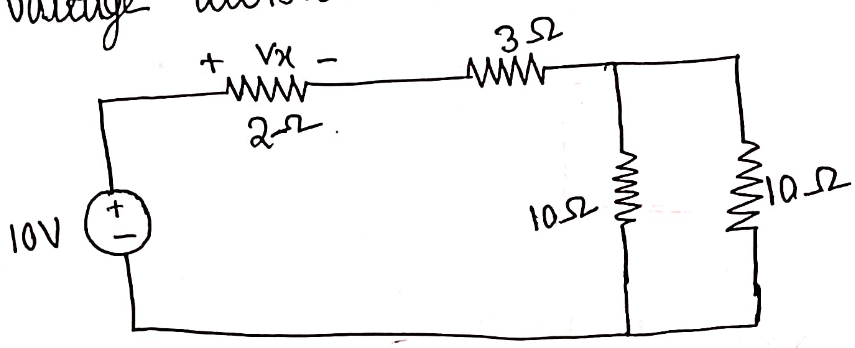
The voltage drop across the resistor R_1 is,

$$V_1 = i R_1$$

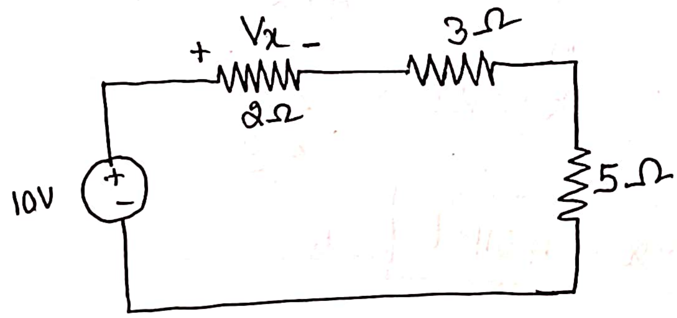
$$= \frac{V}{R_1 + R_2} \cdot R_1$$

$$\boxed{\therefore V_1 = V \cdot \frac{R_1}{R_1 + R_2}}$$

1) use voltage division to determine V_x in the ckt.

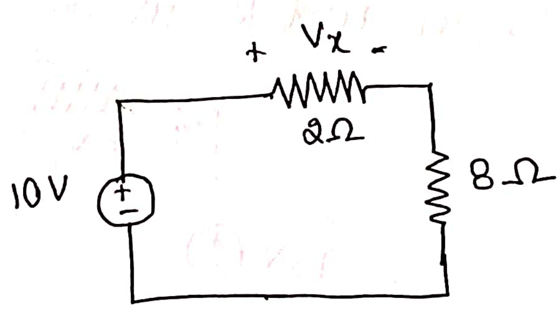


Solu



$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

$$= \frac{\cancel{10} \times \cancel{10}^{5}}{\cancel{20}_{\cancel{2}}}$$



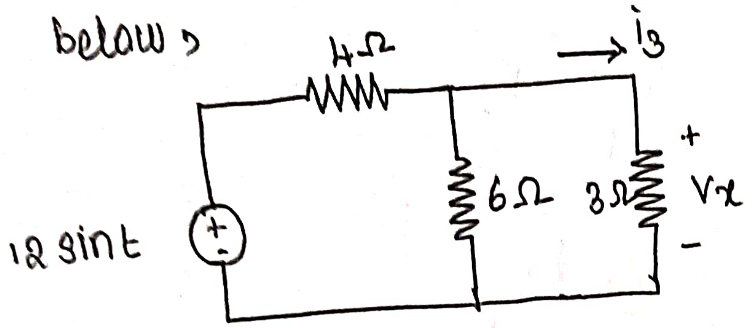
By voltage divider Rule,

$$V_x = 10V \times \frac{2}{2+8} = \cancel{10} \times \left(\frac{2}{\cancel{10}}\right)$$

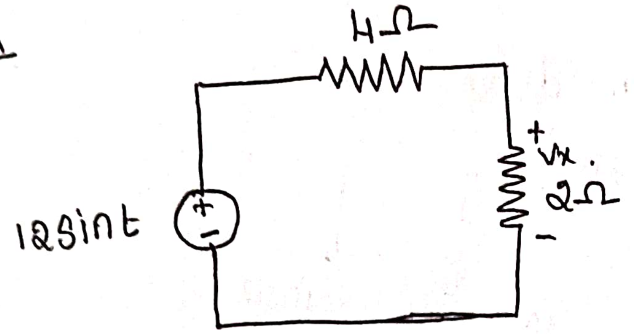
$$\boxed{V_x = 2V}$$

Electric circuit Analysis

2) use voltage division and determine V_x in the ckt below,



soln



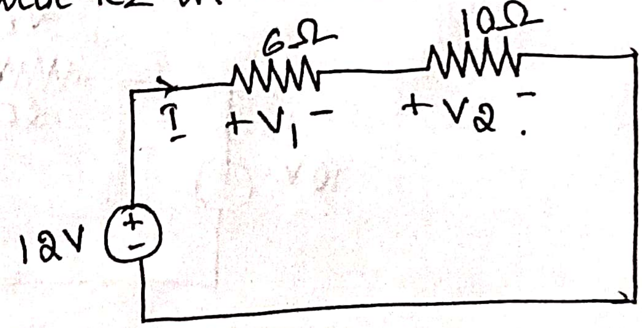
$$R_{eq} = \frac{6\times3}{6+3} = \frac{\cancel{6}\times\cancel{3}}{\cancel{9}} = 2\Omega$$

$$V_x = 12\sin t \times \frac{2}{4+2}$$

$$= \cancel{12}\,4\sin t \times \frac{\cancel{2}}{\cancel{6}}$$

$$\boxed{V_x = 4\sin t}\ \text{volts.}$$

3) Find voltage across R_1 and R_2 in the ckt



$$V_1 = 12\times\frac{6}{6+10}$$

$$= \cancel{12}\,3 \times \frac{\cancel{6}\,3}{\cancel{16}\,\cancel{8}\,\cancel{4}\,2} = \frac{9}{2}$$

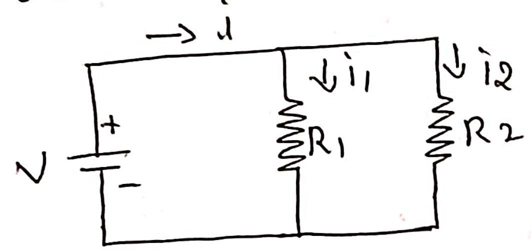
$$\boxed{V_1 = 4.5\,V}$$

$$V_2 = \cancel{12}\,3 \times \frac{\cancel{10}\,5}{\cancel{16}\,\cancel{8}\,\cancel{4}\,2} = \frac{15}{2} = 7.5\,V$$

$$\boxed{V_2 = 7.5\,V}$$

current division

current division rule states that the current in any of the parallel branches is equal to the ratio of opposite branch resistance to the sum of all resistances multiplied by the total circuit current.



The current through R_2 is

$$i_2 = \frac{V}{R_2}$$

$$= \frac{i(R_1 \| R_2)}{R_2}$$

$$= i \frac{R_1 \cancel{R_2}}{(R_1+R_2)\cancel{R_2}}$$

$$= i\left(\frac{R_1}{R_1+R_2}\right)$$

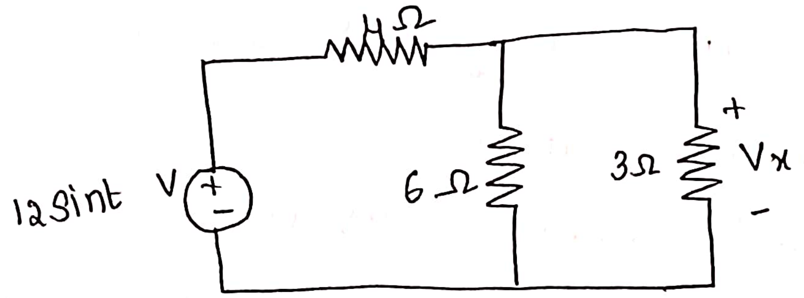
The current through R_1 is

$$i_1 = \frac{V}{R_1}$$

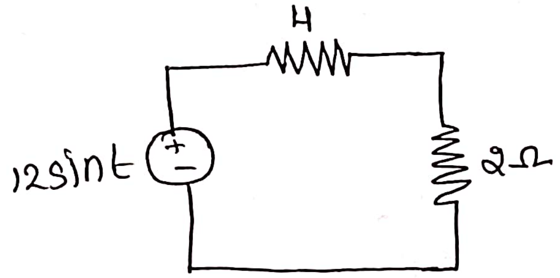
$$= \frac{i(R_1 \| R_2)}{R_1} = \frac{i \cancel{R_1} R_2}{\cancel{R_1}(R_1+R_2)}$$

$$\boxed{i_1 = i \frac{R_2}{R_1+R_2}.}$$

1) ⑥ write an expression for the current through the 3Ω resistor in the circuit of fig shown.



Soln



$$R_{eq} = \frac{6\times3}{6+3} = \frac{18}{9} = 2$$

$$i = \frac{V}{R} = \frac{\cancel{12}^{\,2}\sin t}{\cancel{6}\,\Omega} = 2\sin t.$$

$$\boxed{i = 2\sin t}$$

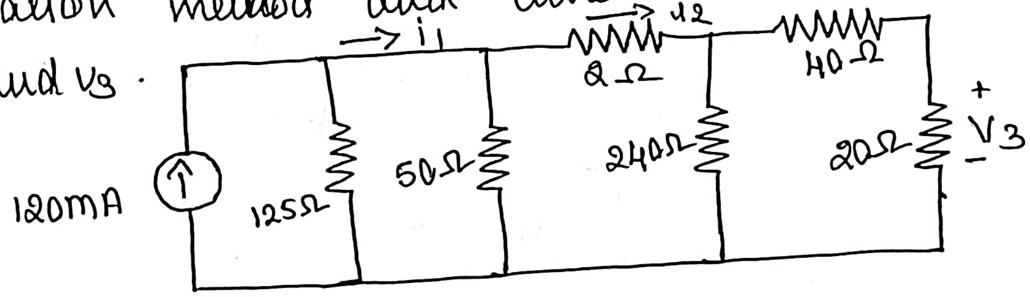
ct through 3Ω is

$$i_{3\Omega}(t) = i \times \frac{6}{6+3}$$

$$= 2\sin t \cdot \frac{\cancel{6}^{\,2}}{\cancel{9}_{\,3}}$$

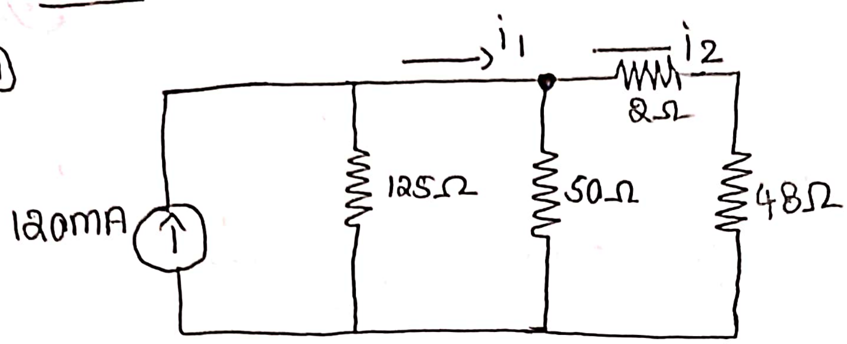
$$\boxed{i_3(t) = \frac{4}{3}\sin t}$$

2) For the following circuit use resistance combination method and current division to find i_1, i_2 and v_3.



soln

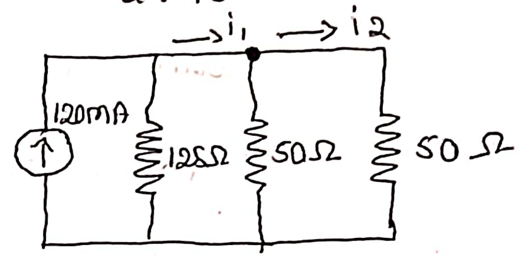
1



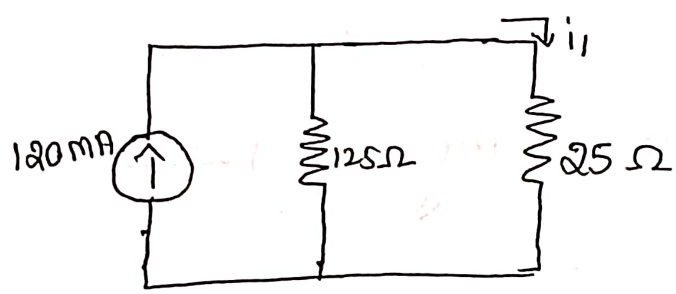
$$\frac{240\times 60}{240+60} = \frac{24\cancel{0}\times \cancel{60}^{2}}{\cancel{30}\cancel{0}} = 48$$

$2+48 = 50\Omega$

2



3



$$\frac{5\cancel{0}\times 5\cancel{0}}{1\cancel{00}} = 25$$

By current divider rule,

$$i_1 = I \times \frac{125}{125+25}$$

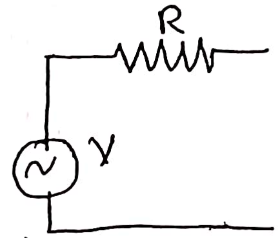
$$= \overset{20m}{\cancel{120\,m}} \times \frac{\cancel{125}}{150}\,\frac{25}{30} = \frac{5}{\cancel{6}}$$

$$\boxed{i_1 = 100\text{mA}}$$

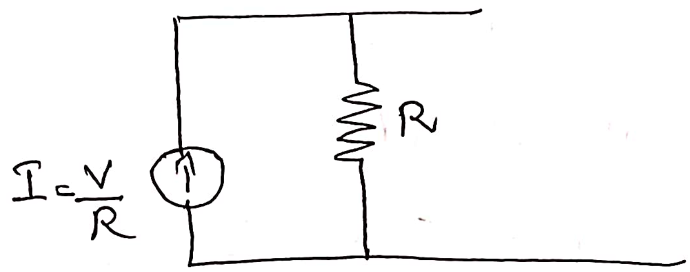
(8)

Source Transformation

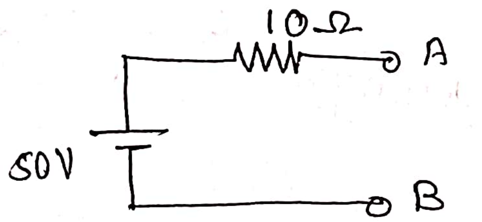
Consider a voltage source in series with a resistance R shown in figure



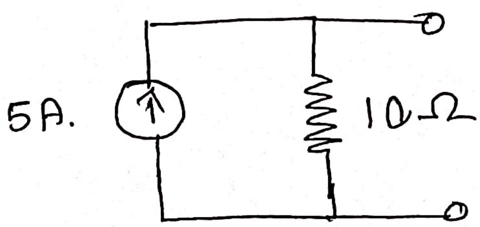
This voltage source in series with a resistor can be converted to a parallel current source in parallel with a resistor



1) Convert the following voltage source into current source

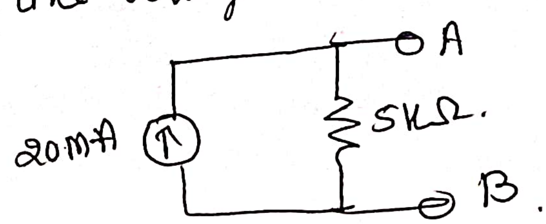


soln

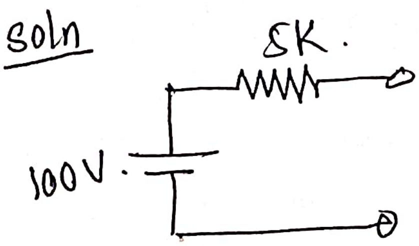


$I = \frac{V}{R} = \frac{50V}{10\Omega} = 5A$

2) Convert the current source into voltage source,



soln

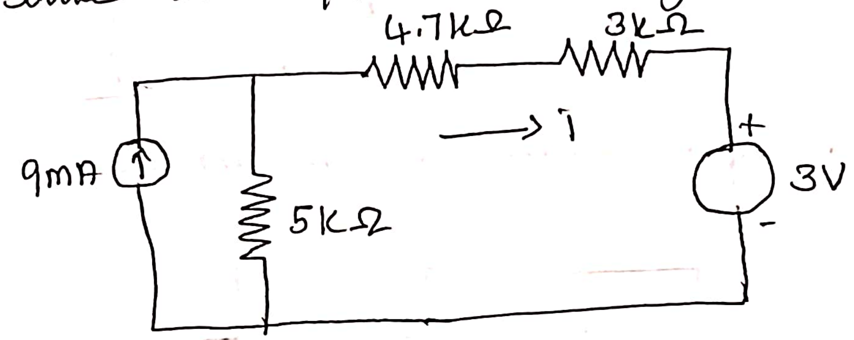


$V = IR$

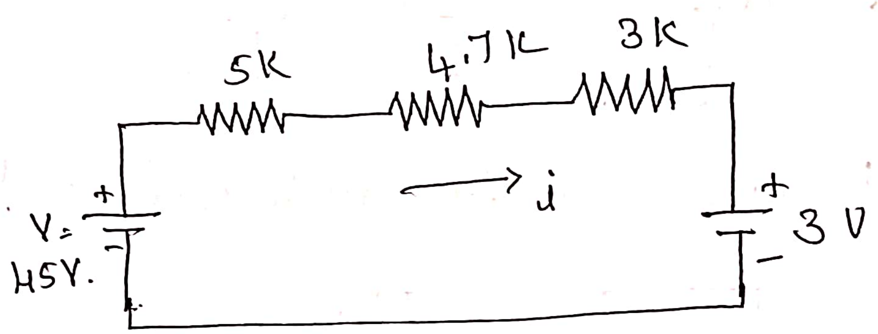
$= 20m \times 5K$

$= 20 \times 5 = 100\ V$

3) Find the current through 4.7kΩ after converting current source into equivalent voltage source.



Soln



$V = IR = 9mA \times 5K.$

$V = 45V$

Total Resistance is 12.7k.

Apply KVL

$45 - i(12.7) - 3 = 0$

$42 - i(12.7)k = 0$

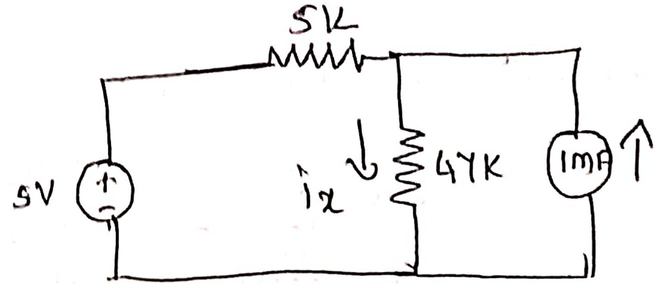
$i(12.7)k = 42$

$i = \frac{42}{12.7k} = 3.307mA$

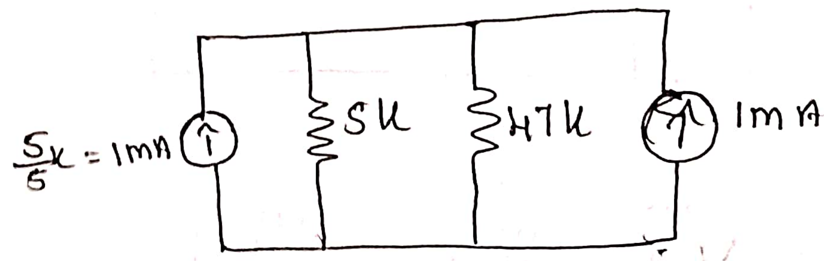
$\boxed{i = 3.307mA}$

(10)

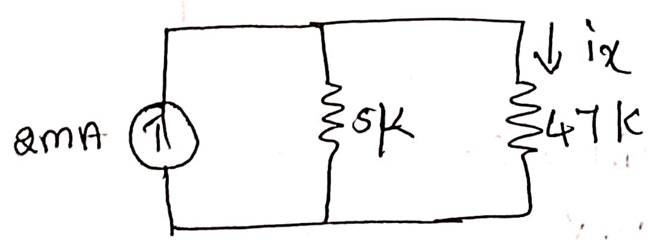
4) Find I_x



Convert v/g source to current source.



current sources in ‖el can be added, $\frac{47K \times 5K}{52K}$.



use ct. divider rule

$$i_x = i \times \frac{5K}{5K+47K}$$

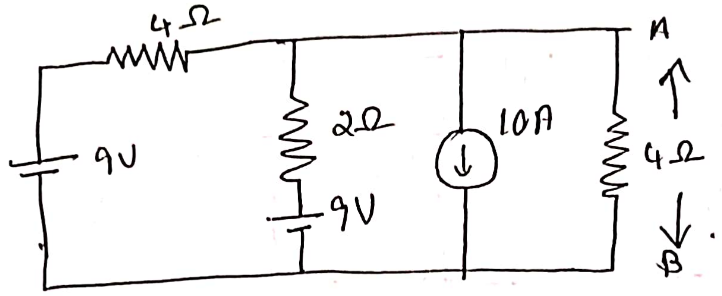
$$= 2m \times$$

$$= 2m \times 0.096 \times 10^{-3}$$

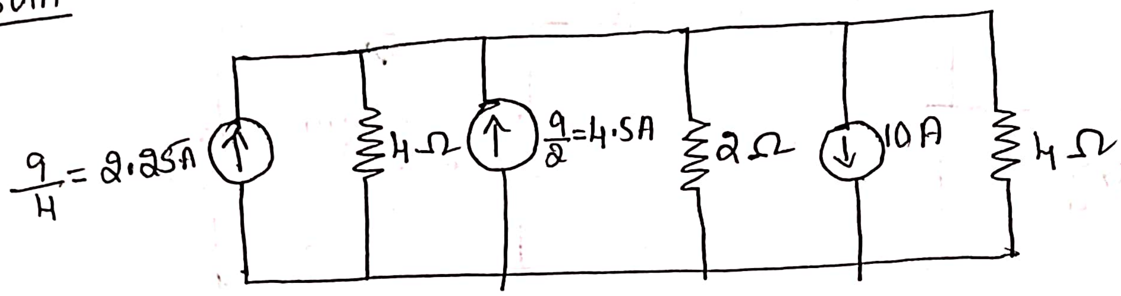
$$= 193 \times 10^{-6}$$

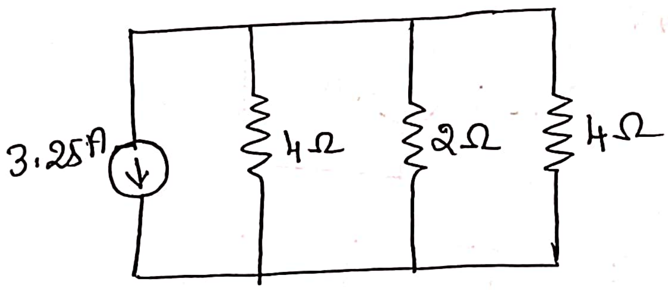
$$\boxed{i_x = 192 \mu A}$$

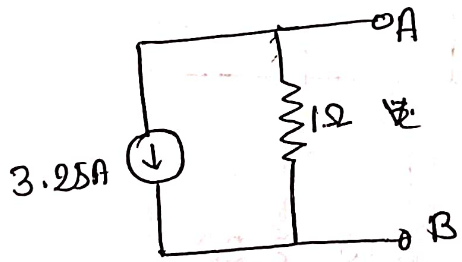
5) Find the total voltage across AB. using source transformation. (11)



soln







$$\frac{1}{R_{eq}} = \frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C}$$

$$= \frac{1}{4} + \frac{1}{2} + \frac{1}{4}$$

$$\frac{1}{R_{eq}} = \frac{1+2+1}{4} = \frac{4}{4} = 1$$

$$\boxed{R_{eq} = 1\Omega}$$

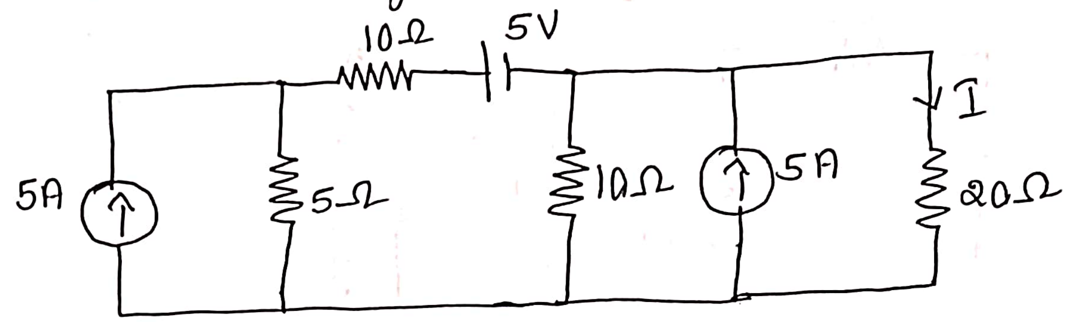
$$V = -I \times R$$

$$= -3.25 \times 1$$

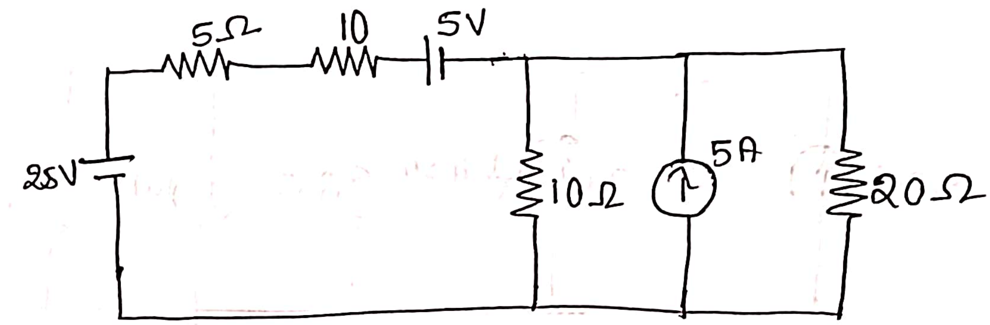
$$\boxed{V_{AB} = -3.25V}$$

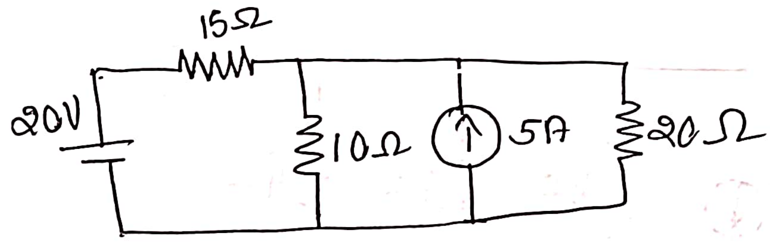
12

6) Find the current through 20Ω using source transformation

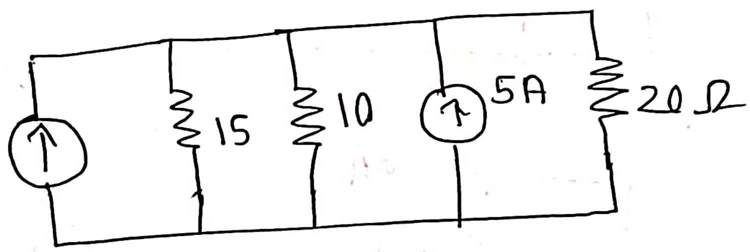


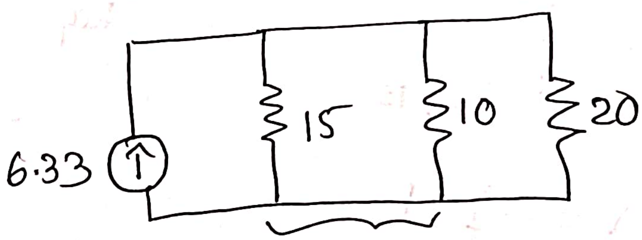
$V = IR$
$= 5 \times 5$
$= 25V$

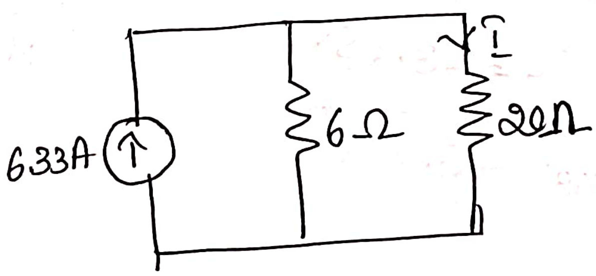




$\frac{20}{15} = 1.33$





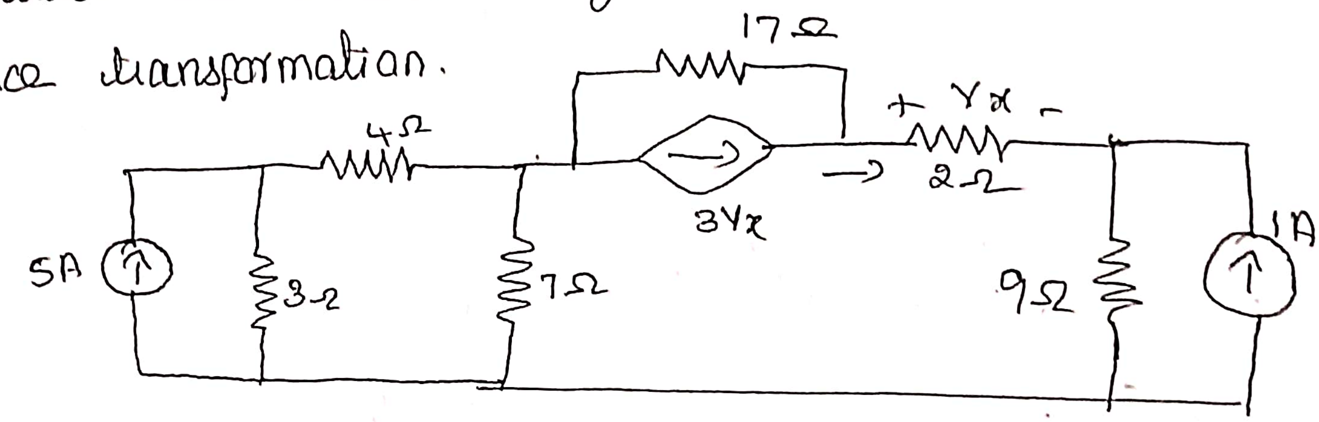


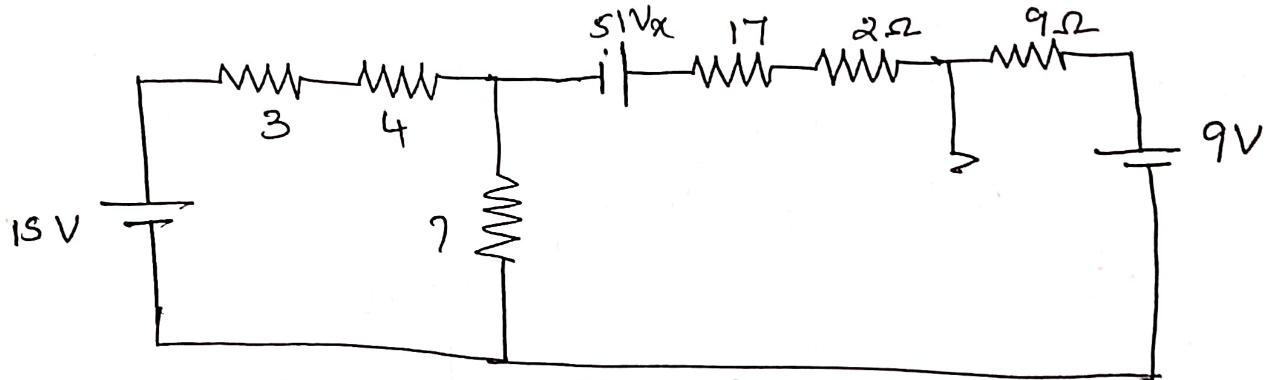
$$\frac{\cancel{15}^{3} \times \cancel{10}^{2}}{\cancel{25}\,\cancel{5}} = 6\Omega$$

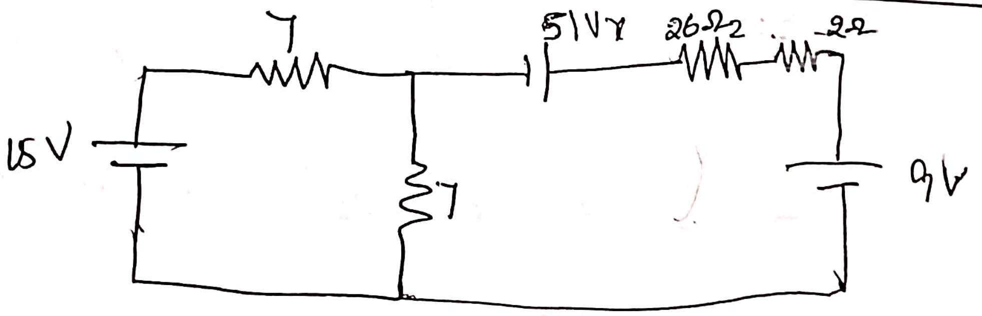
$$I_{20} = 6.33 \times \frac{6}{20+6}$$

$$= 6.33 \times \frac{6}{26} \Rightarrow \boxed{I = 1.46A}$$

Calculate the current through 2Ω resistor using source transformation.







$\frac{3}{17} \times \frac{3}{51 V_x}$.

$15V - 7i_1 - 7(i_1 - i_2) = 0$

$15V - 7i_1 - 7i_1 + 7i_2 = 0$ — (1).

$-7i_1 = -15 \Rightarrow i_1 = \frac{-15}{-7} = 2.1 A$

$7(i_2 - i_1) + 51V_x - 28i_2 - 9 = 0$.

$7i_2 - 7i_1 - 28i_2 = 9 - 51V_x$.

$-7i_1 - 21 i_2 = 9 - 51V_x$. — (2).

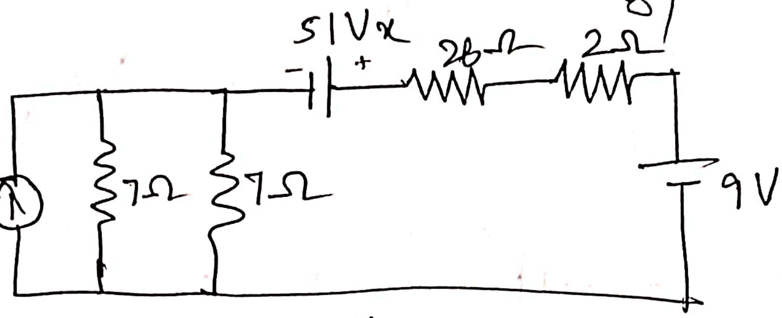
$-14.7 - 21i_2 = 9 - 51 \times 2i_2$ $\quad [\therefore 2i_2 = V_x]$

$-14.7 - 21i_2 = 9 - 102 i_2$

14

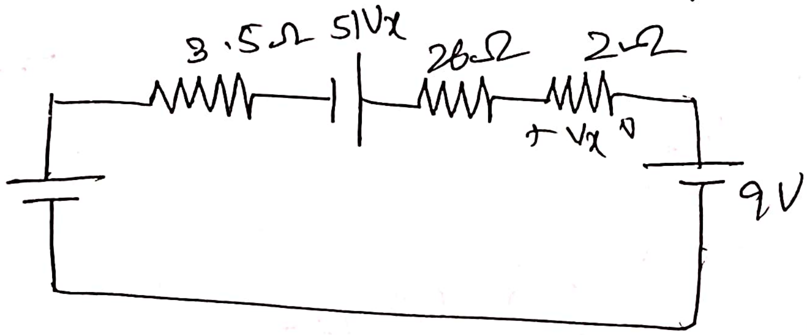
$85 i_2 = 23.7$

$i_2 = \frac{23.7}{81}$



$\frac{15}{7} = 2.142$

$\frac{7 \times 7}{7+7} = \frac{49}{14} = \frac{7}{2} = 3.5\Omega$



$2.142 \times 3.5 = 7.5V$

KVL

$7.5 - 3.5i + 51V_x - 26i - 2i - 9V = 0$

$1.5 - 31.5i + 51V_x = 0 \qquad \because V_x = 2 \times i$

$1.5 - 31.5i + 51(2i) = 0$

$1.5 - 31.5i + 102i = 0$

$1.5 + 70.5i = 0.$

$i = \frac{-1.5}{70.5}$

$i = -0.0212A \Rightarrow i = -21.2mA$

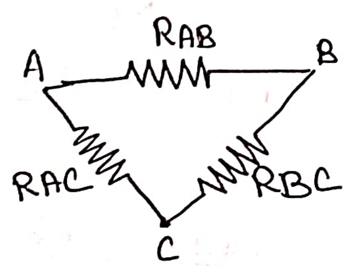
$\therefore$ ~~$V_x = -0.021 \times 2$~~

~~$= -0.042$~~

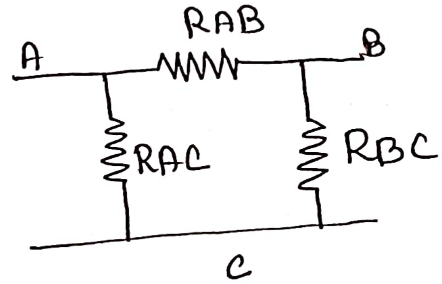
STAR - DELTA Transformation

In some of the circuits there won't be series (or) Parallel combination of resistors to reduce the complexity of the circuit another useful technique called Δ-Y (Delta-Wye) conversion is used.

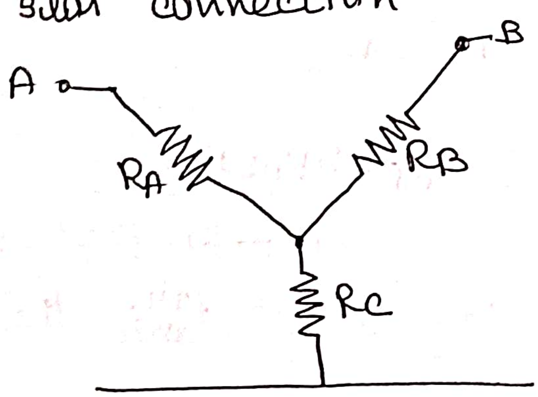
Delta connection.

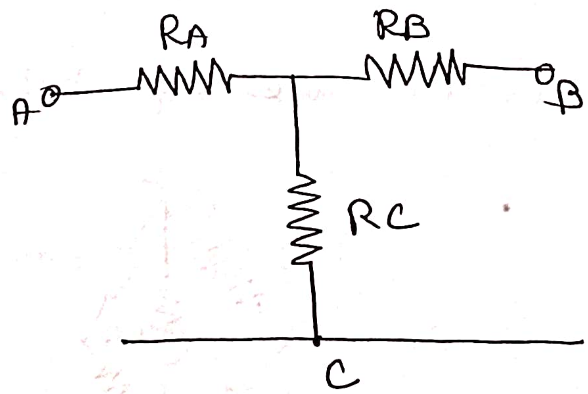


(or)

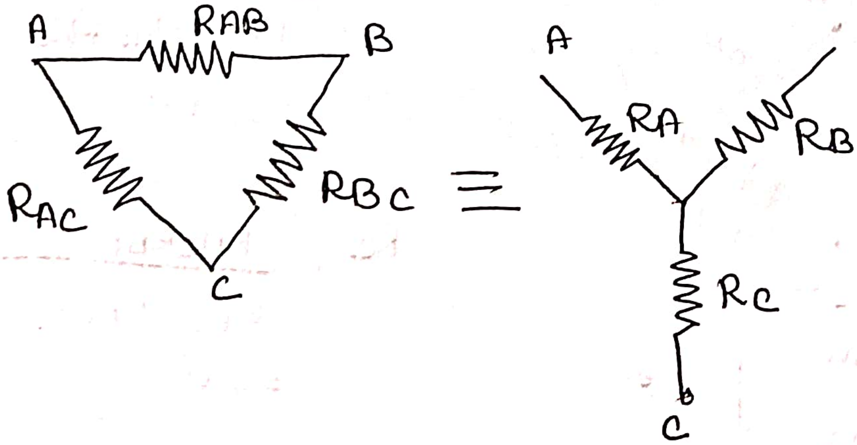


star connection





Δ to Y conversion formula.



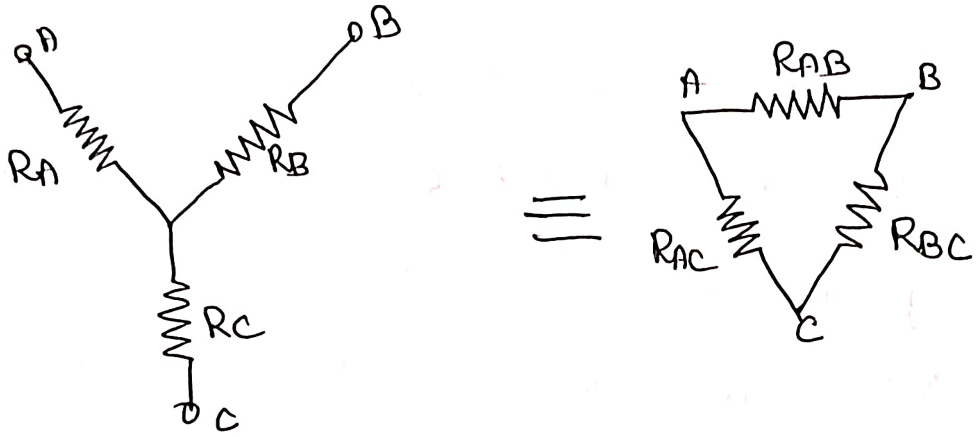
$$R_A = \frac{R_{AB} R_{AC}}{R_{AB} + R_{AC} + R_{BC}}$$

$$R_B = \frac{R_{AB} R_{BC}}{R_{AB} + R_{AC} + R_{BC}}$$

$$R_C = \frac{R_{AC} R_{BC}}{R_{AB} + R_{AC} + R_{BC}}$$

16

Y - Δ conversion formula

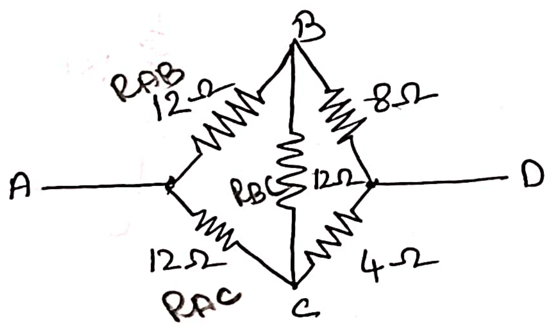


$$R_{AB} = \frac{R_AR_B + R_BR_C + R_CR_A}{R_C}$$

$$R_{BC} = \frac{R_AR_B + R_BR_C + R_CR_A}{R_A}$$

$$R_{AC} = \frac{R_AR_B + R_BR_C + R_CR_A}{R_B}$$

1) calculate the Req between A & D



$$R_A = \frac{R_{AB}R_{AC}}{R_{AB}+R_{AC}+R_{BC}} = \frac{12 \times 12}{12+12+\cancel{4}12}$$

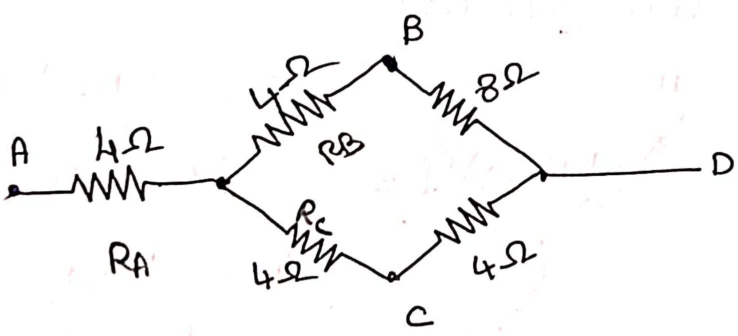
$$R_A = \frac{144}{36} = 4\Omega$$

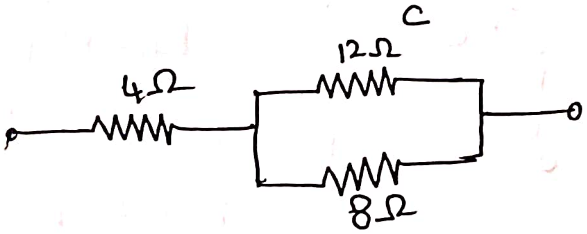
$$R_B = \frac{R_{AB}R_{BC}}{R_{AB}+R_{AC}+R_{BC}} = \frac{12 \times 12}{36}$$

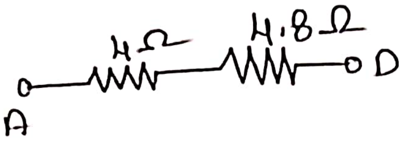
$= 4\Omega$

$$R_C = \frac{R_{AC}R_{BC}}{R_{AB}+R_{AC}+R_{BC}} = \frac{12 \times 12}{36}$$

$= 4\Omega$

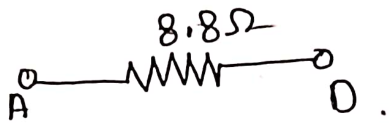




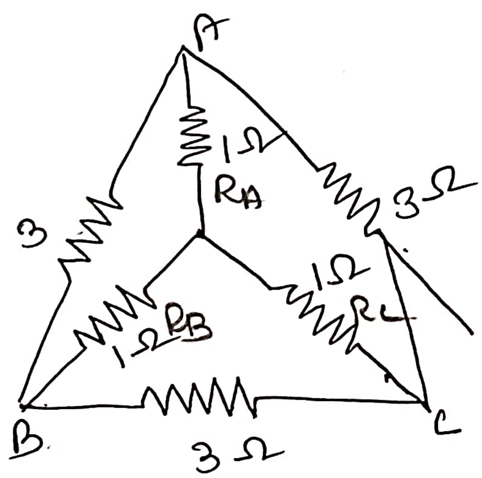


$$\frac{R_1R_2}{R_1+R_2} = \frac{12\times8}{12+8} = \frac{96}{20}$$

$$= 4.8\Omega$$



2) Find Req b/w B & C

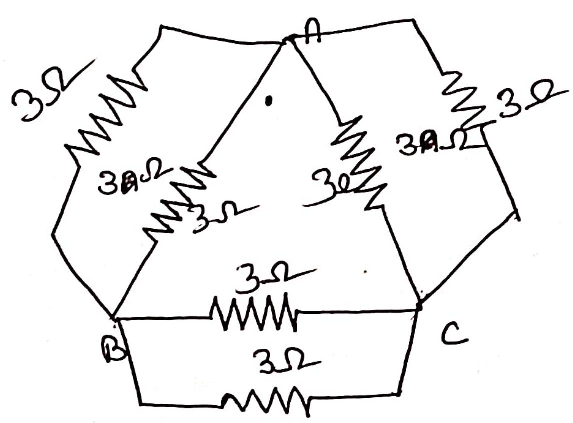


$$R_{AB} = \frac{R_AR_B+R_BR_C+R_CR_A}{R_C} = \frac{1+1+1}{1}$$

$$=3\Omega$$

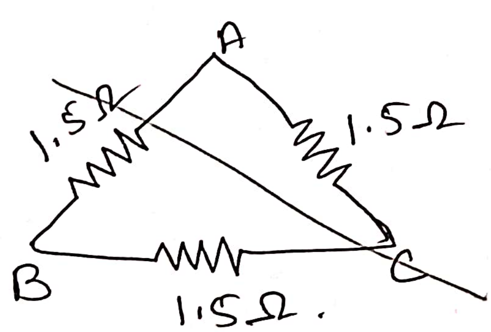
$$R_{BC} = \frac{R_{AB}+R_{BC}+R_{CA}}{R_A} = 3\Omega$$

$$R_{CA} = \frac{R_{AB}+R_{BC}+R_{CA}}{R_B} = 3\Omega$$



$$= \frac{3\times3}{3+3} = \frac{9}{6}$$

$$= \frac{3}{2} = 1.5\Omega$$



Req b/w B & C is 1.5Ω.