ROHININ COLLEGE OF ENGINEERING AND TECHNOLOGY
Approved by AICTE & Affliated to anna university
Accredited with A+ grade by NAAC
DEPARTMENT OF MECHANICAL ENGINEERING

NAME OF THE SUBJECT: ENGINEERING MECHANICS

SUBJECT CODE : ME3351

REGULATION 2021

UNIT V: DYNAMICS OF PARTICLES

Impact of Elastic Bodies:

A collision between two bodies to be an impact, if the bodies are in contact for short interval of a time and exert very large force on a short period of time.

On impact bodies deform first and then recover due to elastic properties and start moving with different velocities.

Types of Impact:

- ❖ Line of impact
- ❖ Direct impact
- ❖ Oblique impact
- ❖ Central impact
- ❖ Eccentric impact

Perfectly Elastic impact: [e=1]

If both of bodies regain to their original shape and size after the impact. Both momentum and energy is conserved.

In elastic impact [e<1]

The collision do not return to their original shape and size completely after the collection. Only the momentum remains conserved, but there is a loss energy.

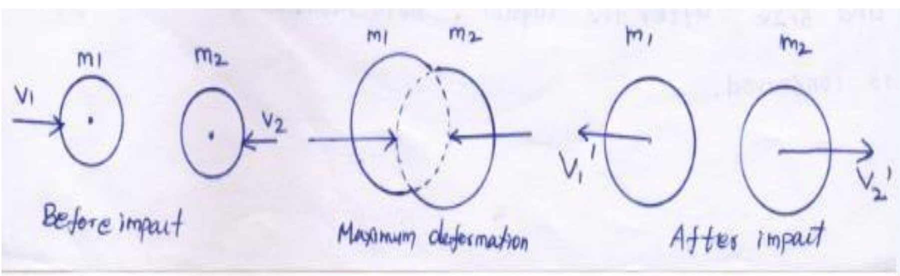
Period of collision:

During the collection, the bodies undergo a deformation for a small time interval and then recover the deformation in a further small interval.

Time elapse b/w initial contact and maximum deformation is called the period of deformation. And the instant of separation is called time of restitution or period of recovery.

Principal of collision:

Consider 2 bodies approach each other with the velocity v_1 and v_2 masses m_1 and m_2 are shown in fig.



Let 'F' be force entered due to collection at a small time. Apply conservation of momentum principal for both bodies

$$m_1v_1 + m_2v_2 = m_1v_{1^1} + m_2v_{2^1}$$

Newton's impact Eqn:

Coefficient of restitution, e=$\frac{relative\ velocity\ of\ separation}{relative\ velocity\ of\ approach}$

$$e = \frac{v_2^1 v_1^1}{v_1 - v_2}$$

Total kinetic energy at before impact

$$= \frac{1}{2}\, m_1 v + \frac{1}{2} m_2 v_2^{2}$$

Total kinetic energy at after impact

$$= \frac{1}{2} m_1 {v_1^{1}}^{2} + \frac{1}{2} m_2 {v_2^{1}}^{2}$$

Loss of K.E=Intial K.E − Final K.E

<u>Oblique:</u>

$V_1 \sin \propto_1 = V_1^{1} \sin \theta_1$

$V_2 \sin \propto_1 = V_2 \sin \theta_1$

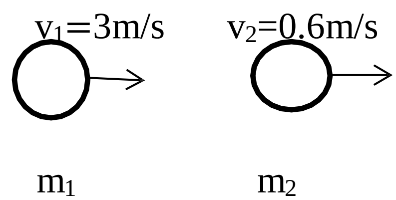
$m_1\, v_1 \cos \propto_1 + m_1 v_1^{1} \cos \theta_1 + m_2 v_2^{1} \cos \theta_2$ $m_1 = m_2$

$$e = \frac{V_2^{1} \cos \theta_2 - V_1^{1} \cos \theta_1}{V_1 \cos \propto_1 - V_2 \cos \propto_2}$$

<u>Problem based on impact of elastic body:</u>

1. A sphere of 1 kg moving at 3 m/s, collides with another sphere of weight of 5 kg in the same Direction at 0.6 m/s. If the collision is perfectly elastic, find the velocity after impact.

<u>Given:</u>



$m_1 = 1$ kg

$m_2 = 5$ kg

$v_1 = 3$ m/s

$v_2 = 0.6$ m/s

Perfectly elastic impact e= 1

<u>To find:</u>

Velocity at after the impact V_1^1 & V_2^1

Soln:l

Law of conservation of momentum

$m_1v_1 + m_2v_2 = m_1v_1^1 + m_2v_2$

$1\times3+5\times0.6=1\ v_1^1+5\ v_2^1$

$V_1^1+5\ V_2^1 = 6$------- > (1)

The coefficient of restitution, $e = \frac{V_2^1 - V_1^1}{V_1 - V_2}$

$e = 1$[perfectly Elastic Impact]

$1 = \frac{V_2^1 - V_1^1}{3 - 0.6}$

$V_2^1 - V_2^1 = 1\times[3-0.6]$

$V_2^1 - V_1^1 = 2.4$ ------> (2)

Solve Eqn (1) & (2)

$V_1^1+5\ V_2^1 = 6$

$V_2^1 - V_1^1 = 2.4$

$6\ V_2^1 = 8.4$

$V_2^1 = 8.4/6$

$V_2^1 = 1.4$ m/s

V_2^1 value sub in Eqn (1)

$V_1^1+5\ V_2^1 = 6$ --------> $V_1^1 = 6 - [5\times V_2^1]$

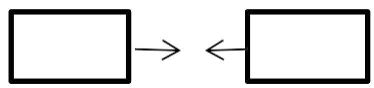
$V_1^1 += 6 - [5\times1.4]$

$V_1^1 = -1$ m/s

$V_1^1 = 1$ m/s

2. A car weighting 5 KN is moving east with a velocity of 54 k m p h and collide with a second car weighting 12 KN is moving west with a velocity of 72 k m p h If the impact is perfectly plastic, what will be the velocities of the cars. V_1=54km/h v_2=-72km/h

Given:



$W_1 = 5KN$ $W_2 = 12KN$

$M_1 = {}^{5}/_{9.81}$ $M_2 = {}^{12}/_{981}$

$W_1 = 5$ KN $= {}^{5}/_{9.81} = 0.509$ kg $= m_1$

$W_2 = 12$ KN $= {}^{12}/_{9.81} = 1.22$ kg $= m_2$

V_1=54 km/hr

$V_2 = -72$ km/hr

To Find:

Velocity of car

Soln:

Law of conservation momentum

$m_1 v_1 + m_2 v_2 = m_1 v_1^1 + m_2 v_2^1$

Perfectly plastic means e=0

$\therefore v_1^1 = v_2^2 = e$

$0.509 \times 54 + 1.22 \times [-72] = 0.509 \times v_1^1 + 1.22 \times v_2^1$

$27.486 - 87.84 = [0.509\, Vc + 1.22]$

$-60.354 = Vc \times 1.729$

$V_c = \dfrac{-60.354}{1.729}$

$Vc = -34.90\ km/hr$

3. Direct central impact occurs between 300N body moving to right with a velocity of 6 m/s and 150N body moving to the left with a velocity of 10 m/s. Find the velocity of each body after the impact if the coefficient of restitution is 0.8.

Same as problem No:1

Ans: $V_2^1 = 9.2$ m/s

$V_1^1 = -3.6$ m/s

4. Two bodies, one of which 20N and velocity 10 m/s and the other of 100N with a velocity of m/s downward,each other and implinges centerlly. Find the velocity of each body of the impact if the coefficient of restitution is 0.6. Find also the loss in kinetic energy due to impact.

<u>Given data:</u>

$W_1 = 20N$ $\quad m_1 = \dfrac{20}{9.81}$ $\quad m_1 = 2.038$ kg

$V_1 = 10$ m/s

$W_2 = 100N$ $\quad m_1 = \dfrac{100}{9.81}$ $\quad m_2 = 10.19$ kg

$V_2 = -10$ m/s

Coefficient of restitution, $e = 0.6$

<u>To find:</u>

Final velocity After impact V_1^1 & V_2^1

Loss of kinetic Energy.

Soln:

Law of conservation of Energy

$m_1 v_1 + m_2 v_2 = m_1 v_1^1 + m_2 v_2^1$

$(2.038 \times 10) + (10.19 \times -10) = 2.038\, v_1^1 + 10.19 \times v_2^1$

$20.38 - 101.9 = 2.038 v_1^1 + 10.19\, v_2^1$

$-81.52 = 2.038\, v_1^1 + 10.19\, v_2^1$

$2.038\; v_1^1 + 10.19\, v_2^1 = -81.52$ ---------> (1)

If coefficient of restitution Eg 'e' is given

$$e = \frac{V_2^1 - V_1^1}{V_1 - V_2}$$

$$0.6 = \frac{V_2^1 - V_1^1}{10 - (-10)} \qquad \frac{V_2^1 - V_1^1}{20}$$

$V_2^1 - V_1^1 = 20 \times 0.6$

$V_2^1 - V_1^1 = 12$ --------> (2)

Solve Eqn (1) & (2)

	$2.038\, V_1^1 + 10.19\, V_2^1$	=	$-$ 81.52 ------> (1)
Eqn (2) × 2.037	$2.038\, V_2^1 - 2.038\, V_1^1$	=	24.456
	$12.228\, V_2^1$	=	-57.06

$V_2^1 = \frac{-57.06}{12.228}$

$V_2^1 = -4.66$ m/s

V_2^1 value sub in Eqn (1)

$2.038\, V_1^1 + 10.19\, V_2^1 = -81.52$ --------> (1)

$2.038\ V_1^1 + 10.19 \times (-4.66) = -81.52$

$2.038\ V_1^1 + [-47.55] = -81.52$

$V_1^1 = \frac{-81.52+47.55}{2.038}$

$V_1^1 = \frac{-33.96}{2.038}$

$V_1^1 = 16.66$ m/s

Loss of kinetic Energy:

= Initial kinetic Energy – Final kinetic Energy

[before Impact] [after impact]

Total kinetic Energy before impact

$= \frac{1}{2} m_1 v_1^2 + \frac{1}{2}(10.19)\ (-10)^2$

$= \frac{1}{2} (20038)\ (10)^2 + \frac{1}{2}(10.19)\ (-10)^2$

Before K.E = 611.4 N.m

Total kinetic at after impact [find K.E]

$= \frac{1}{2}\ V_1\ {}^{1\ 2} + \frac{1}{2}\ m_2{}^{1\ 2}$

$= \frac{1}{2} \times (2.038)(-16.66)^2 + \frac{1}{2} \times (10.19)(-4.66)^2$

After K.E= 394.26 N.m

Loss of kinetic energy during impact

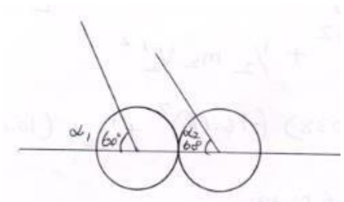
$= \text{After K.E} = \text{Before K.E}$

$= 611.4 - 394.26$

Loss= 217.11 N.m

<u>Problem 5</u>

A ball of weight 500g moving with velocity of im/sec impings on a bar of mass 1kg moving with velocity 0.75m/s at the time of impact the velocity of the body are parallel and inclined at $60^0$to the line joining there centers. Determine the velocity direction of the ball after the impact where e= 0.6 also find the loss of kinetic energy due to impact.



$\propto_1 = \propto_2 = 60°$

m_1=500g= $\frac{500}{1000} = 0.5kg$

m_2= 1kg

$v_1 = 1m/s$

$v_2 = 0.75$

Coefficient of restitution e= 0.6

<u>To find:</u>

1. final velocity $v1^1$&$v\ 2^1$

2. Direction θ_1& θ_2

3. loss of kinetic energy

<u>Soln:</u>

Law of conservation of momentum

$m_1 v_1 \cos \propto_1 + m_2 v_2 \cos \propto_2 = m_1 v1^1 cos\theta_1 + 1\ v2^1 cos\theta_2$

$0.635 = 0.5 v1^1 cos\theta_1 + v2^1 cos\theta_2$

If coefficient of restitution is given

$e = \frac{v2^1 cos\theta_2 - v1^1 cos\theta_1}{v_1 \cos\propto_{1} - v_2 \cos\propto_2}$

$0.6 = \frac{v2^1 cos\theta_2 - v1^1 cos\theta_1}{1 \cos 60 - 0.75\ os\ 60}$

$0.6[\cos - 0.7\times \cos 60] = v2^1 cos\theta_2 - v1^1 cos\theta_1$ ----------------- (2)

Solve Eqn (1) &(2)

$0.625 = 0.5 v1^1 cos\theta_1 + v2^1 cos\theta_2$

$0.075 = v2^1 \cos\theta_2 \mp v1^1 \cos\theta_1$

$0.55 = 1.5\ v1^1 \cos\theta_1$

$v1^1 \cos\theta_1 = \frac{0.55}{1.5}$

$\therefore v1^1 \cos\theta_1 = 0.366$

$v1^1 \sin\theta_1 = v_1 \sin \propto 1$

$= 1 \sin 60$

$= 0.866$

$\frac{V_1^1 \sin \theta_1}{V_1^1 \cos\ _1} = \frac{0.866}{0.366}$

$\tan \theta_1 = 2.366$

$\theta_1 = \tan^{-1}(2.366)$

$\theta = 67°$

$V_1^1 \cos \theta_1 = 0.366$

$V_1^1 \cos 67° = 0.366$

$V_1^1 = \frac{0.366}{\cos 67°}$

$V_1^1 = 0.94 m/s$

------> (2) $0.075 = V_2^1 \cos\theta_2 - V_1^1 \cos\theta_1$

$0.075 = V_2^1 \cos\theta_2 - 0.94 \cos 67°$

$0.075 + 0.94 \cos 67° = V_2^1 \cos\theta_2$

$0.442 = V_2^1 \cos\theta_2$

$V_2^1 \sin\theta_2 = 0.6495$

$\underline{V_1^1 \sin\theta_1} \quad = \frac{0.6495}{0.442}$

$V_1^1 \cos \quad _1$

$\tan\theta_2 = 1.469$

$\theta_2 = 55°$

$V_2^1 \cos\theta_2 = 0.442$

$V_2^1 \cos 55° = 0.442$

$V_2^1 = \frac{0.442}{\cos 55°}$

$V_2^1 = 0.785\ m/s$

Loss of kinetic Energy = Before K.E− After K.E

Before K.E= $\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$

$= \frac{1}{2} \times 0.5 \times 1^2 + \frac{1}{2} \times 1 \times (0.75)^2$

Before K.E=0.25+0.281

Before K.E=0.531 N.m

After kinetic Energy= $\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$

$= \frac{1}{2} \times 0.5 \times (0.94)^2 + \frac{1}{2} \times 1 \times (0.785)^2$

$= 0.2209 + 0.308$

After K.E= 0.528 N.m

Loss of K.E=before K.E−After K.E

$=0.531-0.528$

Loss K.E=2.1×10^{-3}N.m