

LANGUAGES OF PDA

A language can be accepted by Pushdown automata using two approaches:

1. Acceptance by Final State: The PDA is said to accept its input by the final state if it enters any final state in zero or more moves after reading the entire input.

Let $P = (Q, \Sigma, \Gamma, \delta, q_0, Z, F)$ be a PDA. The language acceptable by the final state can be defined as:

$$1. L(PDA) = \{w \mid (q_0, w, Z) \vdash^* (p, \varepsilon, \varepsilon), q \in F\}$$

2. Acceptance by Empty Stack: On reading the input string from the initial configuration for some PDA, the stack of PDA gets empty.

Let $P = (Q, \Sigma, \Gamma, \delta, q_0, Z, F)$ be a PDA. The language acceptable by empty stack can be defined as:

$$1. N(PDA) = \{w \mid (q_0, w, Z) \vdash^* (p, \varepsilon, \varepsilon), q \in Q\}$$

Equivalence of Acceptance by Final State and Empty Stack

- If $L = N(P_1)$ for some PDA P_1 , then there is a PDA P_2 such that $L = L(P_2)$. That means the language accepted by empty stack PDA will also be accepted by final state PDA.
- If there is a language $L = L(P_1)$ for some PDA P_1 then there is a PDA P_2 such that $L = N(P_2)$. That means language accepted by final state PDA is also acceptable by empty stack PDA.

Example:

Construct a PDA that accepts the language L over $\{0, 1\}$ by empty stack which accepts all the string of 0's and 1's in which a number of 0's are twice of number of 1's.

Solution:

There are two parts for designing this PDA:

- If 1 comes before any 0's
- If 0 comes before any 1's.

We are going to design the first part i.e. 1 comes before 0's. The logic is that read single 1 and push two 1's onto the stack. Thereafter on reading two 0's, POP two 1's from the stack. The δ can be

1. $\delta(q_0, 1, Z) = (q_0, 11, Z)$ Here Z represents that stack is empty
2. $\delta(q_0, 0, 1) = (q_0, \epsilon)$

Now, consider the second part i.e. if 0 comes before 1's. The logic is that read first 0, push it onto the stack and change state from q_0 to q_1 . [Note that state q_1 indicates that first 0 is read and still second 0 has yet to read].

Being in q_1 , if 1 is encountered then POP 0. Being in q_1 , if 0 is read then simply read that second 0 and move ahead. The δ will be:

1. $\delta(q_0, 0, Z) = (q_1, 0Z)$
2. $\delta(q_1, 0, 0) = (q_1, 0)$
3. $\delta(q_1, 0, Z) = (q_0, \epsilon)$ (indicate that one 0 and one 1 is already read, so simply read the second 0)
4. $\delta(q_1, 1, 0) = (q_1, \epsilon)$

Now, summarize the complete PDA for given L is:

1. $\delta(q_0, 1, Z) = (q_0, 11Z)$
2. $\delta(q_0, 0, 1) = (q_1, \epsilon)$
3. $\delta(q_0, 0, Z) = (q_1, 0Z)$
4. $\delta(q_1, 0, 0) = (q_1, 0)$
5. $\delta(q_1, 0, Z) = (q_0, \epsilon)$
6. $\delta(q_0, \epsilon, Z) = (q_0, \epsilon)$ ACCEPT state