Steady State refers to a condition where the current & voltage waveforms are stable repeating sinusoids with a constant amplitude, frequency & phase shift.
(i.e.) There are no transient changes occuring & the circuit has reached a predictable, unchanging state after any initial fluctuations have subsided.

Transient response is the initial response of a system to a sudden change in i/p, before it settles down to a steady state.

Steady State (current) is a constant current that does not change over time. It is a type of Direct current (DC)

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Unit III

Sinusoidal Steady State Analysis.

All the electrical systems operate with alternating voltages & current. Hence it is important to analyze the steady state conditions.

AC fundamentals

Sinusoidal Alternating Quantity

* An Alternating quantity which varies according to the sine of the angle θ is known as sinusoidal alternating quantity.
* In our world sinusoidal voltages & currents are selected for generation of electrical power.

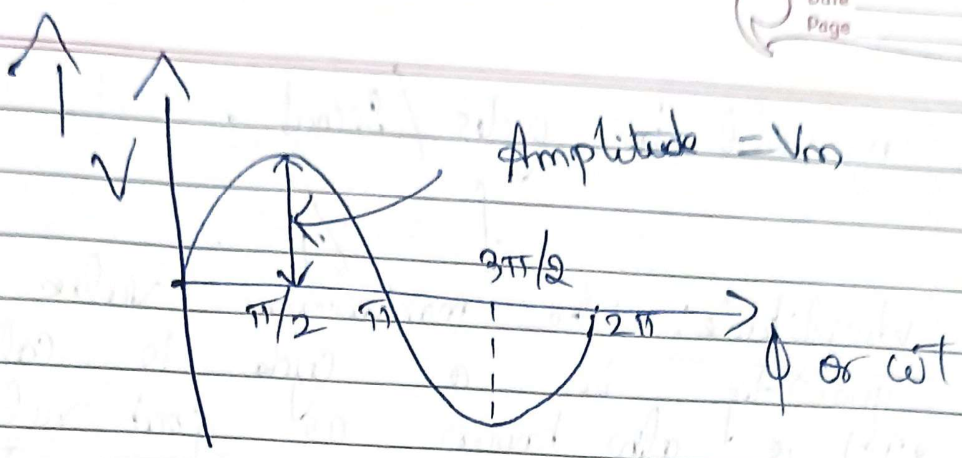
Reason for using Sinusoidal voltages / Currents

* Produces low iron & coil losses in AC rotating machines & transformers.
* It improves efficiency.
* Offer less interference to nearby telephone lines.
* Produce less disturbance in electrical circuit

Characteristics of Sinusoids.

Sinusoidal voltages: $V = V_m(\sin\omega t + \phi)$

Sinusoidal Current: $I = I_m(\sin\omega t + \phi)$



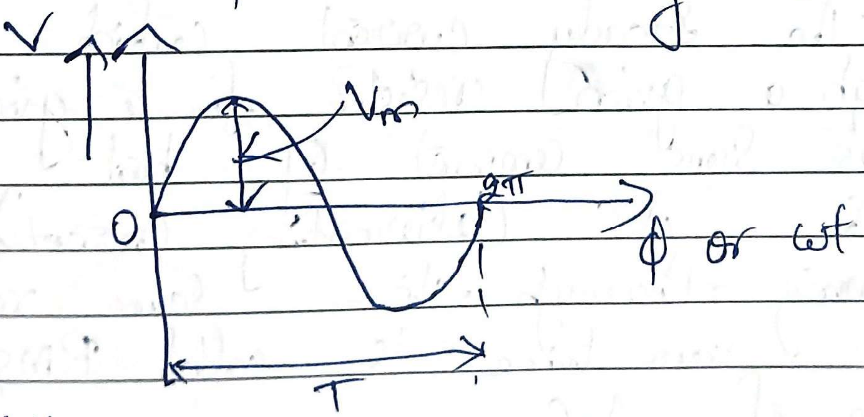
$V = V_m \sin\phi$

Sinusoidal Signal

Where ω = Angular frequency in rad/sec.
ϕ = Phase angle.
V = Instantaneous Voltage in Volts.
I = Instantaneous Current in A.
V_m = Maximum Voltage.
I_m = Maximum Current.

Cycle:
It is a set of positive & negative portions of a waveform.

Time Period:
The time required for an alternating quantity to complete one cycle is called time period denoted by T.



Frequency: The number of cycles per second is frequency & it is denoted by f. It is measured in cycles / second.

$$f = 1/T$$

Amplitude: The maximum value of alternating quantity in a cycle is called amplitude.

* It is also known as peak value / crest value.
* In voltage waveform peak value is V_m. It occurs at $\pi/2$ & $3\pi/2$.

Relation between ω & f

Angular distance = Angular velocity × Time

$\theta = \omega \times t$

$\theta = 360^\circ = 2\pi$ radians, one cycle is completed.

$2\pi = \omega t$

$t \rightarrow$ Time taken for completion of one cycle.

$$\omega = 2\pi / T$$

$$\omega = 2\pi f$$

AC $\xrightarrow{\text{time}(t)}$ heat

DC $\xrightarrow{\text{time}(t)}$ heat

Root Mean Square / Effective value

(average value of AC waveform)

The steady current which when flowing through a given resistor for a given time produces same amount of heat as produced by the AC (alternating current), when flowing through the same resistor for the same time is called RMS / effective value of A.C.

Methods to find RMS Value: Analytical Method.

$$I_{RMS}^2 = \frac{1}{2\pi}\int_0^{2\pi} i^2\, d\theta = \frac{1}{T}\int_0^T i^2\, dt \Rightarrow$$ Unsymmetrical waveform.

$$I_{RMS}^2 = \frac{1}{\pi}\int_0^{\pi} i^2\, d\theta = \frac{2}{T}\int_0^{T/2} i^2\, dt \Rightarrow$$ Symmetrical waveform.

RMS Value of Sinusoidal Alternating Quantity

$i = I_m \sin\theta \Rightarrow$ Symmetrical Waveform.

$$I_{RMS}^2 = \frac{1}{\pi}\int_0^{\pi} i^2\, d\theta = \frac{1}{\pi}\int_0^{\pi} I_m^2 \sin^2\theta\, d\theta.$$

$$= \frac{1}{\pi} I_m^2 \int_0^{\pi} \left(\frac{1 - \cos 2\theta}{2}\right) d\theta.$$

$$= \frac{1}{\pi} I_m^2 \left[\frac{1}{2}\left[\theta\right]_0^{\pi} - \frac{1}{2}\left[\sin 2\theta\right]_0^{\pi}\right]$$

$$\left[\because \sin 2\pi = \sin 0 = 0\right]$$

$$= \frac{1}{\pi} I_m^2 \frac{1}{2}\left[(\pi - 0) - 0\right]$$

$$I_{RMS}^2 = I_m^2/2$$

$$I_{RMS} = I_m/\sqrt{2} \Rightarrow 0.707\, I_m.$$

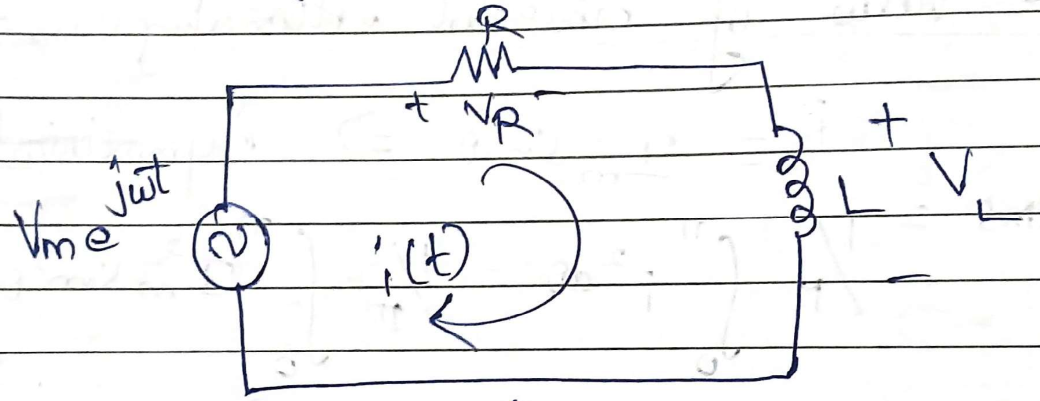
RMS Value = 0.707 × Max value.

RMS value = $\sqrt{\dfrac{\text{Area of the squared curve}}{\text{Base}}}$.

3.2 Complex Forcing Function

Euler's identity gives Sinusoidal & exponentials are related through Complex numbers.

$$e^{j\theta} = \cos\theta + j\sin\theta$$



Basic first Order circuit.

$$V_m e^{j\omega t} = V_m \cos(\omega t) + j V_m \sin\omega t.$$

Apply KVL to the above circuit:

$$V_L + V_R - V_m e^{j\omega t} = 0$$

$$L\frac{di(t)}{dt} + Ri(t) = V_m e^{j\omega t} \quad \text{——} \; (1)$$

Let $i(t) = I_m e^{j(\omega t + \phi)}$

$$\frac{di(t)}{d(t)} = j\omega I_m e^{j(\omega t + \phi)} \quad \text{——} \; (2)$$