UNIT I

1.4 ELASTIC CONSTANTS

1.4.1 LONGITUDINAL STRAIN

When a body is subjected to an axial tensile or compressive load, there is an axial deformation in the length of the body. The ratio of axial deformation to the original length of the body is known as longitudinal strain or linear strain. The longitudinal strain is also defined as the deformation of the body per unit length in the direction of the applied load.

Let L= length of the body,

P=Tensile force acting on the body,

δL=Increase in the length of the body in the direction of load P,

Then, longitudinal strain = $\frac{\delta L}{L}$

1.4.2. LATERAL STRAIN

The strain at right angles to the direction of applied load is known as lateral strain. Let a rectangular bar of length L, breadth b and depth d is subjected to an axial tensile load P as shown in fig. The length of the bar will increase while the breadth and depth will decrease.

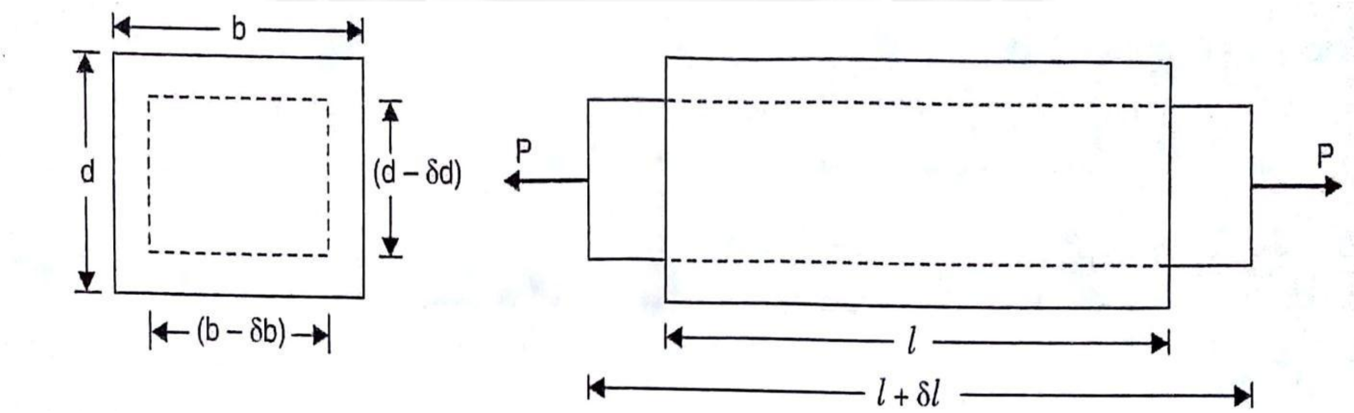
Let δL = Increase in length,

δb = Decrease in breadth and

δd = Decrease in depth.

Then longitudinal strain = $\frac{\delta L}{L}$ and

lateral strain = $\frac{\delta b}{b}$ or $\frac{\delta d}{d}$



1.4.3. ELASTIC CONSTANTS:

There are three types of elastic constants. They are

1. Modulus of Elasticity or Young's Modulus (E),
2. Shear Modulus or Modulus of Rigidity (C) or G or (N) and
3. Bulk Modulus (K)

1.4.4. MODULUS OF ELASTICITY OR YOUNG'S MODULUS:

The ratio of linear stress (tensile or compressive) to the corresponding linear strain is a constant. This ratio is known as Young's Modulus or Modulus of Elasticity and is denoted by E.

$$E = \frac{\text{Linear stress}(\sigma)}{\text{Linear strain}(e)}$$

1.4.5. MODULUS OF RIGIDITY OR SHEAR MODULUS:

The ratio of shear stress to the corresponding shear strain within the elastic limit, is known as Modulus of Rigidity or Shear Modulus. This is denoted by C or G or N.

$$\text{C or G or N} = \frac{\text{Shear stress}(\tau)}{\text{Shear strain}(\emptyset)}$$

1.4.6. BULK MODULUS:

When a body is subjected to the mutually perpendicular like and equal direct stresses, the ratio of direct stress to the corresponding volumetric strain is found to be constant for a given material when the deformation is within a certain limit. This ratio is known as bulk modulus and is usually denoted by K. Mathematically bulk modulus is given by

$$K = \frac{\text{Direct stress}}{\text{Volumetric Strain}} = \frac{\sigma}{\left(\frac{V}{V}\right)}$$

1.4.7. POISSON'S RATIO:

The ratio of lateral strain to the longitudinal strain is a constant for a given material, when the material is stressed within the elastic limit. This ratio is called Poisson's Ratio.

$$\text{Poisson's ratio},(\mu) = \frac{\text{Lateral Strain}}{\text{Linear Strain}}$$

$$\text{Lateral strain} = \mu \times \text{Linear strain}$$

The value of Poisson's ratio varies from 0.25 to 0.33. For rubber, its value ranges from 0.45 to 0.50

Problem1.4.8. Determine the changes in length, breadth and thickness of a steel bar which is 4 m long, 30 mm wide and 20 mm thick and is subjected to an axial pull of 30KN in the direction of its length. Take $E = 2\times10^5$ N/mm^2 and poisson's ratio = 0.3

Given Data

Length of the bar, L=4m =4000mm

Breadth of the bar, b=30mm

Thickness of the bar, t= 20mm

Axial, pull P= 30KN = 30000N

Young's Modulus E= $E=2\times10^5$ N/mm^2

Poisson's ratio $\mu = 0.3$

To Find

The changes in length, changes in breadth and changes in thickness.

Solution

Stress $= \frac{Load}{Area}$

Area $= b\times t = 30\times20 = 600mm^2$

Stress $(\sigma) = \frac{30000}{600} = 50\ N/mm^2$

We know that,

Young's Modulus $= \frac{Stress}{Strain}$

Strain $(e) = \frac{Strss}{Foung's\ Modulus} = \frac{50}{2\times10^5} = 2.5\times10^{-4}$

Changes in length(δL)

Strain (e) $= \frac{\delta L}{L}$

$\delta L = L \times e$

$= 4000 \times 2.5\times10^{-4}$ =**1.0mm**

Changes in breadth (δb)

We know that,

Poisson's ratio (μ) $= \frac{\text{Lateral Strain}}{\text{Linear Strain}}$

Lateral strain = Linear strain ×Poisson's Ratio

$= 2.5\times10^{-4} \times 0.3 = 7.5\times10^{-5}$

Lateral Strain $= \frac{\text{Change in breadth}}{\text{Original breadth}}$

Changes in breadth $(\delta b) = b \times$ lateral strain

$= 30 \times 7.5\times10^{-5} =$ **0.00225mm**

Changes in thickness (δt)

Lateral Strain $= \frac{Change\ in\ thickness}{Original\ thickness}$

Changes in thickness (δt) = t × lateral strain

= 20 ×7.5×10^{-5}

= **0.0015mm**

Problem.1.4.8. Determine the value of Young's Modulus and poisson's ratio of a metallic bar of length 30cm, breadth 4cm and depth 4cm when the bar is subjected to an axial compressive load of 400KN. The decrease in length is given as 0.075cm and increase in breadth is 0.003cm

Given Data

Length of the bar, L=30cm =300mm

Breadth of the bar, b=4cm =40mm

Thickness of the bar, t= 4cm=40mm

Compressive Load P= 400KN =400000N

Decrease in length (δL) = 0.075cm = 0.75mm

Increase in breadth (δb) = 0.003cm = 0.03mm

To Find

The Young's Modulus and Poisson's ratio.

Solution

Young's Modulus

We Know that, Stress = $\frac{Load}{Area}$

Area = b×t = 40×40=1600mm^2

Stress(σ) = $\frac{400000}{1600}$ = 250N/mm^2

Also, Strain(e) = $\frac{\delta L}{L} = \frac{0.75}{300}$ =2.5×10^{-3}

Young's Modulus = $\frac{\text{Stress}}{\text{Strain}} = \frac{250}{2.5\times10^{-3}}$ =**1×10^{5}N/mm^2.**

Poisson's ratio (μ)

We know that, $Poisson's\ ratio\ (\mu) = \frac{\text{Lateral Strain}}{\text{Linear Strain}}$

Lateral Strain = $\frac{\text{Change in breadth}}{\text{Original breadth}} = \frac{0.03}{40}$ = 7.5×10^{-4}

∴ Poisson's ratio $(\mu) = \frac{7.5\times10^{-4}}{2.5\times10^{-3}}$ = **0.3**

Problem1.4.9.A steel bar 300 mm long, 50mm wide and 40mm thick is subjected to a pull of 300KN in the direction of its length. Determine the change in volume. Take E=2×10^5N/mm^2 and $\mu = 0.25$

Given Data

Length of the bar, L=300mm

Breadth of the bar, b=50mm

Thickness of the bar, t= 40mm

Pull P= 300KN =300000N

Young's Modulus E=2×10^5 N/mm^2

Poisson's ratio μ = 0.25

To Find

The changes in volume (δv).

Solution

Original Volume(V) = L×b×t

= 300 × 50 × 40 = 600000mm^3.

$$\text{Linear strain}\left(\frac{dL}{L}\right) = \frac{Stress}{Foung's\ \ Modulus}$$

$$\text{Stress} = \frac{Load}{Area} = \frac{300000}{(50\times40)} = 150 \text{ N/mm}^2.$$

$$\frac{dL}{L} = \frac{150}{2\times10^5} = 0.00075$$

Now, Volumetric strain given by Rectangular bar

$$e_v = \frac{dL}{L}\times(1-2\mu)$$

$$=0.00075\times(1-2\times0.25) = 0.000375$$

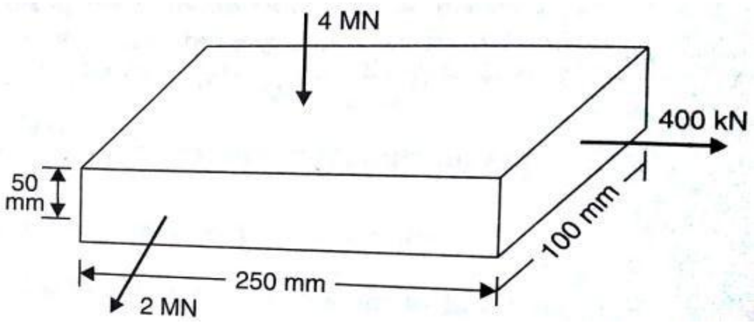
$$e_v = \frac{dV}{V}$$

$$dV = e_v \times v$$

$$= 0.000375\times600000$$

$$= \mathbf{225mm^3}$$

Problem1.4.10.A metallic bar 250mm×100mm×50mm is loaded as shown in Figure. Find the change in volume. Take E=2×10^5N/mm^2 and poisson's ratio = 0.25



Stress in X-direction $(\sigma_x) = \frac{\text{Load in x-direction}}{\text{Area of cross section}} = \frac{\text{Load in x-direction}}{y \times z}$

$= \frac{400000}{100 \times 50} = 80\text{N/mm}^2$ (tension)

Similarly, $\sigma_y = \frac{\text{Load in y-direction}}{x \times z}$

$= \frac{2000000}{250 \times 50} = 160\text{N/mm}^2$.(tensile)

and $\sigma_z = \frac{\text{Load in z-direction}}{x \times y}$

$= \frac{4000000}{250 \times 100} = 160\text{N/mm}^2$ (compression)

Volumetric Strain of a Rectangular bar subjected to three forces which are mutually perpendicular

$$e_v = \frac{1}{E}(\sigma_x + \sigma_y + \sigma z)\ (1-2\mu)$$

$$\frac{dv}{v} = \frac{1}{2 \times 10^5}(80+160-160)(1-2 \times 0.25)$$

$= 0.0002$

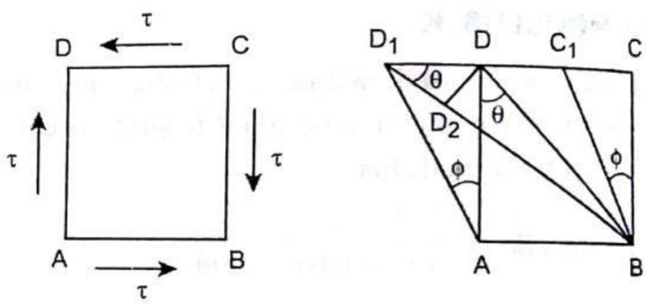
∴ change in volume, $dv = 0.0002 \times v$

$= 0.0002 \times 250 \times 100 \times 50$

$=$ **250mm³**.

1.4.11. RELATIONSHIP BETWEEN MODULUS OF ELASTICITY(E) AND MODULUS OF RIGIDITY(C)

Consider a cube of length L subjected to a shear stress of (τ) shown in Fig. Due to their stresses, the cube will be subjected to a deformation in such a way that the diagonal BD is elongated and the diagonal (AC) will be shortened. Let this shear stress(τ) cause shear strain $(\emptyset)$ as shown in Fig.



(a) Before distortion *(b) After distortion*

Longitudinal strain (e) $= \frac{Change\ in\ length}{0riginal\ length}$

$= \frac{BD1 - BD}{BD}$

$= \frac{D1D2}{BD}$

$= \frac{DD1\ C0S45^0}{BD}$ $\quad \lfloor since \cos 45^o = \frac{D1D2}{D1D} \lceil$

$= \frac{DD1\ C0S45^0}{\frac{AD}{C0S45^0}}$

$= \frac{DD1\ C0S45^0}{\frac{AD}{\frac{1}{\sqrt{2}}}} = \frac{DD1\ C0S45^0}{AD\sqrt{2}}$ $\quad \lfloor since \cos 45^o = \frac{AD}{BD} \lceil$

$= \frac{DD1}{AD\sqrt{2} \times 2}$

$e = \frac{DD1}{2AD}$ …(i)

We know that,

$\tan\emptyset = \frac{DD1}{AD}$

$\emptyset = \frac{DD1}{AD}$ (since$\emptyset$ *is very small*)

Substituting the value of Ø in equation (i) we get,

Longitudinal strain **e =** $\frac{\emptyset}{2}$ …(ii)

From that we know, Longitudinal strain of the diagonal BD is half of the shear strain

Modulus of rigidity, $C = \frac{shearing\ stress(\tau)}{shearing\ strain(\emptyset)}$

Or, $\emptyset = \frac{\tau}{c}$

Substituting the value of Ø in equation ii we get,

Linear strain, e = $\frac{\tau}{2C}$ …(iii)

Tensile strain on the diagonal BD is due to tensile stress and is given by Young's modulus,

$$\text{Young's modulus, } E = \frac{Tensile\ Stress}{Tensile\ Strain}$$

$$E = \frac{\tau}{e}$$

$$\textbf{Tensile strain (e)} = \frac{\tau}{E}$$

Tensile strain on the diagonal BD is due to compressive stress on the diagonal AC which is given by Poisson's ratio.

$$\text{Poission's Ratio, } \mu = \frac{Lateral\ strain}{Longitudinal\ strain\ or\ Tensile\ strain}$$

Lateral strain = Poisson ratio × Tensile strain

$$\textbf{Lateral strain, } e_{lat} = \mu\ X\ \frac{\tau}{E}$$

Total strain = Linear strain + Lateral strain

$$= \frac{\tau}{E} + \mu \times \frac{\tau}{E}$$

$$\text{Total strain} = \frac{\tau}{E}(1+\mu) \quad \text{...(iv)}$$

Equating equation (iii) & (iv) we get,

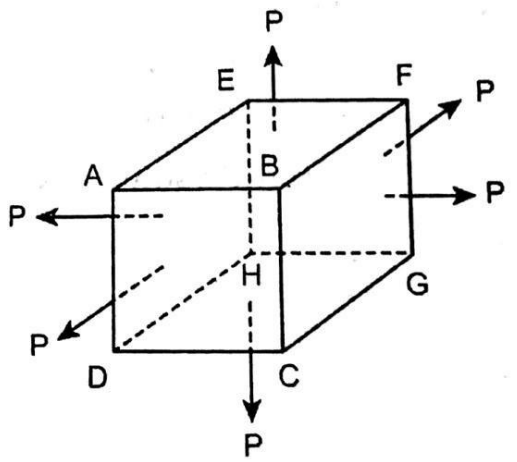
$$\frac{\tau}{2C} = \frac{\tau}{E}(1+\mu)$$

$$\frac{\tau}{2C} = \frac{1}{E}(1+\mu)$$

$$\mathbf{E = 2C(1+\mu)}$$

1.4.12 RELATION BETWEEN BULK MODULUS (K) AND YOUNG'S MODULUS (E)

Consider a cube ABCDEFGH subjected to three mutually perpendicular tensile stresses of equal intensity as shown in Figure



Let, σ = Stress on each face

E = Young's modulus

K= Bulk modulus

$\frac{1}{m}$ or μ = Poisson's ratio

L = Length

Now consider the deformation of one side

(AB) of cube under the action of three mutually perpendicular stresses. This side will suffer the following three strains.

1. Tensile strain of AB is equal to $\frac{\sigma}{E}$ due to stresses on the faces AEHD and BFGC.

Young's modulus, E = $\frac{Tensile\ Stress}{Tensile\ Strain}$

$$E = \frac{\sigma}{e}$$

Tensile strain (e) = $\boldsymbol{\frac{\sigma}{E}}$

2. Compressive lateral strain of AB is equal to $\mu \times \frac{\sigma}{E}$ due to stresses on the faces AEFB and DHGC.

Poission's Ratio, μ = $\frac{\text{Lateral strain}}{\text{Longitudinal strain or Tensile strain}}$

Lateral strain = Poisson ratio × Tensile strain

Lateral strain,$\boldsymbol{e_{lat} = \mu \times \frac{\tau}{E}}$

3. Compressive lateral strain of AB is equal to $\mu \times \frac{\sigma}{E}$ due to stresses on the faces ABCD and EFGH,

Total Strain of AB is given by,

$$\frac{dL}{L} = \frac{\sigma}{E} - \mu\frac{\sigma}{E} - \mu\frac{\sigma}{E}$$

$$\frac{dL}{L} = \frac{\sigma}{E}(1 - \mu - \mu)$$

$$= \frac{\sigma}{E}(1 - 2\mu)$$

$$\frac{dL}{L} = \frac{\sigma}{E}(1 - 2\mu)$$

We know that, original volume of cube $V = L^3$

Differentiating with respect to L,

$$dV = 3L^2 dL$$

$$\frac{dV}{V} = \frac{3L^2 dL}{V}$$

$$\frac{dV}{V} = \frac{3L^2 dL}{L^3}$$

$$\frac{dV}{V} = \frac{3dL}{L}$$

Substituting dL/L value We get

$$\frac{dV}{V} = 3[\frac{\sigma}{E}(1 - 2\mu)[$$

We know that,

$$\text{Bulk Modulus (K)} = \frac{Direct\ Stress}{Volumetric\ strain} = \frac{\sigma}{\frac{d7}{7}}$$

$$= \frac{\sigma}{\frac{3\sigma}{E}(1-2\mu)}$$

$$K = \frac{E}{3(1-2\mu)}$$

$$\mathbf{E = 3K\ (1\text{-}2\mu)}$$

Problem1.4.13. A bar of 30mm diameter is subjected to a pull of 60KN. The measured extension on gauge length of 200mm is 0.1mm and change in diameter is 0.004mm. Calculate the Young's modulus, Poisson's ratio, Shear modulus and Bulk Modulus.

Given Data

Diameter of the bar, d=30mm

Pull P=60KN =60000N

Length of the bar, L= 200mm

Extension in length (δL) = 0.1mm

Changes in diameter (δd) = 0.004mm

To Find

The Young's Modulus, Poisson's ratio, Shear modulus and Bulk Modulus.

Solution

Young's Modulus

We Know that, $\text{Stress} = \frac{Load}{Area}$

$$\text{Area} = \frac{\pi}{4} d^2 = \frac{\pi}{4} 30^2 = 706.86 mm^2$$

$$\text{Stress}(\sigma) = \frac{60000}{706.86} = 84.88\ N/mm^2$$

Also, $\text{Strain(e)} = \frac{\delta L}{L} = \frac{0.1}{200} = 0.0005$

$$\text{Young's Modulus} = \frac{Stress}{Strain} = \frac{84.88}{0.0005} = \mathbf{1.6976 \times 10^5\ N/mm^2.}$$

Poisson's ratio (μ)

We know that, $Poisson's\ ratio\ (\mu) = \frac{Lateral\ Strain}{Linear\ Strain}$

$$\text{Lateral Strain} = \frac{Change\ in\ diameter}{Original\ \ diameter} = \frac{0.004}{30} = 0.00013$$

∴ Poisson's ratio $(\mu) = \frac{0.00013}{0.0005}$ = **0.27**

Shear Modulus(C)

We Know that , $E = 2C(1+\mu)$

Or $C = \frac{E}{2(1+\mu)}$

$= \frac{1.6976 \times 10^5}{2 \times (1+0.27)}$ = $\mathbf{66.835 \times 10^3 N/mm^2}$.

Bulk Modulus (K)

We Know that, $E = 3K(1-2\mu)$

Or $K = \frac{E}{3 \times (1-2\mu)}$

$= \frac{1.6976 \times 10^5}{3 \times (1-2 \times 0.27)}$

= $\mathbf{1.230 \times 10^5 N/mm^2}$.