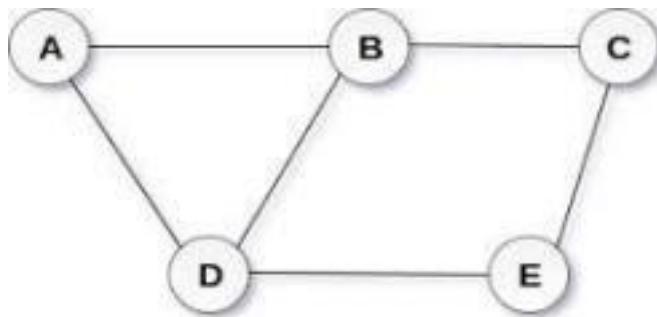


## UNIT IV – GRAPHS

Graphs – Definitions – Traversals – BFS, DFS – Shortest-Path Algorithms – Dijkstra's Algorithm – Minimum Spanning Tree: Prim's Algorithm- Kruskal's Algorithm – Applications: Biconnectivity.

### 4.1 GRAPHS – DEFINITIONS

A graph  $G$  is defined as an ordered set  $(V, E)$ , where  $V(G)$  represents the set of vertices and  $E(G)$  represents the edges that connect these vertices.



#### Undirected Graph

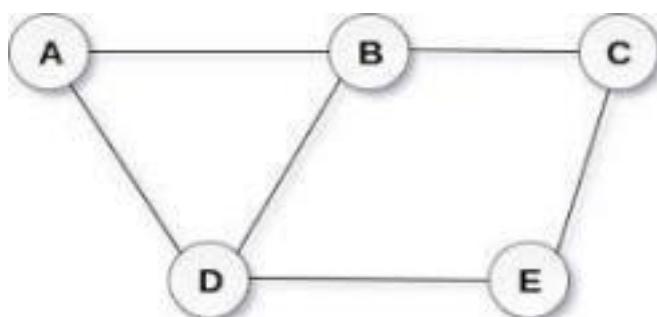
$$V(G) = \{A, B, C, D \text{ and } E\}$$

$$E(G) = \{(A, B), (B, C), (C, E), (E, D), (D, A), (D, B)\}.$$

#### Types of Graphs

##### DIRECTED AND UNDIRECTED GRAPHS

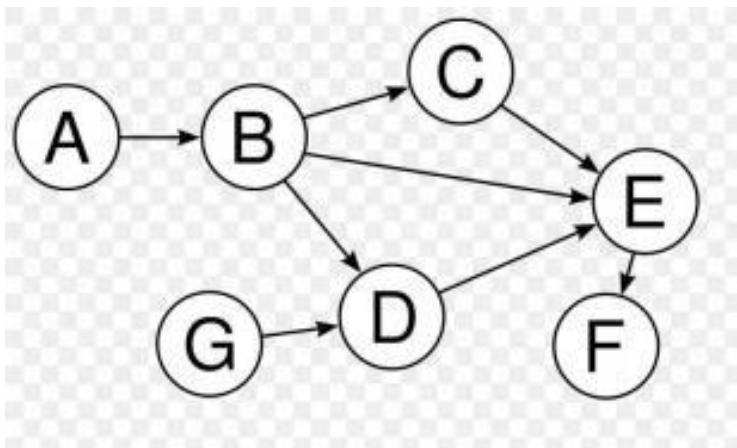
In an undirected graph, edges do not have any direction associated with them.



#### Undirected Graph

##### DIRECTED GRAPHS

In a directed graph, edges have direction associated with them.  $\vee$  If there is an edge from A to B, then there is a path from A to B but not from B to A.



## GRAPH TERMINOLOGY

### Adjacent nodes or neighbours

$e = (u, v)$  that connects nodes  $u$  and  $v$ , the nodes  $u$  and  $v$  are the end points and are said to be the adjacent nodes or neighbours.

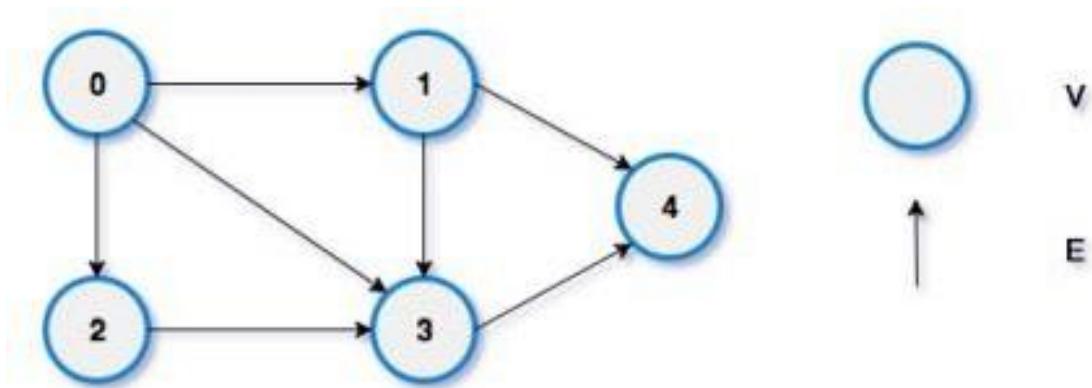
### Degree of a node

Total number of edges containing the node  $u$

### Cycle

A path in which the first and the last vertices are same.

### Degree of a node

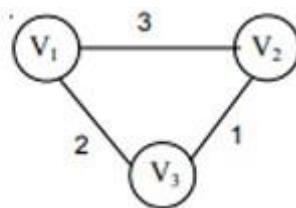
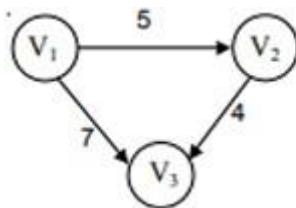


In-degree: In-degree of a vertex is the number of edges coming to the vertex

Out-degree- Out-degree of a vertex is the number edges which are coming out from the vertex.

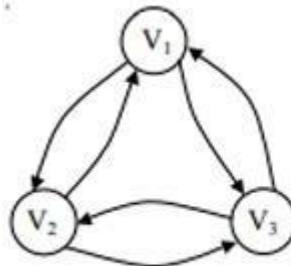
### Weighted Graph

A graph is said to be weighted graph if every edge in the graph is assigned a weight or value. It can be directed or undirected graph

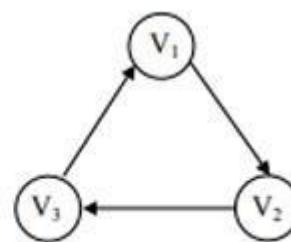


## Strongly Connected Graph

If there is a path from every vertex to every other vertex in a directed graph then it is said to be strongly connected graph. Otherwise, it is said to be weakly connected graph.



Strongly Connected Graph



Weakly Connected Graph

## Graph Representation

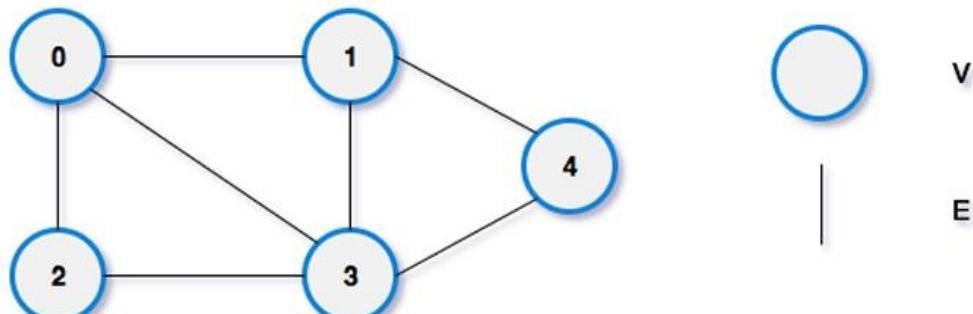
We can easily represent the graphs using the following ways,

1. Adjacency matrix
2. Adjacency list

### Adjacency Matrix Representation of Graph

- Adjacency matrix is a sequential representation.
- If a graph has  $n$  vertices, we use  $n \times n$  matrix to represent the graph. Let's assume the  $n \times n$  matrix as  $\text{adj}[n][n]$ .
- if there is an edge from vertex  $i$  to  $j$ , mark  $\text{adj}[i][j]$  as 1. i.e.  $\text{adj}[i][j] == 1$
- if there is no edge from vertex  $i$  to  $j$ , mark  $\text{adj}[i][j]$  as 0. i.e.  $\text{adj}[i][j] == 0$

### Undirected Graph



Adjacency Matrix of Undirected Graph

	0	1	2	3	4
0	0	1	1	1	0
1	1	0	0	1	1
2	1	0	0	1	0
3	1	1	1	0	1
4	0	1	0	1	0

### Adjacency List Representation of Graph

- Adjacency list is a linked representation.
- In this representation, for each vertex in the graph, we maintain the list of its neighbors. It means, every vertex of the graph contains list of its adjacent vertices.
- We have an array of vertices, which is indexed by the vertex number and for each vertex  $v$ , the corresponding array element points to a singly linked list of neighbors of  $v$ .

