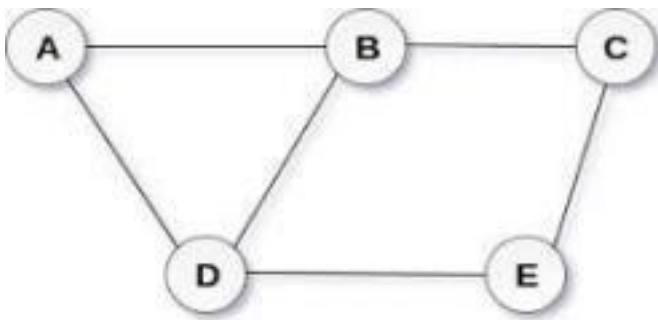


UNIT IV – GRAPHS

Graphs – Definitions – Traversals – BFS, DFS – Shortest-Path Algorithms – Dijkstra's Algorithm – Minimum Spanning Tree: Prim's Algorithm- Kruskal's Algorithm – Applications: Biconnectivity.

4.1 GRAPHS – DEFINITIONS

A graph G is defined as an ordered set (V, E) , where $V(G)$ represents the set of vertices and $E(G)$ represents the edges that connect these vertices.



Undirected Graph

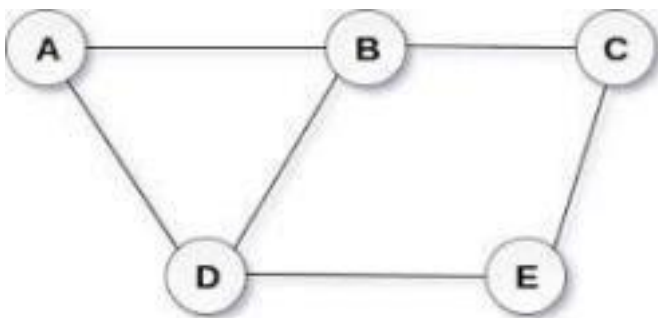
$$V(G) = \{A, B, C, D \text{ and } E\}$$

$$E(G) = \{(A, B), (B, C), (A, D), (B, D), (D, E), (C, E)\}.$$

Types of Graphs

DIRECTED AND UNDIRECTED GRAPHS

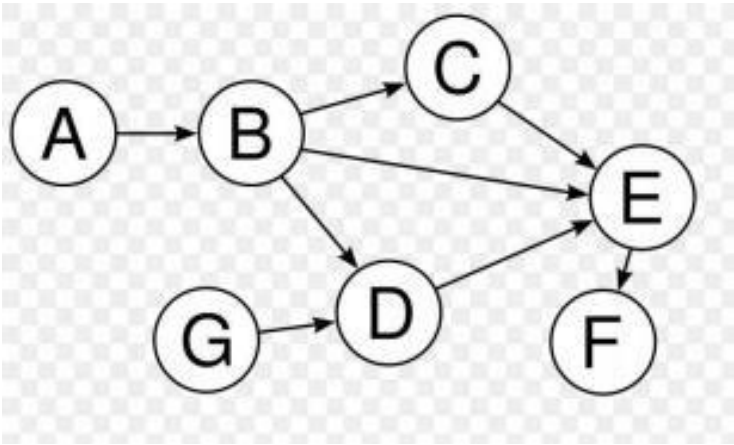
In an undirected graph, edges do not have any direction associated with them.



Undirected Graph

DIRECTED GRAPHS

In a directed graph, edges have direction associated with them. ∪ If there is an edge from A to B, then there is a path from A to B but not from B to A.



GRAPH TERMINOLOGY

Adjacent nodes or neighbours

$e = (u, v)$ that connects nodes u and v , the nodes u and v are the end points and are said to be the adjacent nodes or neighbours.

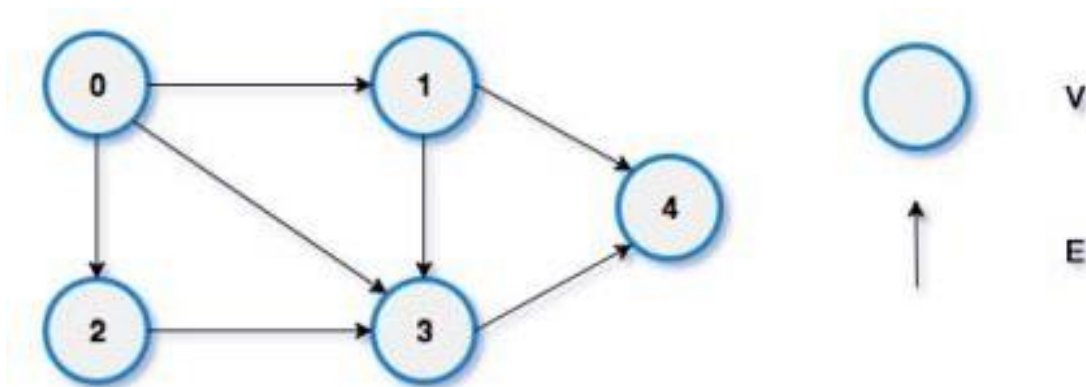
Degree of a node

Total number of edges containing the node u

Cycle

A path in which the first and the last vertices are same.

Degree of a node

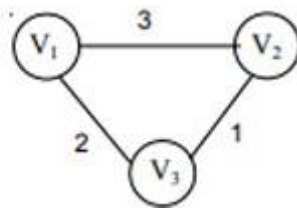
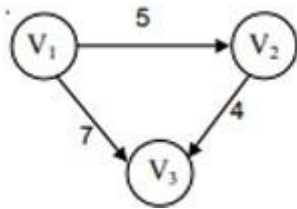


In-degree: In-degree of a vertex is the number of edges coming to the vertex

Out-degree- Out-degree of a vertex is the number edges which are coming out from the vertex.

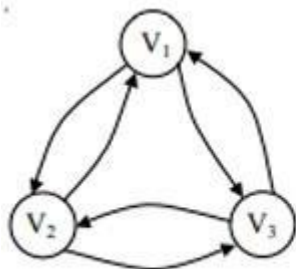
Weighted Graph

A graph is said to be weighted graph if every edge in the graph is assigned a weight or value. It can be directed or undirected graph

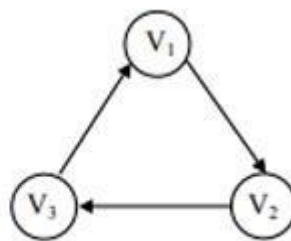


Strongly Connected Graph

If there is a path from every vertex to every other vertex in a directed graph then it is said to be strongly connected graph. Otherwise, it is said to be weakly connected graph.



Strongly Connected Graph



Weakly Connected Graph

Graph Representation

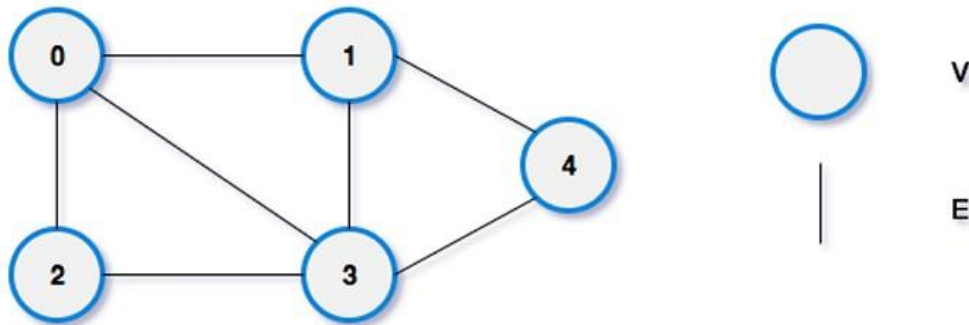
We can easily represent the graphs using the following ways,

1. Adjacency matrix
2. Adjacency list

Adjacency Matrix Representation of Graph

- Adjacency matrix is a sequential representation.
- If a graph has n vertices, we use $n \times n$ matrix to represent the graph. Let's assume the $n \times n$ matrix as $\text{adj}[n][n]$.
- if there is an edge from vertex i to j , mark $\text{adj}[i][j]$ as 1. i.e. $\text{adj}[i][j] == 1$
- if there is no edge from vertex i to j , mark $\text{adj}[i][j]$ as 0. i.e. $\text{adj}[i][j] == 0$

Undirected Graph



Adjacency Matrix of Undirected Graph

	0	1	2	3	4
0	0	1	1	1	0
1	1	0	0	1	1
2	1	0	0	1	0
3	1	1	1	0	1
4	0	1	0	1	0

Adjacency List Representation of Graph

- Adjacency list is a linked representation.
- In this representation, for each vertex in the graph, we maintain the list of its neighbors. It means, every vertex of the graph contains list of its adjacent vertices.
- We have an array of vertices, which is indexed by the vertex number and for each vertex v , the corresponding array element points to a singly linked list of neighbors of v .

