

DESIGN OF EXPERIMENTS

ANOVA – Analysis of Variance

Working Rule (One – Way Classification)

Set the null hypothesis H_0 : There is no significance difference between the treatments.

Set the alternative hypothesis H_1 : There is a significance difference between the treatments.

Step: 1 Find N = number of observations

Step: 2 Find T = The total value of observations

Step: 3 Find the Correction Factor $C.F = \frac{T^2}{N}$

Step: 4 Calculate the total sum of squares and find the total sum of squares

$$TSS = (\sum X_1^2 + \sum X_2^2 + \sum X_3^2 + \dots) - C.F$$

Step: 5 Column sum of squares $SSC \left(\frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_2} + \frac{(\sum X_3)^2}{N_3} + \dots \right) - C.F$

Where N_i = Total number of observation in each column ($i = 1, 2, 3, \dots$)

Step: 6 Prepare the ANOVA to calculate F – ratio

Source of variation	Sum of Degrees	Degrees of Freedom	Mean Square	F - Ratio
Between Samples	SSC	$K - 1$	$MSC = \frac{SSC}{K-1}$	$F_c = \frac{MSC}{MSE}$ if $MSC > MSE$
Within Samples	SSE	$N - K$	$MSE = \frac{SSE}{N-K}$	$F_c = \frac{MSE}{MSC}$ if $MSE > MSC$

Step: 7 Find the table value (use chi square table)

Step: 8 Conclusion:

Calculated value $<$ Table value, then we accept null hypothesis.

Calculated value $>$ Table value, then we reject null hypothesis.

PROBLEMS ON ONE WAY ANOVA

1.A completely randomised design experiment with 10 plots and 3 treatments gave the following results.

Plot No	1	2	3	4	5	6	7	8	9	10
Treatment	A	B	C	A	C	C	A	B	A	B
Yield	5	4	3	7	5	1	3	4	1	7

Analyse the result for treatment effects.

Solution:

Set the null hypothesis H_0 : There is no significance difference between the treatments.

Set the alternative hypothesis H_1 : There is a significance difference between the treatments.

Treatments	Yields from plots			
A	5	7	3	1
B	4	4	7	-
C	3	5	1	-

TABLE:

Treatment A		Treatment B		Treatment C	
X_1	X_1^2	X_2	X_2^2	X_3	X_3^2
5	25	4	16	3	9
7	49	4	16	5	25
3	9	7	49	7	7
1	1	-	-	-	-
$\sum X_1 = 16$	$\sum X_1^2 = 84$	$\sum X_2 = 5$	$\sum X_2^2 = 81$	$\sum X_3 = 9$	$\sum X_3^2 = 35$

Step: 1 $N = 10$

Step: 2 Sum of all the items (T) = $\sum X_1 + \sum X_2 + \sum X_3 = 16 + 15 + 9 = 40$

Step: 3 Find the Correction Factor C . $F = \frac{T^2}{N} = \frac{(40)^2}{10} = 160$

Step: 4 TSS = Total sum of squares

= sum of squares of all the items – C. F

$$\begin{aligned} \text{TSS} &= (\sum X_1^2 + \sum X_2^2 + \sum X_3^2 + \dots) - C.F \\ &= (84 + 81 + 35) - 160 = 40 \end{aligned}$$

Step: 5 SSC = Sum of squares between samples

$$\begin{aligned} \text{SSC} &= \left(\frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_2} + \frac{(\sum X_3)^2}{N_3} + \dots \right) - C.F \\ \text{SSC} &= \left(\frac{(16)^2}{4} + \frac{(15)^2}{3} + \frac{(9)^2}{3} + \dots \right) - 160 \\ &= 64 + 75 + 27 - 160 = 6 \end{aligned}$$

Step: 6 MSC = Mean squares between samples

$$\begin{aligned} &= \frac{\text{Sum of squares between samples}}{d.f} \\ &= \frac{6}{2} = 3 \end{aligned}$$

SSE = Sum of squares within samples

$$\begin{aligned} &= \text{Total sum of squares} - \text{Sum of squares between samples} \\ &= 40 - 6 = 34 \end{aligned}$$

Step: 7 MSE = Mean squares within samples

$$\begin{aligned} &= \frac{\text{Sum of squares within samples}}{d.f} \\ &= \frac{34}{7} = 4.86 \end{aligned}$$

ANOVA TABLE

Source of variation	Sum of Degrees	Degrees of Freedom	Mean Square	F - Ratio
Between Samples	SSC = 6	$K - 1 = 3 - 1 = 2$	$\text{MSC} = \frac{\text{SSC}}{K-1} = 3$	
Within Samples	SSE = 34	$N - K = 10 - 3 = 7$	$\text{MSE} = \frac{\text{SSE}}{N-K} = 4.86$	$F_c = \frac{\text{MSE}}{\text{MSC}} = 1.62$

d.f for (7, 2) at 5% level of significance is 19.35

Step: 8 Conclusion:

Calculated value < Table value, then we accept null hypothesis.

2. Three different machines are used for a production. On the basis of the outputs, set up one – way ANOVA table and test whether the machines are equally effective.

Outputs		
Machine I	Machine II	Machine III
10	9	20
15	7	16
11	5	10
10	6	14

Given that the value of F at 5% level of significance for (2, 9) d. f is 4.26

Solution:

Set the null hypothesis H_0 : The machines are equally effective.

TABLE:

Treatment A		Treatment B		Treatment C	
X_1	X_1^2	X_2	X_2^2	X_3	X_3^2
10	100	9	81	20	400
15	225	7	49	16	256
11	121	5	25	10	100
20	400	6	36	14	196
$\sum X_1 = 56$	$\sum X_1^2 = 846$	$\sum X_2 = 27$	$\sum X_2^2 = 191$	$\sum X_3 = 60$	$\sum X_3^2 = 952$

Step: 1 $N = 12$

Step: 2 Sum of all the items (T) = $\sum X_1 + \sum X_2 + \sum X_3 = 56 + 27 + 60 = 143$

Step: 3 Find the Correction Factor C . $F = \frac{T^2}{N} = \frac{(143)^2}{12} = 1704.08$

Step: 4 TSS = Total sum of squares

= sum of squares of all the items – C. F

$$\begin{aligned} \text{TSS} &= (\sum X_1^2 + \sum X_2^2 + \sum X_3^2 + \dots) - C.F \\ &= (846 + 191 + 952) - 1704.08 = 284.92 \end{aligned}$$

Step: 5 SSC = Sum of squares between samples

$$\begin{aligned} \text{SSC} &= \left(\frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_2} + \frac{(\sum X_3)^2}{N_3} + \dots \right) - C.F \\ \text{SSC} &= \left(\frac{(56)^2}{4} + \frac{(27)^2}{4} + \frac{(60)^2}{4} + \dots \right) - 1704.08 \\ &= 784 + 182.25 + 900 - 1704.08 = 162.17 \end{aligned}$$

Step: 6 MSC = Mean squares between samples

$$\begin{aligned} &= \frac{\text{Sum of squares between samples}}{d.f} \\ &= \frac{162.17}{2} = 81.085 \end{aligned}$$

SSE = Sum of squares within samples

$$\begin{aligned} &= \text{Total sum of squares} - \text{Sum of squares between samples} \\ &= 284.92 - 162.17 = 122.75 \end{aligned}$$

Step: 7 MSE = Mean squares within samples

$$\begin{aligned} &= \frac{\text{Sum of squares within samples}}{d.f} \\ &= \frac{122.75}{9} = 13.63 \end{aligned}$$

ANOVA TABLE

Source of variation	Sum of Degrees	Degrees of Freedom	Mean Square	F - Ratio
Between Samples	SSC = 162.17	$K - 1 = 3 - 1 = 2$	$\text{MSC} = \frac{\text{SSC}}{K-1} = 81.085$	
Within Samples	SSE = 122.75	$N - K = 12 - 3 = 9$	$\text{MSE} = \frac{\text{SSE}}{N-K} = 13.63$	$F_c = \frac{\text{MSE}}{\text{MSC}} = 5.95$

d.f for (2, 9) at 5% level of significance is 4.26.

Step: 8 Conclusion:

Calculated value $>$ Table value, then we reject the null hypothesis.

i.e., the three machines are not equally effective.



Randomised Block Design (RBD)

Working Rule:

Set the null hypothesis H_0 : There is no significance difference between the treatments.

Step: 1 Find T = The total value of observations

Step: 2 Find the Correction Factor C . $F = \frac{T^2}{N}$

Step: 3 Calculate the total sum of squares and find the total sum of squares

$$TSS = (\sum X_1^2 + \sum X_2^2 + \sum X_3^2 + \dots) - C.F$$

Step: 4 Find column sum of squares $SSC = \left(\frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_2} + \frac{(\sum X_3)^2}{N_3} + \dots \right) - C.F$

Where N_i = Total number of observation in each column ($i = 1, 2, 3, \dots$)

Step: 5 Find Column sum of squares $SSR = \left(\frac{(\sum Y_1)^2}{N_1} + \frac{(\sum Y)^2}{N_2} + \frac{(\sum Y_3)^2}{N_3} + \dots \right) - C.F$

Where N_j = Total number of observation in each ROW ($j = 1, 2, 3, \dots$)

Step: 6 $SSE = TSS - (SSC + SSR)$

Step: 7 Prepare the ANOVA to calculate F – ratio

Source of variation	Sum of Degrees	Degrees of Freedom	Mean Square	F - Ratio
Between Columns	SSC	c - 1	$MSC = \frac{SSC}{c-1}$	$F_c = \frac{MSC}{MSE}$ if $MSC > MSE$ $F_c = \frac{MSE}{MSC}$ if $MSE > MSC$
Between Rows	SSR	r - 1	$MSR = \frac{SSE}{r-1}$	$F_c = \frac{MSR}{MSE}$ if $MSR > MSE$ $F_c = \frac{MSE}{MSR}$ if $MSE > MSR$
Error	SSE	(r - 1)(c - 1)	$MSE = \frac{SSE}{(r-1)(c-1)}$	

Step: 8 Find the table value (use chi square table)

Step: 9 Conclusion:

Calculated value < Table value, then we accept null hypothesis.

Calculated value > Table value, then we reject null hypothesis.

PROBLEMS ON TWO WAY ANOVA TABLE

1. Three varieties A, B, C of a crop are tested in a randomized block design with four replication. The plot yields in pounds as follows.

A6	C5	A8	B9
C8	A4	B6	C9
B7	B6	C10	A6

Analysis the experiment yield and state your conclusion.

Solution:

Set the null hypothesis H_0 : There is no significance difference between the rows and columns.

Varieties	Yields				
	1	2	3	4	Total
A	6	4	8	6	24
B	7	6	6	9	28
C	8	5	10	9	32
Total	21	15	24	24	84

TEST STATISTIC:

Varieties		1	2	3	4	Total	X_1^2	X_2^2	X_3^2	X_4^2
		X_1	X_2	X_3	X_4					
Y_1	A	6	4	8	6	24	36	16	64	36
Y_2	B	7	6	6	9	28	49	36	36	81
Y_3	C	8	5	10	9	32	64	25	100	81
Total		21	15	24	24	84	149	77	200	198

Step:1 Grand Total $T = 84$

Step: 2 Correction Factor $C.F = \frac{T^2}{N} = \frac{(84)^2}{12} = 588$

Step: 3 Calculate the total sum of squares and find the total sum of squares

$$TSS = (\sum X_1^2 + \sum X_2^2 + \sum X_3^2 + \dots) - C.F$$

$$=(149 + 77 + 200 + 198) - 588$$

$$= 624 - 588 = 36$$

Step: 4 Find column sum of squares $SSC = \left(\frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_2} + \frac{(\sum X_3)^2}{N_3} + \dots \right) - C.F$

$$SSC = \left(\frac{(21)^2}{3} + \frac{(15)^2}{3} + \frac{(24)^2}{3} + \frac{(24)^2}{3} \right) - 588 = 18$$

Step: 5 Find Row sum of squares $SSR = \left(\frac{(\sum Y_1)^2}{N_1} + \frac{(\sum Y)^2}{N_2} + \frac{(\sum Y_3)^2}{N_3} + \dots \right) - C.F$

$$SSR = \left(\frac{(24)^2}{4} + \frac{(28)^2}{4} + \frac{(32)^2}{4} + \dots \right) - 588 = 8$$

Step: 6 SSE = Residual sum of squares

$$= TSS - (SSC + SSR)$$

$$= 36 - (18 + 8) = 10$$

Step: 7 Prepare the ANOVA to calculate F – ratio

Source of variation	Sum of Degrees	Degrees of Freedom	Mean Square	F - Ratio
Between Columns	SSC=18	$c - 1$ $= 4 - 1 = 3$	$MSC = \frac{SSC}{c-1} = 6$	$F_c = \frac{MSC}{MSE} = 3.6$
Between Rows	SSR=8	$r - 1$ $= 3 - 1 = 2$	$MSR = \frac{SSR}{r-1} = 4$	$F_R = \frac{MSR}{MSE} = 2.4$
Error	SSE = 10	$(r - 1)(c - 1)$ $2 \times 3 = 6$	$MSE = \frac{SSE}{(r - 1)(c - 1)} = 1.667$	

Step: 8 d.f for (3, 6) at 5% level of significance is 4.76

d.f for (2, 6) at 5% level of significance is 5.14

Step: 9 Conclusion:

Calculated value $F_c < \text{Table value}$, then we accept null hypothesis.

There is no significance difference between the columns.

Calculated value $F_R < \text{Table value}$, then we accept null hypothesis.

There is no significance difference between the rows.

2. Four varieties A, B, C, D of a fertilizer are tested in a randomized block design with four replication. The plot yields in pounds as follows.

A 12	D 20	C 16	B 10
D 18	A 14	B 11	C 14
B 12	C 15	D 19	A 13
C 16	B 11	A 15	D 20

Analysis the experimental yield.

Solution:

Set the null hypothesis H_0 : There is no significance difference between the rows and columns.

Varieties	Yields				
	1	2	3	4	Total
A	12	14	15	13	54
B	12	11	11	10	44
C	16	15	16	14	61
D	18	20	19	20	77
Total	58	60	61	57	236 (T)

TEST STATISTIC:

Varieties		1	2	3	4	Total	X_1^2	X_2^2	X_3^2	X_4^2
		X_1	X_2	X_3	X_4					
Y_1	A	12	14	15	13	54	144	196	225	169
Y_2	B	12	11	11	10	44	144	121	121	100
Y_3	C	16	15	16	14	61	256	225	256	196
Y_4	D	18	20	19	20	77	324	400	361	400
Total		58	60	61	57	236	868	942	963	865

Step:1 Grand Total $T = 236$

Step: 2 Correction Factor $C.F = \frac{T^2}{N} = \frac{(236)^2}{16} = 3481$

Step: 3 Calculate the total sum of squares and find the total sum of squares

$$\begin{aligned}
 TSS &= (\sum X_1^2 + \sum X_2^2 + \sum X_3^2 + \dots) - C.F \\
 &= (868 + 942 + 963 + 865) - 3481 \\
 &= 3638 - 3481 = 157
 \end{aligned}$$

Step: 4 Find column sum of squares $SSC = \left(\frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_2} + \frac{(\sum X_3)^2}{N_3} + \dots \right) - C.F$

$$SSC = \left(\frac{(58)^2}{4} + \frac{(60)^2}{4} + \frac{(61)^2}{4} + \frac{(57)^2}{4} \right) - 3481$$

$$= 841 + 900 + 930 + 812 - 3481 = 2$$

Step: 5 Find Row sum of squares $SSR = \left(\frac{(\sum Y_1)^2}{N_1} + \frac{(\sum Y)^2}{N_2} + \frac{(\sum Y_3)^2}{N_3} + \dots \right) - C.F$

$$SSR = \left(\frac{(54)^2}{4} + \frac{(44)^2}{4} + \frac{(61)^2}{4} + \frac{(77)^2}{4} \right) - 3481$$

$$= 729 + 484 + 930.25 + 1482.25 - 3481$$

$$= 144.5$$

Step: 6 SSE = Residual sum of squares

$$= TSS - (SSC + SSR)$$

$$= 157 - (2 + 144.5) = 10.5$$

Step: 7 Prepare the ANOVA to calculate F – ratio

Source of variation	Sum of Degrees	Degrees of Freedom	Mean Square	F - Ratio
Between Columns	SSC=2	$c - 1$ $= 4 - 1 = 3$	$MSC = \frac{SSC}{c-1}$ $= 0.666$	$F_c = \frac{MSE}{MSC} = 1.74$
Between Rows	SSR=144.5	$r - 1$ $= 4 - 1 = 3$	$MSR = \frac{SSR}{r-1}$ $= 48.16$	$F_R = \frac{MSR}{MSE} = 41.51$
Error	SSE = 10.5	$(r - 1)(c - 1)$ $= 3 \times 3 = 9$	$MSE = \frac{SSE}{(r - 1)(c - 1)}$ $= 1.6$	

Step: 8 d.f for (9, 3) at 5% level of significance is 8.82

d.f for (3, 9) at 5% level of significance is 3.86

Step: 9 Conclusion:

Calculated value $F_c < \text{Table value}$, then we accept null hypothesis.

There is no significance difference between the columns.

Calculated value $F_R > \text{Table value}$, then we reject null hypothesis.

There is a significance difference between the rows.

LATIN SQUARE:**Steps in constructing Latin Square****Step:1**

Square the Grand total (T) and divide it by the number of observations (N).

i.e), Find $\frac{T^2}{N}$ which is called the correction factor (C.F)

Step:2

Add the squares of the individual observations (X_i 's) and subtract the C.F from it to get the total sum of squares. i.e), Find Total sum of squares TSS

$$\text{i.e), TSS} = \sum_i (X_i)^2 - \frac{T^2}{N}$$

Step:3

Add the squares of the row sums (R_i) divide it by the number of items in a row and subtract the C.F from the result to get the row sum of squares.

$$\text{Row sum of squares } SSR = \frac{(\sum R_i)^2}{n_1} - C.F$$

Where n_1 is the number of items in a row.

Step:4

Add the squares of the columns sums (C_i) divide it by the number of items and subtract the C.F from the result to get the column sum of squares.

$$\text{Column sum of squares } SSC = \frac{(\sum C_j)^2}{n_2} - C.F$$

Where n_2 is the number of items in a column.

Step:5

Sum of the squares of the treatment sums (T_i) divide it by the number of treatments and subtract the C.F from it to get the treatment sum of squares, i.e., Treatment sum of squares.

$$SST = \frac{(\sum T_i)^2}{n_i} - C.F$$

Where n_i is the number of treatments.

Step:6

Subtract the sum obtained in steps 3, 4, and 5 from 2 we get residual.

i.e., Residual $SSE = TSS - (SSR + SSC + SST)$

Step:7

Prepare the ANOVA table using all these and calculate the various mean squares as follows.

Source of variation	Sum of Degrees	Degrees of Freedom	Mean Square	F - Ratio
Between Rows	SSR	$n - 1$	$MSR = \frac{SSR}{n-1}$	$F_R = \frac{MSR}{MSE}$ if $MSR > MSE$ $F_R = \frac{MSE}{MSR}$ if $MSE > MSR$
Between Columns	SSC	$n - 1$	$MSC = \frac{SSC}{n-1}$	$F_c = \frac{MSC}{MSE}$ if $MSC > MSE$ $F_c = \frac{MSE}{MSC}$ if $MSE > MSC$
Treatments	SST	$n - 1$	$MST = \frac{SST}{n-1}$	$F_T = \frac{MST}{MSE}$ if $MST > MSE$ $F_T = \frac{MSE}{MST}$ if $MSE > MST$
Residual or Error	SSE	$(n - 1)(n - 2)$	$MSE = \frac{SSE}{(n - 1)(n - 2)}$	

Step:8

Compute the F-ratio and find out whether the differences are significant or not according to the given level of significance.

1. Set up the analysis of variance for the following results of a Latin square design.

A 12	C 19	B 10	D 8
C 18	B 12	D 6	A 7
B 22	D 10	A 5	C 21
C 12	A 7	C 27	B 17

Solution:

Set the null hypothesis H_0 : There is no significance difference between the rows, columns and treatments.

Table I (To find TSS, SSR and SSC)

	C_1	C_2	C_3	C_4	Row Total R_i	$R_i^2/4$
R_1	12	19	10	8	49	600.25
R_2	18	12	6	7	43	462.25
R_3	22	10	5	21	58	841
R_4	12	7	27	17	63	992.25
Column Total C_j	64	48	48	53	213 (T)	2895.75
$C_j^2/4$	1024	576	576	702.25	2895.75	$\sum R_i^2/4$
					$\sum C_j^2/4$	

Table II (To find SST)

	1	2	3	4	Row Total T_i	$T_i^2/4$
A	12	7	5	7	31	240.25
B	10	12	22	17	61	930.25
C	19	18	21	27	85	1806.25
D	8	6	10	12	36	324
						3300.75 = $\sum T_i^2/4$

Step:1

Grand total (T) = 213

Step:2

$$\text{Correction factor (C.F.)} = \frac{T^2}{N} = \frac{(213)^2}{16} = 2835.56$$

Step:3

Sum of squares of individual observations

$$\begin{aligned}
 &= (12)^2 + (7)^2 + (5)^2 + (7)^2 + (10)^2 + (12)^2 + (22)^2 + (17)^2 + \\
 &\quad (19)^2 + (18)^2 + (21)^2 + (27)^2 + (8)^2 + (6)^2 + (10)^2 + (12)^2 \\
 &= 3483
 \end{aligned}$$

Step:4

TSS = sum of squares of individual observations – C.F

$$= \sum_i (X_i)^2 - \frac{T^2}{N} = 3486 - 2835.56 = 647.44$$

Step:5

Row sum of squares $SSR = \frac{(\sum R_i)^2}{4} - C.F = 2895.75 - 2835.56 = 60.19$

Step:6

Column sum of squares $SSC = \frac{(\sum C_j)^2}{4} - C.F = 2878.25 - 2835.56$
 $= 42.69$

Step:7

Sum of squares of Treatment

$$SST = \frac{(\sum T_i)^2}{n_i} - C.F = 3300.75 - 2835.56 = 465.19$$

Step:8

$$\begin{aligned}
 \text{Residual } SSE &= TSS - (SSR + SSC + SST) \\
 &= 647.44 - (60.19 + 42.69 + 465.19) = 79.37
 \end{aligned}$$

Step:9

Prepare the ANOVA table using all these and calculate the various mean squares as follows.

Source of variation	Sum of Degrees	Degrees of Freedom	Mean Square	F - Ratio
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Between Rows	SSR=60.19	$4 - 1 = 3$	$MSR = \frac{SSR}{n-1} = 20.06$	$F_R = \frac{MSR}{MSE} = 1.52$
Between Columns	SSC=42.69	$4 - 1 = 3$	$MSC = \frac{SSC}{n-1} = 14.23$	$F_C = \frac{MSC}{MSE} = 1.08$
Treatments	SST=465.19	$4 - 1 = 3$	$MST = \frac{SST}{n-1} = 155.06$	$F_T = \frac{MST}{MSE} = 11.73$
Residual or Error	SSE=79.37	$(4 - 1)(4 - 2) = 6$	$MSE = \frac{SSE}{(n - 1)(n - 2)} = 13.22$	

Step: 10

d.f for (3, 6) at 5% level of significance is 4.76

Step: 11 Conclusion:

Calculated value $F_C < \text{Table value}$, then we accept null hypothesis.

There is no significance difference between the columns.

Calculated value $F_R < \text{Table value}$, then we accept null hypothesis.

There is no significance difference between the rows.

Calculated value $F_T > \text{Table value}$, then we reject null hypothesis.

There is a significance difference between the rows.