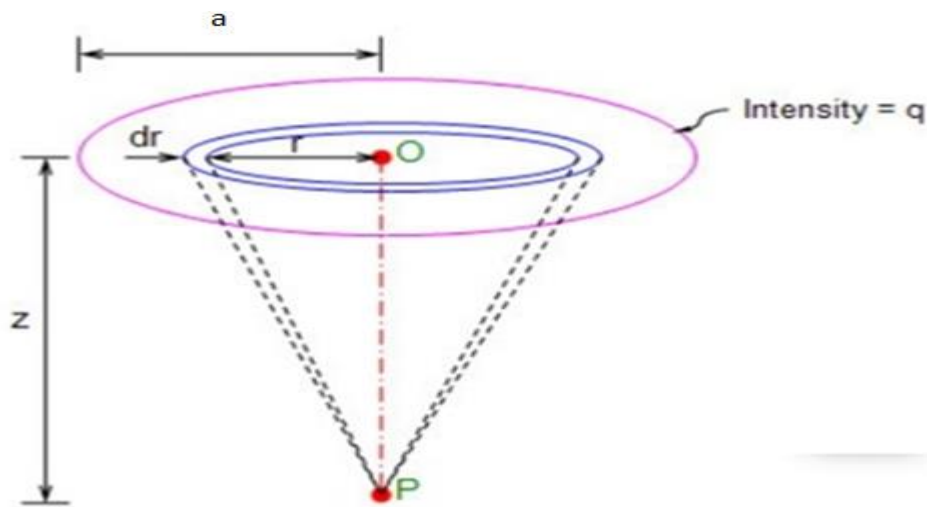


2) Stress due to uniformly loaded circular area:



The Boussinesq equation for vertical stress due to a point load can be extended to find the vertical stress at any point beneath the centre of a uniformly loaded circular area.

Let q = intensity of the load per unit area

and R = the radius of the loaded area

Let us consider an elementary ring of radius r and thickness ' dr ' or δr of the loaded area.

The load on the elementary ring = $q \delta a = q(2\pi r)dr$

But we know that

$$\sigma_z = \frac{3Q}{2\pi z^2} \left[\frac{1}{\left(1 + \frac{r^2}{z^2}\right)} \right]^{5/2}$$

$$\Delta\sigma_z = \frac{3q(2\pi r)dr}{2\pi z^2} \left[\frac{1}{\left(1 + \frac{r^2}{z^2}\right)} \right]^{5/2}$$

$$\Delta\sigma_z = \frac{3q(r)dr}{(r^2 + z^2)^{5/2}} z^3$$

The vertical stress due to full load is given by

$$\sigma_z = 3qz^3 \int_0^a \frac{rdr}{(r^2 + z^2)^{5/2}} \text{-----}(1)$$

According to Boussinesq equation

$$R = \sqrt{r^2 + z^2} = (r^2 + z^2)^{1/2}$$

Replacing R=n

$$n = (r^2 + z^2)^{1/2}$$

$$n^2 = (r^2 + z^2) \text{-----}(2)$$

Diff eqn (2) $2n \cdot dn = 2rdr$

$$n \cdot dn = rdr$$

$$\text{when } r=0 \text{ eqn (2)} \rightarrow n^2 = z^2$$

$$\text{when } r=a \text{ eqn (2)} \rightarrow n^2 = (a^2 + z^2)$$

$$n = (a^2 + z^2)^{1/2}$$

Substituting the new limit in terms of n in equation(1)

$$\sigma_z = 3qz^3 \int_z^{(a^2+z^2)^{1/2}} \frac{rdr}{(r^2 + z^2)^{5/2}}$$

$$\sigma_z = 3qz^3 \int_z^{(a^2+z^2)^{1/2}} \frac{n \cdot dn}{(n^2)^{5/2}}$$

$$\sigma_z = 3qz^3 \int_z^{(a^2+z^2)^{1/2}} \frac{ndn}{(n)^5}$$

$$\sigma_z = 3qz^3 \int_z^{(a^2+z^2)^{1/2}} \frac{dn}{(n)^4}$$

$$\sigma_z = 3qz^3 \int_z^{(a^2+z^2)^{1/2}} n^{-4} dn$$

$$\sigma_z = 3qz^3 \left[\frac{n^{-4+1}}{-4+1} \right]_z^{(a^2+z^2)^{1/2}}$$

$$\sigma_z = 3qz^3 \left[\frac{n^{-3}}{-3} \right]_z^{(a^2+z^2)^{1/2}}$$

$$\sigma_z = \frac{3qz^3}{-3} [n^{-3}]_z^{(a^2+z^2)^{1/2}}$$

$$\sigma_z = \frac{3qz^3}{-3} \left[\left((a^2 + z^2)^{\frac{1}{2}} \right)^{-3} - z^{-3} \right]$$

$$\sigma_z = \frac{3qz^3}{-3} \left[\frac{1}{(a^2 + z^2)^{\frac{3}{2}}} - \frac{1}{z^3} \right]$$

$$\sigma_z = \frac{3qz^3}{-3} \left[\frac{1}{\left[z^2 \left(\frac{a^2}{z^2} + 1 \right)^{3/2} \right]} - \frac{1}{z^3} \right]$$

$$\sigma_z = \frac{3qz^3}{-3z^3} \left[\frac{1}{\left[\left(\frac{a^2}{z^2} + 1 \right)^{3/2} \right]} - 1 \right]$$

$$\sigma_z = \frac{3qz^3}{3z^3} \left[1 - \frac{1}{\left[\left(\frac{a^2}{z^2} + 1 \right)^{3/2} \right]} \right]$$

$$\sigma_z = q \left[1 - \frac{1}{\left[\left(\frac{a^2}{z^2} + 1 \right)^{3/2} \right]} \right]$$

$$\sigma_z = q K_B$$

$$K_B = \left[1 - \frac{1}{\left[\left(\frac{a^2}{z^2} + 1 \right)^{3/2} \right]} \right]$$

K_B = Influence factor (Boussinesq) for uniformly distributed circular area.

Problem:

1) A circular area 6m in diameter, carries uniformly distributed load of 10KN/m². Determine the vertical stress at a depth of 2m, 4m, 8m. Plot the variation of vertical stress with depth

Solution:

$$\sigma_z = q \left[1 - \frac{1}{\left[\left(\frac{a^2}{z^2} + 1 \right)^{3/2} \right]} \right]$$

$$a = 3\text{m}, q = 10\text{KN/m}^2$$

at $z = 2\text{m}$,

$$\sigma_z = 10 \left[1 - \frac{1}{\left[\left(\frac{3^2}{2^2} + 1 \right)^{3/2} \right]} \right]$$

$$\sigma_z = 8.29\text{KN/m}^2$$

at $z = 4\text{m}$

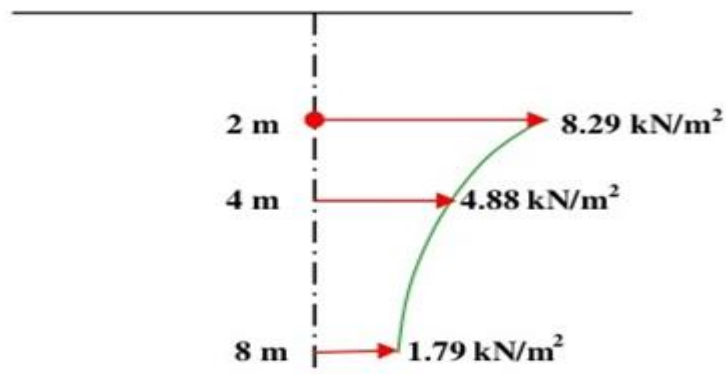
$$\sigma_z = 10 \left[1 - \frac{1}{\left[\left(\frac{3^2}{4^2} + 1 \right)^{\frac{3}{2}} \right]} \right]$$

$$\sigma_z = 4.88 \text{ kN/m}^2$$

At z=8m

$$\sigma_z = 10 \left[1 - \frac{1}{\left[\left(\frac{3^2}{8^2} + 1 \right)^{\frac{3}{2}} \right]} \right]$$

$$\sigma_z = 1.79 \text{ kN/m}^2$$



OBSERVE OPTIMIZE OUTSPREAD