DIGITAL IMAGE PROCESSING

Unit 2– Image Restoration: Restoration Filters (Gonzalez & Woods)

Topic: Restoration Filters

1. Restoration Filters

Restoration filters recover an estimate $\hat{f}(x,y)$ of the original image from a degraded observation

$$g(x,y) = h(x,y)*f(x,y) + \eta(x,y).$$

2. Mean Filters (Spatial Domain)

These filters operate on the grey levels g(s,t) of an m × n neighbourhood Sxy centred at (x,y), and are used when the only degradation present is noise (H(u,v)=1):

Property	Statement
Arithmetic mean filter	$\hat{f}(x,y) = (1/mn)\ \Sigma\ g(s,t)$ — simple averaging; smooths noise but blurs edges
Geometric mean filter	$\hat{f}(x,y) = [\ \Pi\ g(s,t)\ ]^{(1/mn)}$ — comparable smoothing to the arithmetic mean, but loses less fine detail
Harmonic mean filter	$\hat{f}(x,y) = mn / \Sigma[1/g(s,t)]$ — effective against salt noise and Gaussian noise, but fails for pepper noise
Contraharmonic mean filter, order Q	$\hat{f}(x,y) = \Sigma\ g(s,t)^{(Q+1)} / \Sigma\ g(s,t)^{Q}$ — removes pepper noise for Q > 0 and salt noise for Q < 0 (Q=0 gives arithmetic mean; Q=−1 gives harmonic mean); the sign of Q must be chosen correctly or the noise is made worse

3. Order-Statistics Filters (Spatial Domain)

These filters rank (order) the grey levels within Sxy and select a value based on that ranking, and are particularly effective against impulse noise:

4. Adaptive Filters

Adaptive filters change their behaviour depending on the statistical characteristics of the image inside the filter window Sxy, giving noticeably better performance than the non-adaptive filters above — at the cost of additional filter complexity.

Property	Statement
Adaptive, local noise reduction filter	Compares the local variance σ^2_{Sxy} of Sxy with the overall noise variance σ^2_η: $\hat{f}(x,y) = g(x,y) - (\sigma^2_\eta/\sigma^2_{Sxy})\cdot[g(x,y)-\bar{z}_{Sxy}]$. If σ^2_η is small relative to σ^2_{Sxy}, the filter returns a value close to g(x,y) (preserving edges/detail); if the two variances are approximately equal, it returns close to the local average $\bar{z}_{Sxy}$ (maximal smoothing).
Adaptive median filter	Changes the size of Sxy during operation depending on certain conditions, distinguishing impulse noise from edges/detail. It can remove impulse noise with probabilities much higher than the standard median filter, while

Property	Statement
	simultaneously preserving sharpness by smoothing non-impulse noise that a fixed-size median filter cannot handle.

5. Inverse Filtering (Frequency Domain)

When the degradation function H(u,v) is known, the most direct restoration approach is inverse filtering — dividing the transform of the degraded image by H(u,v):

$$\hat{F}(u,v) = G(u,v)/H(u,v) = F(u,v) + N(u,v)/H(u,v)$$

This reveals the central weakness of inverse filtering: since N(u,v) is unknown, and H(u,v) typically approaches zero away from the origin, the noise term N(u,v)/H(u,v) can dominate and swamp the restored image. In practice, the filter is therefore only applied within a limited radius around the origin, excluding frequencies where H(u,v) is very small.

6. Minimum Mean Square Error (Wiener) Filtering

The Wiener filter overcomes the noise-sensitivity problem of inverse filtering by incorporating the statistical properties of both the image and the noise, minimising the mean square error $e^2 = E\{(f-\hat{f})^2\}$:

$$\hat{F}(u,v) = [\ |H(u,v)|^2 \,/\, (\ H(u,v)\cdot[|H(u,v)|^2 + Sn(u,v)/Sf(u,v)]\ )\ ] \cdot G(u,v)$$

where $Sn(u,v)=|N(u,v)|^2$ is the noise power spectrum and $Sf(u,v)=|F(u,v)|^2$ is the power spectrum of the original image. A single adjustable constant K commonly replaces the ratio Sn/Sf in practice:

$$\hat{F}(u,v) = [\ |H(u,v)|^2 \,/\, (\ H(u,v)\cdot(|H(u,v)|^2 + K)\ )\ ] \cdot G(u,v)$$

Because $|H(u,v)|^2 + K$ never approaches zero, the Wiener filter avoids the catastrophic noise amplification of naive inverse filtering, giving much more stable restorations.

7. Constrained Least Squares Filtering

The Wiener filter requires the noise-to-signal power ratio, which is often not available except as an average value. Constrained least squares filtering needs only the mean and variance of the noise, and optimises the restoration subject to a smoothness constraint based on the Laplacian operator ∇^2:

$$\hat{F}(u,v) = [\ H^*(u,v) \,/\, (\ |H(u,v)|^2 + \gamma|P(u,v)|^2\ )\ ] \cdot G(u,v)$$

where P(u,v) is the Fourier transform of the Laplacian operator, and γ is a regularisation parameter that is adjusted (typically iteratively) so that the residual satisfies the known noise variance.

8. Periodic-Noise Restoration Filters (Frequency Domain)

When the corrupting noise is periodic rather than random, it is best isolated and removed selectively in the frequency domain, since it produces sharp, symmetric impulse-like spikes in the Fourier spectrum.

9. Summary — Selecting a Restoration Filter

Situation	Recommended filter
Gaussian noise only	Arithmetic / geometric mean filter
Salt-and-pepper (impulse) noise	Median filter, or contraharmonic filter with correct sign of Q
Salt noise only / pepper noise only	Min filter / Max filter respectively
Mixed random noise types	Midpoint or alpha-trimmed mean filter
Noise varies spatially / edges must be preserved	Adaptive local-noise-reduction or adaptive median filter
Periodic noise	Band-reject or notch filtering
Known blur H, negligible noise	Inverse filtering (radially limited)
Known blur H and noise statistics	Wiener (minimum mean square error) filtering
Known blur H, only noise mean/variance known	Constrained least squares filtering