1.3 BASIC ELEMENTARY OR STANDARD TIME SIGNALS

1.3.1 BASIC (ELEMENTARY OR STANDARD) CONTINUOUS TIME SIGNALS

Step signal

Unit Step signal is defined as

$$u(t) = 1 \;\; for\; t \geq 0$$
$$= 0 \; for\; t < 0$$

Ramp signal

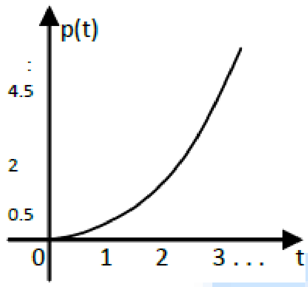
Unit ramp signal is defined as

$$r(t) = t \;\; for\; t \geq 0$$
$$= 0 \; for\; t < 0$$

Parabolic signal

Unit Parabolic signal is defined as

$$x(t) = \frac{t^2}{2} \;\; for\; t \geq 0$$
$$= 0 \; for\; t < 0$$



Unit Parabolic signal

Relation between Unit Step signal, Unit ramp signal and Unit Parabolic signal:

- Unit ramp signal is obtained by integrating unit step signal

$$i.e., \int u(t)dt = \int 1dt = t = r(t)$$

- Unit Parabolic signal is obtained by integrating unit ramp signal

$$i.e., \int r(t)dt = \int t\, dt = \frac{t^2}{2} = p(t)$$

- Unit step signal is obtained by differentiating unit ramp signal

$$\frac{d}{dt}(r(t)) = \frac{d}{dt}(t) = 1 = u(t)$$

- Unit ramp signal is obtained by differentiating unit Parabolic signal

$$i.e., \frac{d}{dt}(p(t)) = \frac{d}{dt}\left(\frac{t^2}{2}\right) = \frac{1}{2}(2t) = t = r(t)$$

Unit Pulse signal is defined as

$$\Pi(\mathrm{t}) = 1 \text{ for } |\mathrm{t}| \leq \frac{1}{2}$$

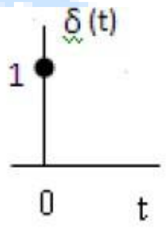
$$= 0 \text{ elsewhere}$$

Impulse signal

Unit Impulse signal is defined as

$$\delta(t) = 0 \; for \; t \neq 0$$

$$\int_{-\infty}^{\infty} \delta(t)dt = 1$$



Unit Impulse signal

Properties of Impulse signal:

Property 1:

$$\int_{-\infty}^{\infty} x(t)\delta(t)\, dt = x(0)\delta(0) = x(0) \qquad [\because \delta(t) \; exists \; only \; at \; t = 0 \; and \; \delta(0) = 1]$$

Hence proved.

Property 2:

$$\int_{-\infty}^{+\infty} \boldsymbol{x(t)\delta(t - t_0)dt = x(t_0) = x(t_0)\delta(0) = x(t_0)}$$

[$\boldsymbol{\delta(t - t_0)}$ exits only $\boldsymbol{at\ t = t_0}$ so $\boldsymbol{\delta(0) = 1}$]

Hence proved.

Sinusoidal signal

Cosinusoidal signal is defined as

$$x(t) = Acos\,(\Omega t + \Phi)$$

Sinusoidal signal is defined as

$$x(t) = A\sin(\Omega t + \Phi)$$

Where $\Omega = 2\pi f = \frac{2\pi}{T}$ *and* Ω *is the angular frequency in rad/sec*

f is frequency in cycles/sec or Hertz and

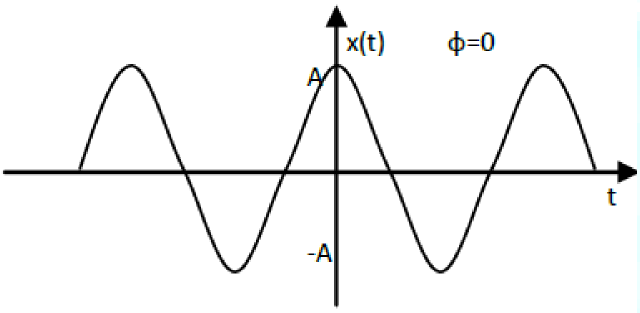
A is amplitude

T is time period in seconds

Φ is phase angle in radians

Cosinusoidal signal

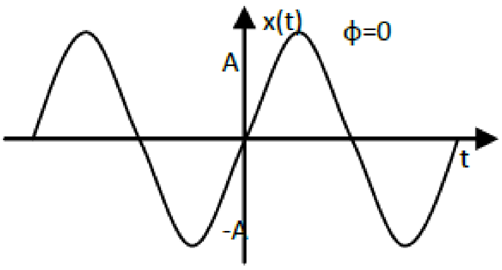
when $\phi = 0$, $x(t) = A\cos(\Omega t)$



Cosinusoidal signal

Sinusoidal signal

when $\phi = 0$, $x(t) = A\sin(\Omega t)$



Sinusoidal signal

Exponential signal

Real Exponential signal is defined as $x(t) = Ae^{at}$, where A is amplitude

Depending on the value of ‘a’ we get dc signal or growing exponential signal or decaying exponential signal.

Complex exponential signal is defined as

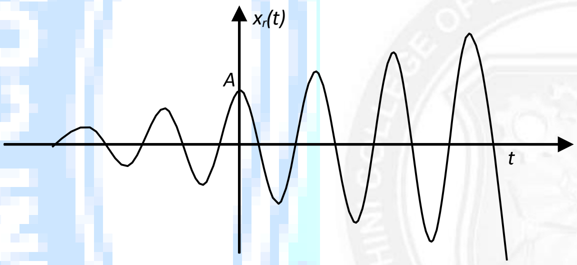
$$x(t) = Ae^{st}$$

where A is amplitude, s is complex variable and $s = \sigma + j\Omega$

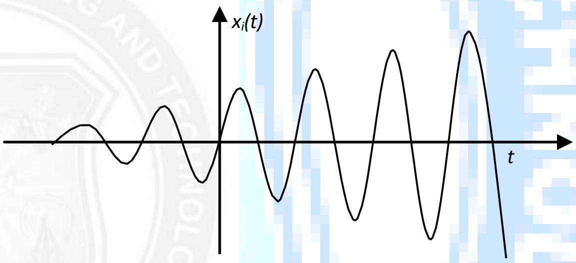
$$x(t) = Ae^{st} = Ae^{(\sigma+j\Omega)\,t} = Ae^{\sigma t}\, e^{j\Omega t} = Ae^{\sigma t}\,(cos\Omega t + jsin\Omega t)$$

$when\ \sigma = +ve, tten\ x(t) = Ae^{\sigma t}\,(cos\Omega t + jsin\Omega t),$

$where\ x_r(t) = Ae^{\sigma t}\,cos\Omega t\ and\ x_i(t) = Ae^{\sigma t}\,sin\Omega t$



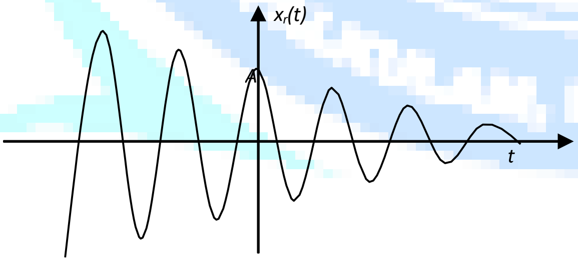
Exponentially growing Cosinusoidal signal



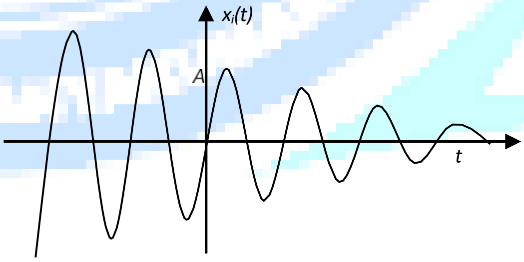
Exponentially growing sinusoidal signal

$when\ \sigma = -ve, then\ x(t) = Ae^{-\sigma t}\,(cos\Omega t + jsin\Omega t),$

$where\ x_r(t) = Ae^{-\sigma t}\,cos\Omega t\ and\ x_i(t) = Ae^{-\sigma t}\,sin\Omega t$



Exponentially decaying Cosinusoidal signal



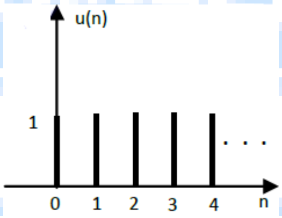
Exponentially decaying sinusoidal signal

1.3.2 BASIC (ELEMENTARY OR STANDARD) DISCRETE TIME SIGNALS

Step signal

Unit Step signal is defined as

$$u(n) = 1 \quad for\ n \geq 0$$
$$= 0 \quad for\ n < 0$$

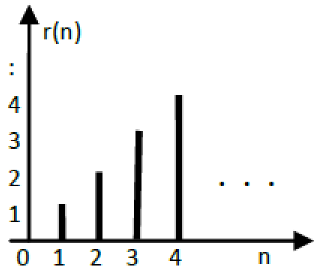


Unit step signal

Unit Ramp signal

Unit Ramp signal is defined as

$$r(n) = n \quad for\ n \geq 0$$
$$= 0 \quad for\ n < 0$$

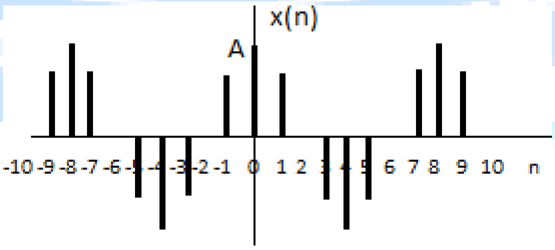


Unit Ramp signal

Sinusoidal signal

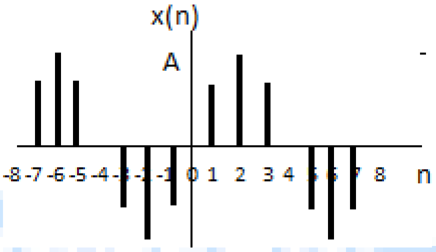
Cosinusoidal signal is defined as

$$x(n) = Acos(\omega n)$$



Sinusoidal signal is defined as

$$x(n) = Asin(\omega n)$$



Sinusoidal signal

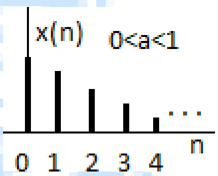
$$where\ \omega = 2\pi f = \frac{2\pi}{N} m\ and\ \omega\ is\ frequency\ in\ radians/sample$$

$m\ is\ the\ smallest\ integer$

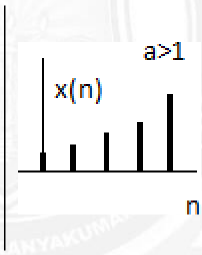
f is frequency in cycles/sample, A is amplitude

Exponential signal

Real Exponential signal is defined as $x(n) = a^n\ for\ n \geq 0$



Decreasing exponential signal

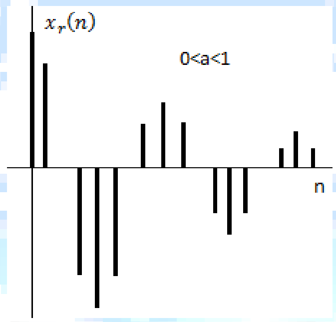


Increasing exponential signal

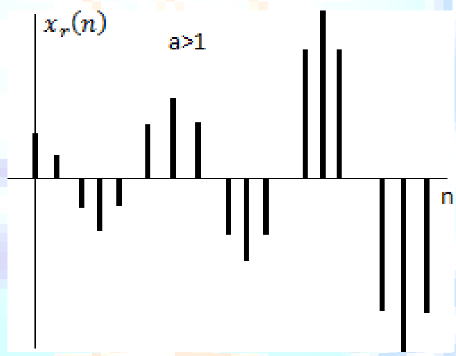
Complex Exponential signal is defined as

$$x(n) = a^n e^{j(\omega 0 n)} = a^n [cos\omega_0 n + jsin\omega_0 n]$$

$$where\ x_r(n) = a^n cos\omega_0 n\ and\ x_i(n) = a^n sin\omega_0 n$$

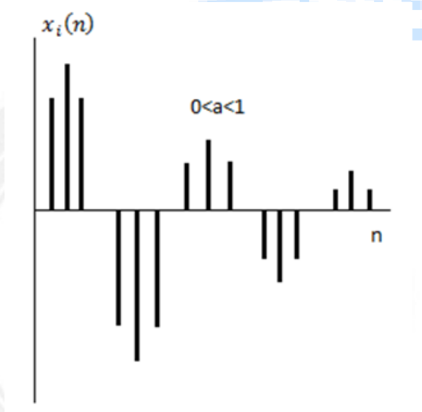


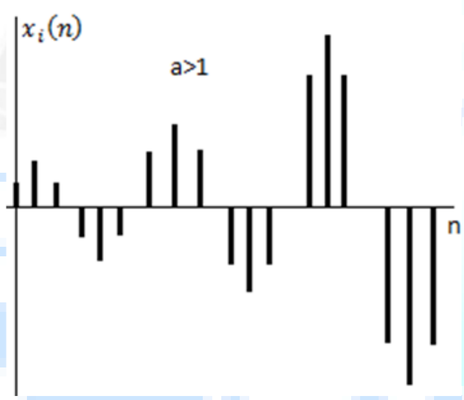
Exponentially decreasing Cosinusoidal signal



Exponentially growing Cosinusoidal signal

Exponentially decreasing sinusoidal signal





Exponentially growing sinusoidal signal