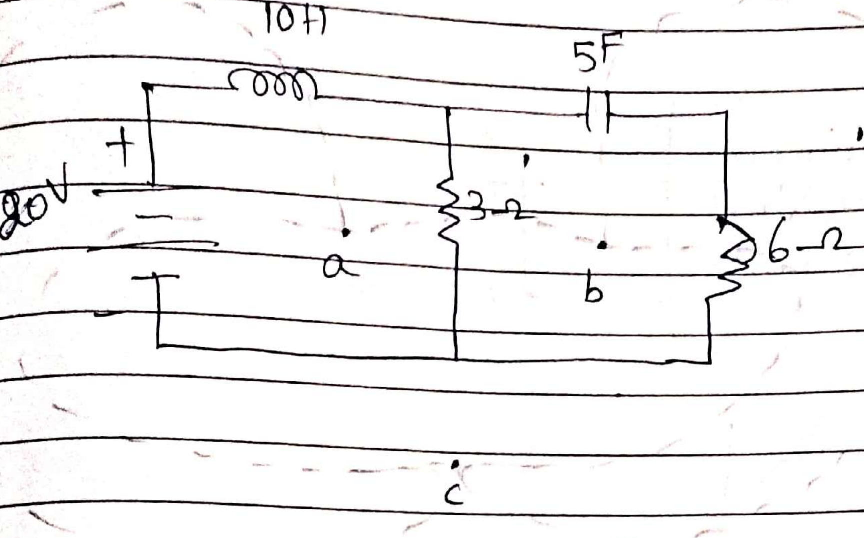
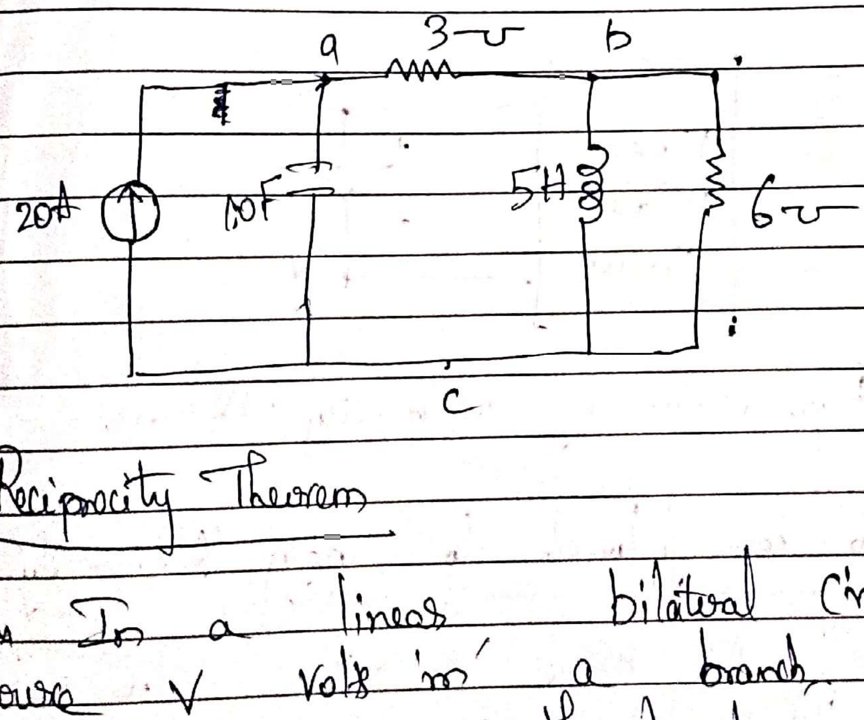
2) Draw the dual n/w for given n/w.



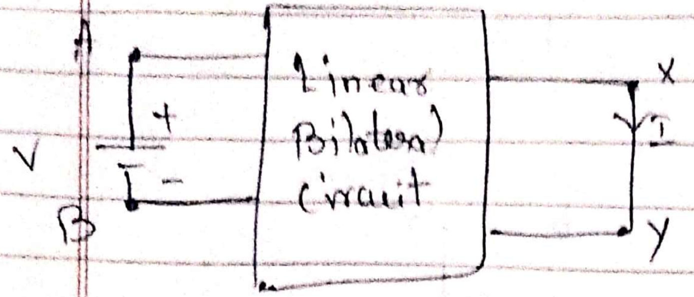
Dual N/W.

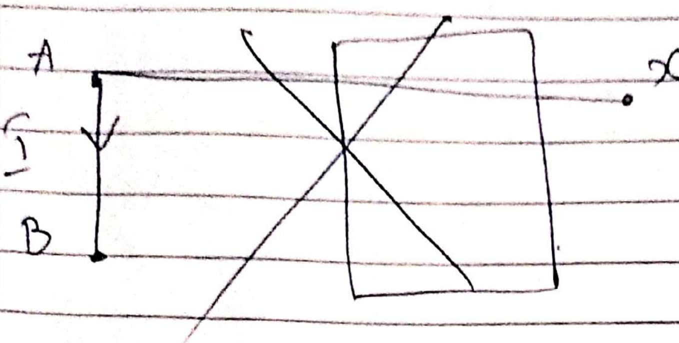


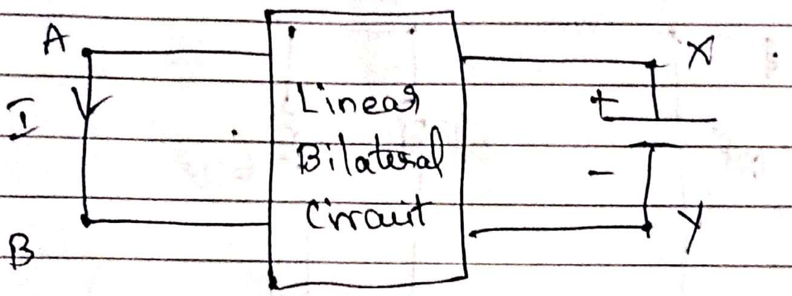
Reciprocity Theorem

* In a linear bilateral circuit a voltage source V volts in a branch gives the current I in another branch.
* The ratio of voltage (V) to current (I) remains constant when the position of V & I are interchanged.
* The ratio (i.e) V/I) is called transfer resistance.
* The circuit which satisfies reciprocity theorem

is called reciprocal circuit







Representation of reciprocity theorem.

Steps to be followed to apply Reciprocity Theorem

(1) Identify the source & the load in the given circuit

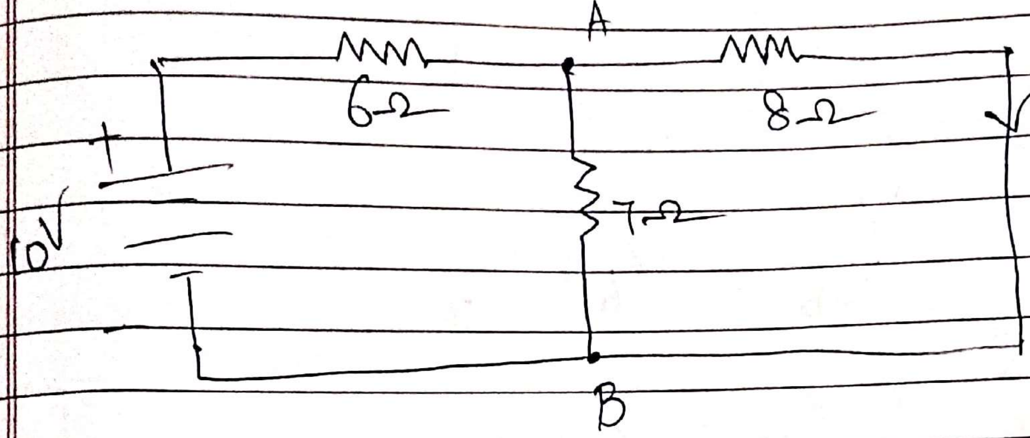
(2) Find the current flowing through the load by using any of the N/W reduction techniques.

(3) Interchange the position of source & load.

(4) Find the current flowing through the load

(5) Verify the ratio (V/I) in both case. It will be the same.

Proof of Reciprocity Theorem with an example circuit



$R_{AB} \Rightarrow 7 \parallel^{el} 8 \qquad 1/R_{AB} = 1/7 + 1/8$

$R_{AB} = 3.73\,\Omega \qquad \frac{1}{R_{AB}} = \frac{15}{56} \Rightarrow R_{AB} = \frac{56}{15}$

$R_{eq} = 6 + R_{AB} = 9.73\,\Omega$

Current $I_T = 10/R_{eq} = \frac{10}{9.73} = 1.028\,A$.

Applying current division at node A,

Current $I_8 = \frac{I_T \times 7}{8+7} = \frac{1.028 \times 7}{15}$

$I = 0.48\,A$.

$\therefore$ ratio $V/I = 10/0.48 = 20.83\,\Omega$

[Transfer resistance]

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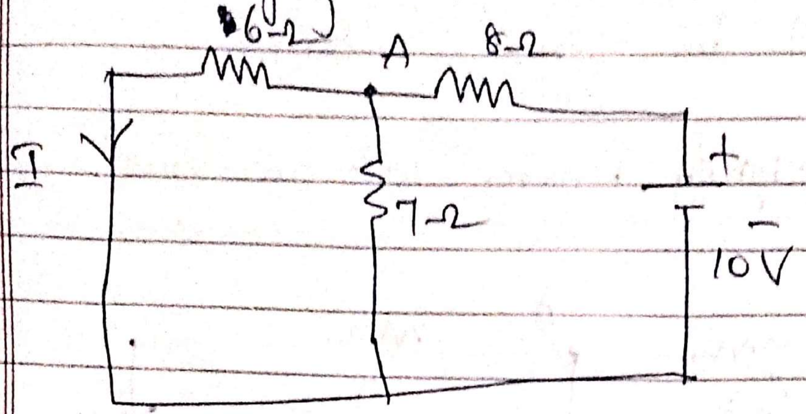
Interchanging source & load.



$$1/R_{AB} = 1/6 + 1/7$$

$$R_{AB} = 3.23\Omega$$

$$R_{eq} = 3.23 + 8 = 11.23\Omega.$$

$$I_1 = \frac{10}{11.23} = 0.89\,A.$$

Applying current division at A.

$$I_6 = I = \cancel{10 \times 7} \quad \frac{0.89 \times 7}{6+7}$$

$$= \frac{0.89 \times 7}{13}$$

$$I = 0.48\,A$$

∴ The ratio $\left(V/I\right) = \frac{10}{0.48} = 20.83\Omega.$

⇒ Transfer Resistance.

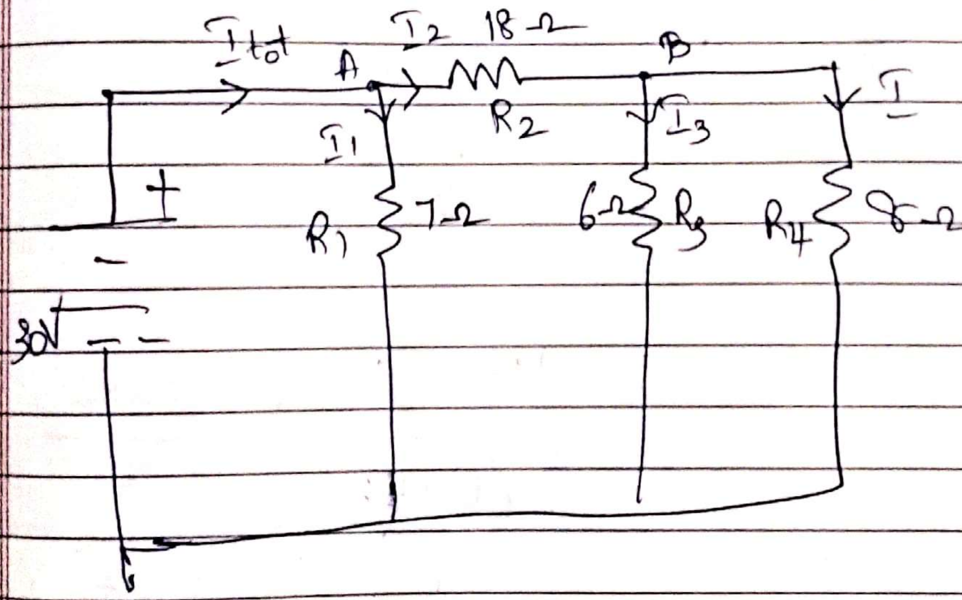
Thus, the ratio is found to be same in both cases & hence reciprocity theorem is proved.

Limitations

1. This theorem cannot be applicable for Circuits having more than one source.
2. This theorem will not be suitable for circuits having time-varying elements.
3. This theorem will be satisfied only for Circuits which does not have dependent sources.

Example

In the N/w shown in figure, find the current in the 8 Ω resistor, verify the reciprocity theorem.



To find the equivalent resistance,

$$R_{eq}' = (8 \parallel 6 + 18) \parallel^{el} 7$$

$$= \left[\left(\frac{6\times 8}{6+8}\right) + 18\right] \parallel^{el} 7 = 5.28\,\Omega$$

$$I_{tot}' = V/R_{eq}' = \frac{30}{5.28} = 5.68\,A.$$

Applying current division technique at node A

$$I_2' = \frac{I_{tot}' \times R_1}{R_1 + R_{234}}$$

$$= \frac{5.68 \times 7}{7 + 21.43}$$

$$I_2' = 1.4\,A.$$

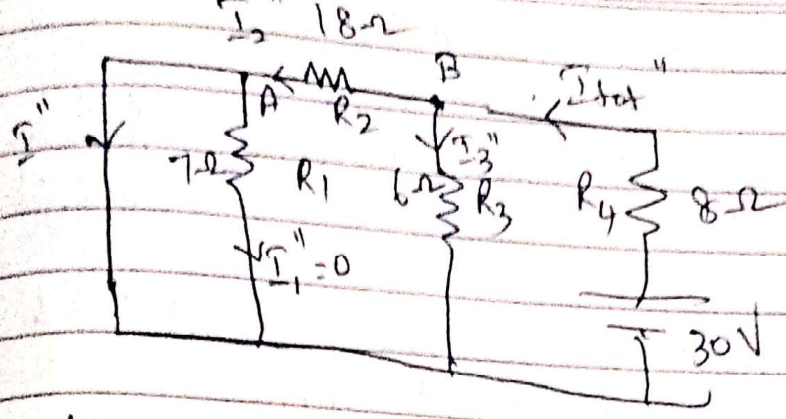
Applying current division technique at node B

$$I' = \frac{I_2' \times R_3}{(R_3 + R_4)}$$

$$= \frac{1.4 \times 6}{(6+8)} = \frac{1.4 \times 6}{14}$$

$$I = 0.6\,A$$

On changing the 30V voltage source to the 8 Ω resistor branch, circuit will be changed as follows,



As 7Ω resistor (R_1) is short circuited, it cannot be considered for calculating I''.

So $I'' = I_2''$,

Now,

R_2 & R_3 are connected in ‖el,

$$\frac{1}{R_{23}} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{18} + \frac{1}{6}$$

$$R_{23} = 4.5\,\Omega$$

Total current,

$$I_{tot}'' = \frac{V}{R_{23} + R_4}$$

$$= \frac{30}{4.5+8} \quad [\text{Series Connection}]$$

$$= \frac{30}{4.5+8}$$

$$I_{tot}'' = 2.4\,A$$

Apply current division technique at node 'B'.

$$I_2'' = I_{tot}'' \times R_3 / (R_2 + R_3)$$

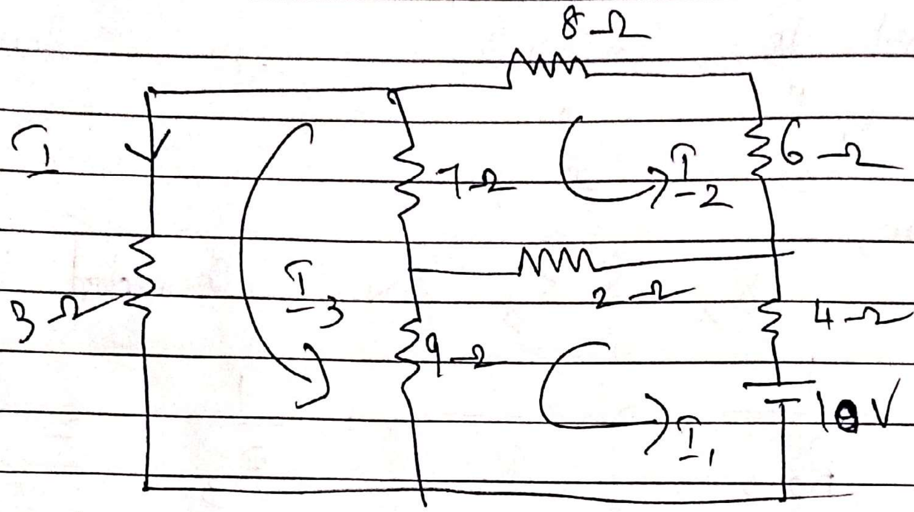
$$= \frac{2.4 \times 6}{(18+6)}$$

$I_2'' = I_L'' = 0.6A$

$I_L' = I_L''$ ⇒ Thus, reciprocity theorem is verified.

Example

In the circuit of figure, find I & verify reciprocity theorem.



In matrix forms,

$$[R][I] = [V]$$

$$\begin{bmatrix} 15 & -2 & -9 \\ -2 & 23 & -7 \\ -9 & -7 & 19 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} 15 & -2 & -9 \\ -2 & 23 & -7 \\ -9 & -7 & 19 \end{bmatrix} = 3629$$

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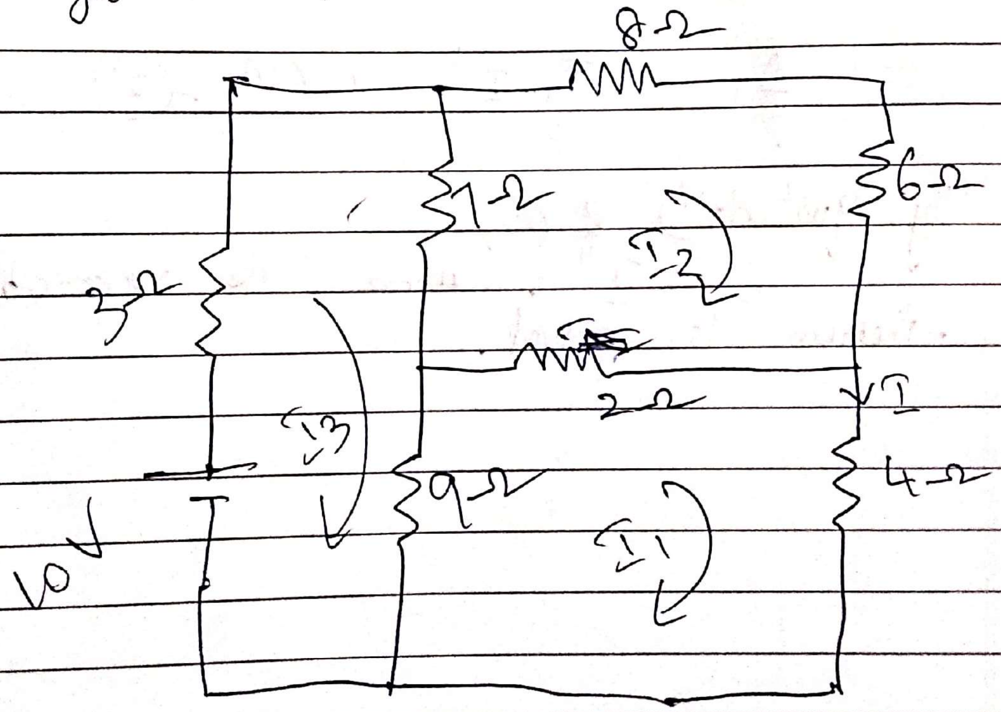
$$\Delta_3 = \begin{bmatrix} 15 & -2 & 10 \\ -2 & 23 & 0 \\ -9 & -7 & 0 \end{bmatrix} = 2210$$

$$I_3 = \frac{\Delta_3}{\Delta} = \frac{2210}{3629}$$

$$I_3 = I' = 0.61A \quad \text{— (1)}$$

On changing the 10V battery source to the 3Ω resistor branch, circuit will be changed.



In matrix form,

$$[R][I] = [V]$$

$$\begin{bmatrix} 15 & -2 & -9 \\ -2 & 23 & -7 \\ -9 & -7 & 19 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} 15 & -2 & -9 \\ -2 & 23 & -7 \\ -9 & -7 & 19 \end{bmatrix} = 3629.$$

$$\Delta_1 = \begin{bmatrix} 0 & -2 & -9 \\ 0 & 23 & -7 \\ 10 & -7 & 19 \end{bmatrix} = 2210.$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{2210}{3629}.$$

$$\Delta_1 = I_1 = I'' = 0.61A. \quad (2)$$

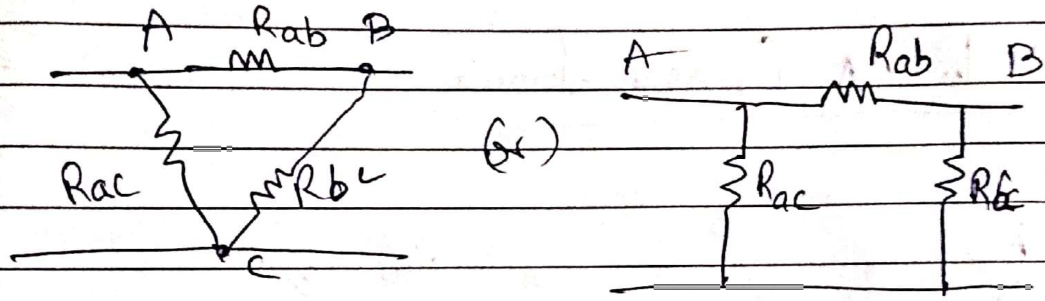
By equation (1) & (2)

$I' = I''$. Hence the reciprocity theorem is proved.

Star-Delta Transformation

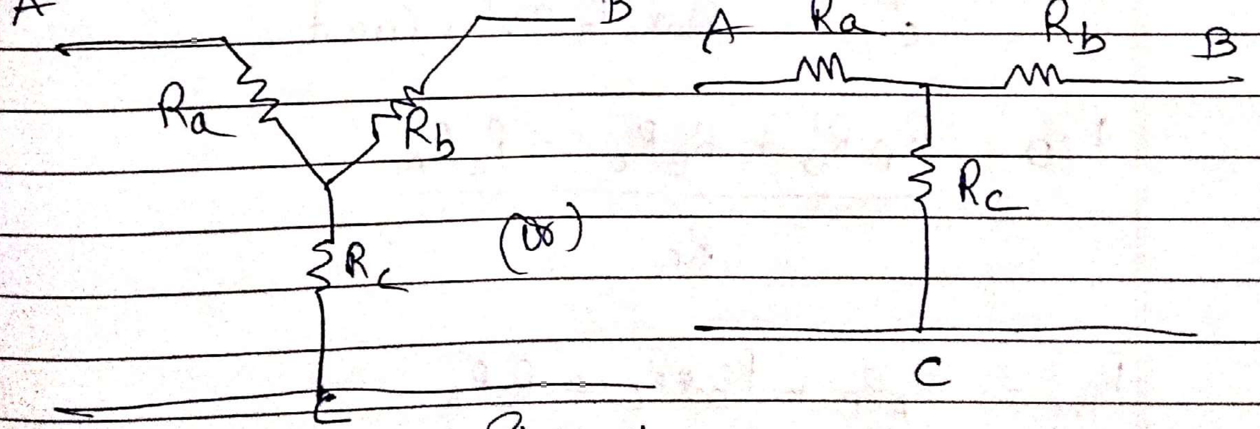
In some of the circuits, there can't be series (or) parallel combination of resistors in that case to reduce the complexity of a circuit another useful technique called (Δ-Y) (Delta-Wye) Conversion is used.

Delta Conversion (Δ ~~Convert~~ Connection).



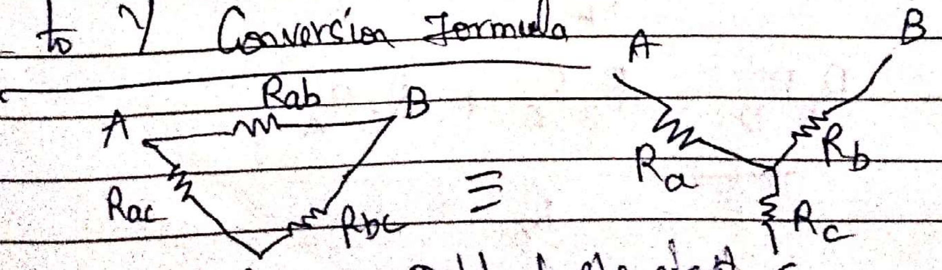
Delta circuit.

Star Connection (Y Connection)



Star circuit.

Δ to Y Conversion Formula



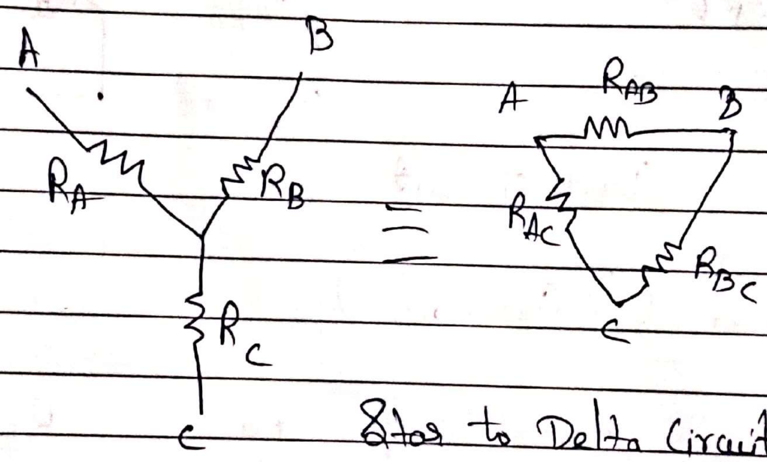
Delta to star circuit

$$R_a = \frac{R_{ab} R_{ac}}{R_{ab} + R_{ac} + R_{bc}}$$

$$R_b = \frac{R_{ab} R_{bc}}{R_{ab} + R_{bc} + R_{ac}}$$

$$R_c = \frac{R_{ac} R_{bc}}{R_{ab} + R_{bc} + R_{ac}}$$

Y to Δ Conversion formula



Star to Delta Circuit.

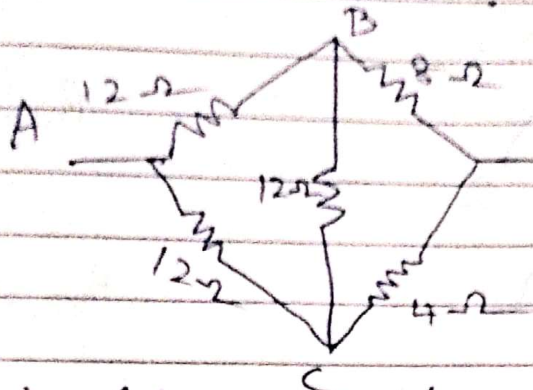
$$R_{AB} = \frac{R_A R_B + R_B R_C + R_C R_A}{R_C}$$

$$R_{BC} = \frac{R_A R_B + R_B R_C + R_C R_A}{R_A}$$

$$R_{AC} = \frac{R_A R_B + R_B R_C + R_C R_A}{R_B}$$

Problems based on Δ - Y Conversion.

Example ① Calculate the R_{eq} (Equivalent Resistance) between A & D.



$R_{AB} = R_{BC} = R_{AC} = 12\,\Omega$

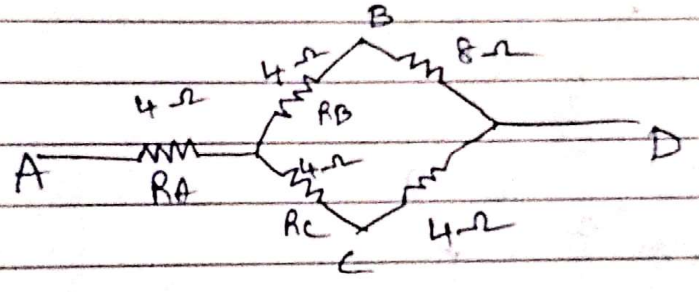
Step1: ΔABC ⇒ YABC.

$R_A = \frac{12 \times 12}{12+12+12}$

$= \frac{144}{36} = 4\,\Omega$

$R_B = 4\,\Omega$

$R_C = 4\,\Omega$



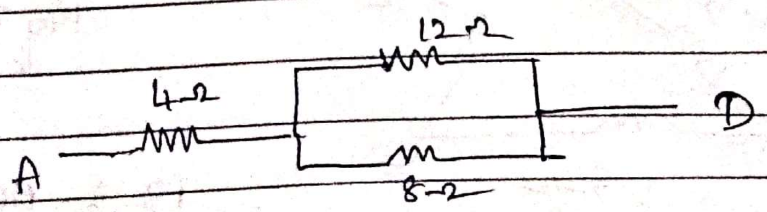
Step2: Since 8 Ω and 4 Ω are in series,

$\therefore R_{eq1} = 8 + 4 = 12\,\Omega$.

$|||^{ly}$ 4 Ω & 4 Ω are in series.

$\therefore R_{eq2} = 4 + 4 = 8\,\Omega$.

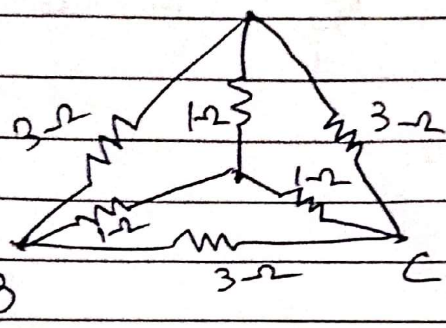
∴



$12 \parallel 8 \Rightarrow \frac{12 \times 8}{12+8} = 4.8\,\Omega$.

$R_{eq} = 4.8 + 4 = 8.8\,\Omega$ //

Example: Find R_{eq} between terminal B & C



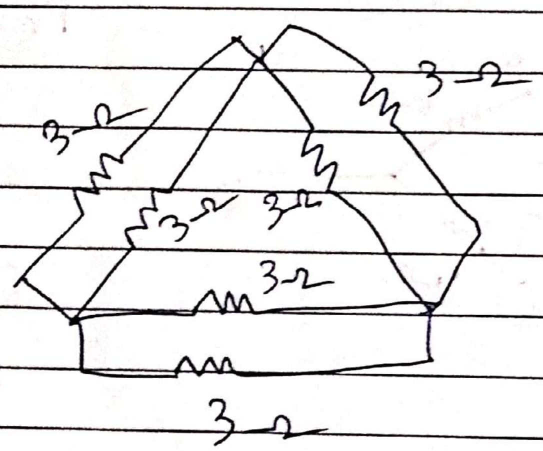
$$R_{AB} = \frac{R_A R_B + R_B R_C + R_C R_A}{R_C}$$

$\Rightarrow$ $R_{AB} = \frac{1+1+1}{1\Omega}$

$= 3\Omega$

$R_{BC} = R_{CA} = 3\Omega$

Step 1 : Y to Δ

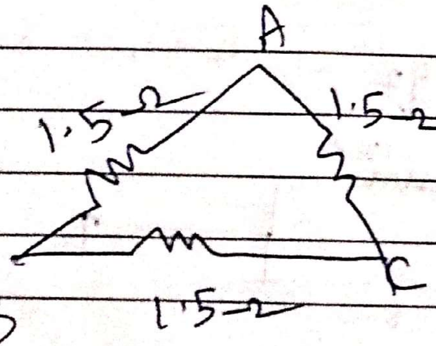


Step 2: Since 3Ω & 3Ω are in parallel

$$R_{eq1} = \frac{3 \times 3}{3+3} = \frac{9}{6} = 1.5\Omega$$

||ly $R_{eq2} = 1.5\Omega$

$R_{eq3} = 1.5\Omega$

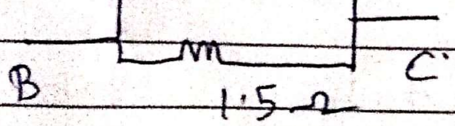


AB Ω & AC Ω are in series

and BC Ω ||el

$1.5 + 1.5 = 3\Omega$

Step 3

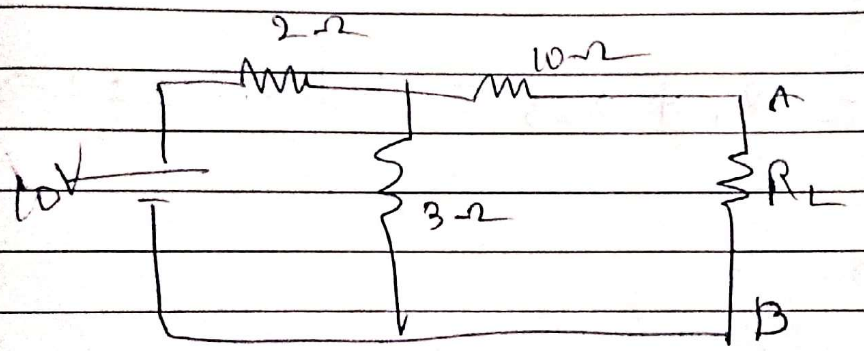


$$R_{eq} = \frac{3\times1.5}{3+1.5} = \frac{4.5}{4.5} = 1\,\Omega$$

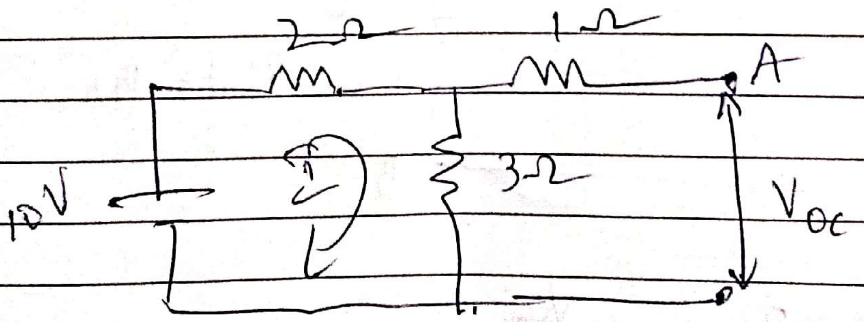
$$R_{eq} = 1\,\Omega$$

Maximum Power transfer Example

Find the value of R_L at which maximum power is transferred to R_L & hence the maximum power transferred to R_L



Find V_{th}.



$$I = \frac{10}{2+3} = 2A = I_{2\Omega} = I_{3\Omega}$$

$$\therefore V_{oc} = V_{th} = V_{1\Omega} + V_{3\Omega}$$

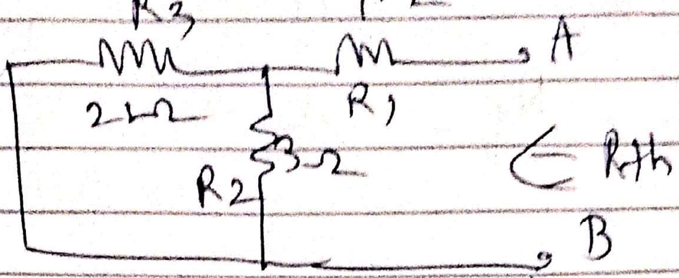
$$= V_{3\Omega}$$

$$= (I_{3\Omega} \times 3) = V_{th} = 2\times3 = 6\,V$$

Series connection

But $V_{1\Omega} = 0$, since $I_{1\Omega} = 0$

To find R_{th}



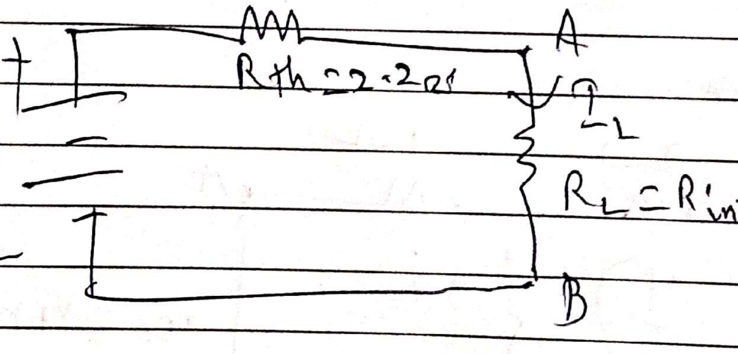
R_2 & R_3 are in $\parallel^{el}$

So $$\frac{1}{R_{23}} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{2} + \frac{1}{3}$$

$$R_{23} = 1.2\,\Omega$$

$$R_{th} = R_1 + R_{23} = 1 + 1.2 = 2.2\,\Omega$$

for MP theorem $R_L = R_{Th}$



$$\Rightarrow I_L = \frac{V_{th}}{R_{th} + R_L} = \frac{V_{th}}{2R_{th}} = \frac{6}{2 \times 2.2}$$

$$I_L = 1.36\,A$$

Max Power at $R_L \Rightarrow P_{max} = I_L^2 R_L$

$$= (1.36)^2 \times 2.2 = 4.069\,W$$

(ii) Max Power across R_L $= (I_L)^2 R_L$

$$P_L^2 = \frac{(V_{th})^2}{4 R_{th}}$$

$$= \frac{6^2}{4 \times 2.2}$$

$P_L^2 = 4.069$ Watts.