ROHINI COLLEGE OF ENGINEERING & TECHNOLOGY

DEPARTMENT OF MATHEMATICS

UNIT II : SIMPLEX METHOD

The Linear Programming with two variables can be solved graphically. The graphical method of solvinglinear programming problem is of limited application in the business problems as the number of variables is substantially large. If the linear programming problem has larger number of variables, the suitable method for solving is Simplex Method. The simplex method is an iterative process, through which it reaches ultimately to the minimum or maximum value of the objective function.

The simplex method also helps the decision maker/manager to identify the following:

- Redundant Constraints
- Multiple Solutions
- Unbounded Solution
- Infeasible Problem

Simplex method – Procedure

1) Simplex method is used to solve problems having any number of decision variables (two or more).
2) First convert the problem to *Standard Form* by converting all the constraints into 'equal to' type. For this, add a *Slack Variable* to the LHS if the constraint is ≤ type. Subtract a *Surplus Variable* and add an *Artificial Variable* to the LHS if the constraint is ≥ type. For = type constraint(s) also, add artificial variable(s). The slack variable(s) and the artificial variable(s) (if any) provide the starting solution in the Simplex method and

are known as *basic variables*. All other variables are given zero value (known as *non-basic variables*).

3) Construct the Simplex table by drawing two vertical lines and a horizontal line at the top which cuts the two vertical lines. Above this horizontal line and between the two vertical lines, write all the variables found in the constraints.
4) To the left side of the left vertical line and above the horizontal line create three columns: 'C_B' , 'B' and 'X_B'. Above the 'X_B' column, create a 'C_j' row. To the right side of the right vertical line and above the horizontal line create a column for replacement ratio(θ).
5) Against the 'C_j' row, enter the coefficients of all the variables found in the objective function above the respective variables found below this row.

A. Problems involving only ≤ type constraints:

6) Take the slack variables as the starting basic variables by giving zero value to all the other variables in the constraints. The number of basic (i.e. slack) variables will equal the number of constraints. Enter these variables under the 'B' column, and the RHS values of the constraints under the 'X_B' column.
7) Enter the coefficients of the constraints in the main body of the Simplex table in appropriate rows and columns. Take zero value for variables that do not appear in a constraint.
8) Under the C_B column, enter the objective function coefficients of the current basic variables.
9) **Optimality check:** To check whether the current solution (the basic variables and their corresponding 'X_B' column values) is optimal, first compute 'Z_j'. Z_j values are obtained by multiplying the C_B column figures with corresponding figures(like X_1, X_2,S_1,S_2) in each column in the main body of the Simplex table, and adding up for each column. Compute the Z_j values column by column, and enter them against a 'Z_j' row created below the row for the bottom-most basic variable.
10) Below the Z_j row, create a C_j-Z_j row. Subtract all the Z_j values from the corresponding C_j values (found at the very top of the Simplex table). Enter these figures in the C_j-Z_j row under the appropriate columns.

11) For a maximization problem, the ***optimality condition*** specifies that if *all* the C_j-Z_j values are either *negative* or *zero (*C_j-$Z_j \leq 0$), the current solution is optimal. If even *one* positive C_j-Z_j is present, the current solution is not optimal. [For a minimization problem, the *reverse* holds good, i.e. if all the C_j-Z_j values are either *positive* or *zero(*C_j-$Z_j \geq 0$),, the current solution is optimal. Otherwise it is not optimal.]
12) We will consider only a maximization problem. If more than one *positive* C_j-Z_j exists, select the *largest* of these positive values. The corresponding variable at the top of that column is the ***entering variable.*** (If there is only *one* positive C_j-Z_j, the corresponding variable at the top automatically becomes the 'entering variable'). By introducing this variable in place of one of the current basic variables, we can get an improved solution.
13) To determine the ***leaving variable*** in the next stage, proceed as follows: Having identified the entering variable, mark an arrow near the corresponding C_j-Z_j value. That column is known as the ***key column.*** Compute ratios by dividing the 'X_B' column figures by the corresponding key column figures (entering variable column). Write these values under the 'θ'(Replacement ratio) column against the appropriate X_B column and key column entries. The *minimum positive ratio* (θ value) indicates the *leaving variable.* Mark an arrow near this minimum ratio. The corresponding row is known as the ***key row.*** The meeting point of the *key row* and *key column* is known as the ***key element.***
14) This concludes the first stage of the simplex table. Go to the second stage by drawing a line below the C_j-Z_j row. Enter the new set of basic variables under the 'Basic' column by including the *entering variable* in place of the *leaving variable,* leaving all other basic variables in their positions unchanged.
15) Under the C_B column, enter the objective function coefficients of the current set of basic variables in their appropriate places.
16) As a first step, convert the *key element* to unity, (i.e. one). To do this, divide all the elements in the *key row* (including the one under the 'X_B' column) by whatever number that appears as the key element.
17) Next, enter the new figures in the remaining rows (including the 'X_B' column figures) by using the formula: **[old relevant row figures] – [common element][new key row figures] = [new relevant row figures].** The common element for a row is the element in that row which appears in the key column.

18) The basic variables under the 'B' column and their corresponding values under the 'X_B' column provide the current solution to the problem. Check if this solution is optimal or not by carrying out the **optimality check** outlined from ***step 9*** onwards.

19) Once optimality is reached (all C_j-Z_j values are zero or negative for maximization problem), write down the optimal solution to the problem by giving the values of the decision variables and the value of the objective function. (No need to show the values of the slack and other such variables).

B. Problems having = type and/or ≥ type constraints:

1. For constraints of = type, add an artificial variable (A) to the LHS of the constraint. For constraints of ≥ type, subtract a surplus variable (S) and then add an artificial variable (A) to the LHS of the constraint. The artificial variables are added to provide a starting feasible solution for the simplex table. In the case of ≤ type constraints, the slack variable (S) itself will provide a starting solution. Hence there is no need to use artificial variables in the case of constraints of ≤ type.
2. Take the *slack variables* and the *artificial variables* as the starting basic variables by giving zero value to all the other variables in the constraints. Then follow the procedure from **step 6** outlined above for ≤ type constraints.
3. If a feasible solution exists for the problem, the artificial variables will not be present as basic variables in the optimal table. No feasible solution for the problem exists if even one artificial variable remains as a basic variable in the optimal table.

Problem : 1

Maximize $60x_1 + 70x_2$

Subject to: $2x_1 + x_2 \leq 300$ $3x_1 + 4x_2 \leq 509$ $4x_1 + 7x_2 \leq 812$ $x_1, x_2 \geq 0$

Solution

First we introduce the variables

$s_3, s_4, s5 \geq 0$

So that the constraints becomes equations, thus

$2x_1 + x_2 + s_3 = 300$

$3x_1 + 4x_2 + s_4 = 509$

$4x_1 + 7x_2 + s_5 = 812$

Corresponding to the three constraints, the variables s3, s4, s5 are called as slack variables. Now, thesystem of equation has three equations and five variables. There are two types of solutions they are basic and basic feasible, which are discussed as follows:

Basic Solution

We may equate any two variables to zero in the above system of equations, and then the system will have three variables. Thus, if this system of three equations with three variables is solvable such a solution is called as basic solution. If we take x_1=0 and x_2=0, the solution of the system with remaining three variables is s_3=300, s_4=509 and s_5=812, this is a basic solution and the variables s_3, s_4, and s_5 are known as basic variables whereas the variables x_1, x_2 are known as non-basic variables. The number of basic solution of a linear programming problem is depends on the presence of thenumber of constraints and variables.

Basic Feasible Solution

A basic solution of a linear programming problem is called as basic feasible solutions if it is feasible it means all the variables are non-negative. The solution s_3=300, s_4=509 and s_5=812 is a basic feasible solution.

The profit Z=60x_1 + 70x_2 i.e. Maximize 60x_1 + 70x_2

In this problem the slack variables s_3, s_4, and s_5 provide a basic feasible solution from which the simplex computation starts. That is s_3==300, s_4=509 and s_5=812. This result follows because of the special structure of the columns associated with the slacks.

If z represents profit then z=0 corresponding to this basic feasible solution. We represent by C_B the coefficient of the basic variables in the objective function and by XB the numerical values of the basic variable. So that the numerical values of the basic variables are: XB_1=300, XB_2=509, XB_3=812. The profitz=60x_1+70x_2 can also expressed as z-60x_1-70x_2=0. The simplex computation starts with the first compact standard simplex table as given below:

C_B	Basic	C_j	60	70	0	0	0
	Variables	XB	x_1	x_2	s_3	s_4	s_5
0	s_3	300	2	1	1	0	0
0	s_4	509	3	4	0	1	0
0	s_5	812	4	7	0	0	1
	z		-60	-70	0	0	0

In the objective function the coefficients of the variables are $CB_1=CB_2=CB_3=0$. The topmost rowof the Table 1 denotes the coefficient of the variables x_1, x_2, s_3, s_4, s_5 of the objective function respectively. The column under x_1 indicates the coefficient of x_1 in the three equations respectively. Similarly the remaining column also formed.

On seeing the equation $z=60x_1+70x_2$ we may observe that if either x_1 or x_2, which is currently non-basic is included as a basic variable so that the profit will increase. Since the coefficient of x_2 is higher we choose x_2 to be included as a basic variable in the next iteration. An equivalent criterion of choosing a new basic variable can be obtained the last row of the above Table i.e. corresponding to z.

Since the entry corresponding to x_2 is smaller between the two negative values, x_2 will be included as a basic variable in the next iteration. However with three constraints there can be only three basic variables.

Thus, by bringing x_2 a basic variable one of the existing basic variables becomes non-basic. The question here is How to identify this variable? The following statements give the solution to this question.

Consider the first equation i.e. $2x_1 + x_2 + s_3 = 300$

From this equation

$2x_1+s_3=300-x_2$

But $x_1=0$. Hence, in order that $s_3\geq 0$

$300-x_2\geq 0$ i.e. $x_2\leq 300$

Similarly consider the second equation i.e. $3x_1 + 4x_2 + s_4 = 509$

From this equation

$3x_1+s_4=509-4x_2$

But, $x_1=0$. Hence, in order that $s_4\geq0$

$509-4x_2\geq0$

i.e. $x_2\leq509/9$

Similarly consider the third equation i.e. $4x_1 + 7x_2+ s_5 = 812$

From this equation

$4x_1+s_5=812-7x_2$

But $x_1=0$.

Hence, in order that $s5\geq0$

$812-7x_2\geq0$

i.e. $x_2\leq812/7$

Therefore the three

equation lead to

$x_2\leq300$, $x_2\leq509/9$, $x_2\leq812/7$

Thus $x2=\text{Min}$ ($x_2\leq300$, $x_2\leq509/9$,

$x_2\leq812/7$) it means$x2=\text{Min}$ ($x_2\leq300/1$,

$x_2\leq509/9$, $x_2\leq812/7$) = 116

Therefore $x_2=116$

If $x_2=116$, you may be note from the

third equation$7x_2+s_5=812$

i.e. $s_5=0$

Thus, the variable s5 becomes non-basic in the

next iteration. So that the revised values of the

other two basic variables are

$S_3=300-$

$x_2=184$

$S_4=509-$

$4*116=45$

Refer to Table 1, we obtain the elements of the next Table i.e. Table 2 using the following rules:

1. We allocate the quantities which are negative in the z-row. Suppose if all the quantities are positive, the inclusion of any non-basic variable will not increase the value of the

objective function. Hence the present solution maximizes the objective function. If there are more than onenegative values we choose the variable as a basic variable corresponding to which the z value is least as this is likely to increase the more profit.

2. Let x_j be the incoming basic variable and the corresponding elements of the j^{th} row column be denoted by $Y1_j$, $Y2_j$ and $Y3_j$ respectively. If the present values of the basic variables are XB_1, XB_2 and XB_3 respectively, then we can compute.

Min [XB_1/Y1j, XB_2/$Y2_j$, XB_3/Y3j] for Y1j, Y2j, Y3j>0.

Note that if any $Y_{ij} \leq 0$, this need not be included in the comparison. If the minimum occurs Corresponding to XBr/Yrj then the r^{th} basic variable will become non-basic in the next iteration.

3. Using the following rules the Table 2 is computed from the Table 1.

i. The revised basic variables are s_3, s_4 and x_2. Accordingly, we make CB_1=0, CB_2=0 and CB_3=70.

ii. As x_2 is the incoming basic variable we make the coefficient of x_2 one by dividing each element of row-3 by 7. Thus the numerical value of the element correspondingto x_1 is 4/7, corresponding to s_5 is 1/7 in Table 2.

iii. The incoming basic variable should appear only in the third row. So we multiply the third-row of Table 2 by 1 and subtract it from the first-row of Table 1 element by element. Thus the element corresponding to x_2 in the first-row of Table 2 is 0.

Therefore the element corresponding to x_1 is

2-1*4/7=10/7 and the element corresponding to s_5 is 0-1*1/7=-1/7

In this way we obtain the elements of the first and the second row in Table 2. In Table 2 the numerical values can also be calculated in a similar way.

C_B	Basic	C_j	60	70	0	0	0
	Variables	X_B	x_1	x_2	s_3	s_4	s_5
0	s_3	184	10/7	0	1	0	-1/7
0	s_4	45	5/7	0	0	1	-4/7
70	x_2	116	4/7	1	0	0	1/7
	zj-cj		-140/7	0	0	0	70/7

Table 2

Let CB_1, CB_2, Cb_3 be the coefficients of the basic variables in the objective function. For example in Table 2 $CB_1=0$, $CB_2=0$ and $CB_3=70$. Suppose corresponding to a variable J, the quantity z_j is defined as $z_j=CB_1, Y_1+CB_2, Y2_j+CB3Y3_j$. Then the z-row can also be represented as Z_j-C_j.

$z_1 - c_1 = 10/7*0+5/7*0+70*4/7-60 =$
$-140/7 z_5 - c_5 = -1/7*0-$
$4/7*0+1/7*70-0 = 70/7$

1. Now we apply rule (1) to Table 2. Here the only negative z_j-c_j is z_1-c_1 = -140/7Hence x_1 should become a basic variable at the next iteration.

2. We compute the minimum of the ratio

$$\text{Min} \left(\frac{184}{\frac{10}{7}}, \frac{45}{\frac{5}{7}}, \frac{116}{\frac{4}{7}} \right) = \text{Min} \left(\frac{644}{5}, 63, 203 \right) = 63$$

This minimum occurs corresponding to s_4, it becomes a non-basic variable in next iteration.

3. Like Table 2, the Table 3 is computerizing the rules (i), (ii), (iii) as described above.

C_B	Basic Variables	C_jX_B	60 x_1	70 x_2	0 s_3	0 s_4	0 s_5
0	s3	94	0	0	1	-2	1
60	x1	63	1	0	0	7/5	-4/5
70	x2	80	0	1	0	-4/5	3/5
	z_j-c_j		0	0	0	28	-6

Table 3

1. $z_5 - c_5 < 0$ should be made a basic variable in the next iteration.

2. Now compute the minimum ratios
3. From the Table 3, Table 4 is calculated following the usual steps.

C_B	Basic Variables	C_jX_B	60 x_1	70 x_2	0 s_3	0 s_4	0 s_5
0	s_5	94	0	0	1	-2	1
60	x_1	691/5	1	0	4/5	-1/5	0
70	x_2	118/5	0	1	-3/5	2/5	0
	z_j-c_j		0	0	6	16	0

Note that $z_j - c_j \geq 0$ for all j, so that the objective function can't be improved any further.
Thus, the objective function is maximized for $x_1 = 691/5$ and $x_2=118/5$ and
The maximum value of the objective function is 9944.

Problem 2

Using Simplex method solve the LPP

Maximize $Z = 3x_1 + 2x_2 + 5x_3$

Subject to $x_1 + 4x_2 \leq 420$

$3x_1 + 2x_3 \leq 460$

$x_1 + 2x_2 + x_3 \leq 430$ *and* $x_1, x_2, x_3 \geq 0$

Soln.

by introducing the slack variables s_1, s_2, s_3, the problem in standard form becomes

Maximize $Z = 3x_1 + 2x_2 + 5x_3 + 0s_1 + 0s_2 + 0s_3$

Subject to $x_1 + 4x_2 + s_1 = 420$

$3x_1 + 2x_3 + s_2 = 460$

$x_1 + 2x_2 + x_3 + 0s_3 = 430$ *and* $x_1, x_2, x_3 \geq 0$

Initial Iteration:

		C_j	(3	2	5	0	0	0)	
C_B	Y_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3	Ratio
0	s_1	420	1	4	0	1	0	0	
0	s_2	460	3	0	(2)	0	1	0	460 / 2 = 230
0	s_3	430	1	2	1	0	0	1	430 / 1 = 430
$Z_j - C_j$			-3	-2	-5	0	0	0	

Since all $(Z_j - C_j) < 0$ the current basic feasible solution is not optimal. The leaving variable is the basic variable s_2 and the corresponding basic variable x_3 enters the basis.

First Iteration:

		C_j	(3	2	5	0	0	0)	
C_B	Y_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3	Ratio
0	s_1	420	1	4	0	1	0	0	420 / 4 = 105
5	x_3	230	3/2	0	1	0	½	0	-
0	s_3	200	-1/2	(2)	0	0	-1/2	1	200 / 2 = 100
$Z_j - C_j$			-3	-2	-5	0	0	0	

Since all $(Z_j - C_j) < 0$ the current basic feasible solution is not optimal. The leaving variable is the basic variable s_3 and enter the variable is x_2.

Second Iteration:

		C_j	(3	2	5	0	0	0)	
C_B	Y_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3	Ratio
0	s_1	20	2	0	0	1	1	-2	
5	x_3	230	3/2	0	1	0	½	0	
2	x_2	100	-1/4	1	0	0	-1/4	1/2	
$Z_j - C_j$		1350	4	0	0	0	2	1	

Since all $(Z_j - C_j) \geq 0$ the current basic feasible solution is optimal. The optimal solution is

Max. Z = 1350, $x_1 = 0$, $x_2 = 100$ and $x_3 = 230$.

Problem : 3

Use Penalty method

Maximize $Z = 2x_1 + x_2 + x_3$

Subject to $4x_1 + 6x_2 + 3x_3 \le 8$

$3x_1 - 6x_2 - 4x_3 \le 1$

$2x_1 + 3x_2 - 5x_3 \ge 4$ *and* $x_1, x_2, x_3 \ge 0$

Soln.

By introducing the non negative slack variable s_1, s_2 and surplus variable s_3, the standard form of the LPP becomes

Maximize $Z = 2x_1 + x_2 + x_3 + 0s_1 + 0s_2 + 0s_3$

Subject to $4x_1 + 6x_2 + 3x_3 + s_1 = 8$

$3x_1 - 6x_2 - 4x_3 + s_2 = 1$

$2x_1 + 3x_2 - 5x_3 - s_3 = 4$ *and* $x_1, x_2, x_3 \ge 0$

To get the basic feasible solution , add the artificial variable R_1 to the left hand side of the constraint equation which does not possess the slack variable and assign M to the artificial variable in the objective function. The LPP becomes

Maximize $Z = 2x_1 + x_2 + x_3 + 0s_1 + 0s_2 + 0s_3 - MR_1$

Subject to $4x_1 + 6x_2 + 3x_3 + s_1 = 8$

$3x_1 - 6x_2 - 4x_3 + s_2 = 1$

$2x_1 + 3x_2 - 5x_3 - s_3 + R_1 = 4$ *and* $x_1, x_2, x_3 \ge 0$

Initial Iteration:

		C_j	(2	1	1	0	0	0	-M	
C_B	Y_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3	R_1	Ratio
0	s_1	8	4	6	3	1	0	0	0	8 / 6 = 1.3
0	s_2	1	3	-6	-4	0	1	0	0	-
-M	R_1	4	2	(3)	-5	0	0	-1	1	4 / 3 = 1.33
$Z_j - C_j$		-4M	-2M-2	-3M-1	5M-1	0	0	M	0	

Since all $(Z_j - C_j) < 0$ the current basic feasible solution is not optimal. The leaving variable is the basic variable R_1 and the corresponding basic variable x_2 enters the basis.

First Iteration:

		C_j	(2	1	1	0	0	0)	
C_B	Y_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3	Ratio
0	s_1	0	0	0	(13)	1	0	2	0
0	s_2	9	7	0	-14	0	1	-2	-
1	x_2	4/3	2/3	1	-5/3	0	0	-1/3	-
$Z_j - C_j$		4/3	-4/3	0	-8/3	0	0	-1/3	

Since all $(Z_j - C_j) < 0$ the current basic feasible solution is not optimal. The leaving variable is variable s_1 and enter the variable is x_3.

Second Iteration:

		C_j	(2	1	1	0	0	0)	
C_B	Y_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3	Ratio
1	x_3	0	0	0	1	1/13	0	2/13	-
0	s_2	9	(7)	0	0	14/13	1	2/13	9/7
1	x_2	4/3	2/3	1	0	5/39	0	-1/13	2
$Z_j - C_j$		4/3	-4/3	0	0	8/39	0	1/13	

Since all $(Z_j - C_j) < 0$ the current basic feasible solution is not optimal. The leaving variable is variable s_2 and enter the variable is x_1.

Third Iteration:

		C_j	(2	1	1	0	0	0)	
C_B	Y_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3	Ratio
1	x_3	0	0	0	1	1/13	0	2/13	
2	x_1	9/7	1	0	0	2/13	7/7	2/91	
1	x_2	10/21	0	1	0	1/39	-2/21	-25/273	
$Z_j - C_j$		64/21	0	0	0	16/39	4/21	29/273	

Since all $(Z_j - C_j) \geq 0$ the current basic feasible solution is optimal.

The optimal solution is Max. Z = 64/21, $x_1 = 9 / 7$, $x_2 = 10 / 21$ and $x_3 = 0$

Problem : 4

Using dual Simplex method solve the LPP

$Minimize\ \ Z = 2x_1 + x_2$

Subject to

$3x_1 + x_2 \geq 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \geq 3 \qquad and \quad x_1, x_2 \geq 0$

Soln.

After converting the objective function in to maximization type and all the constraints in ≤ type, the given LPP becomes

$Max\ \ Z^* = -2x_1 - x_2$

Subject to

$-3x_1 - x_2 \leq -3$

$-4x_1 - 3x_2 \leq -6$

$-x_1 - 2x_2 \leq -3 \qquad and \quad x_1, x_2 \geq 0$

By introducing the non negative slack variable s_1, s_2 and s_3, the LPP becomes

$Max\ \ Z^* = -2x_1 - x_2 + 0s_1 + 0s_2 + os_3$

Subject to

$-3x_1 - x_2 + s_1 = -3$

$-4x_1 - 3x_2 + s_2 = -6$

$-x_1 - 2x_2 + s_3 = -3 \qquad and \quad x_1, x_2 \geq 0$

Initial Iteration:

		C_j	(-2	-1	0	0	0)
C_B	Y_B	X_B	x_1	x_2	s_1	s_2	s_3
0	s_1	-3	-3	-1	1	0	0
0	s_2	-6	-4	(-3)	0	1	0
0	s_3	-3	-1	-2	0	0	1
$Z_j^* - C_j$		0	2	1	0	0	0

Since all $Z_j^* - C_j \geq 0$ and all $X_{Bi}<0$, the current solution is not optimal.

Since $X_{B2} = -6$ is most negative, the corresponding variable s_2 is the leaving variable.

Now $\theta = Max \left\{ \frac{(Z_j - C_j)}{a_{ik}}, a_{ik} < 0 \right\} \Rightarrow Max \left\{ \frac{2}{-4}, \frac{1}{-3} \right\} = \frac{-1}{3}$. Therefore the corresponding variable x_2 enters the basis.

First Iteration:

		C_j	(-2	-1	0	0	0)
C_B	Y_B	X_B	x_1	x_2	s_1	s_2	s_3
0	s_1	-1	(-5/3)	0	1	-1/3	0
-1	x_2	2	4/3	1	0	-1/3	0
0	s_3	1	5/3	0	0	-2/3	1
$Z_j^* - C_j$		-2	2/3	0	0	1/3	0

Since all $Z_j^* - C_j \geq 0$ and $X_{B1}<0$, the current solution is not optimal.

Since X_{B1} = -1 is most negative, the corresponding variable s_1 is the leaving variable.

Now $\theta = Max\left\{\frac{(Z_j - C_j)}{a_{ik}}, a_{ik} < 0\right\} \Rightarrow Max\left\{\frac{2/3}{-5/3}, \frac{1/3}{-1/3}\right\} = \frac{-1}{5}$. Therefore the corresponding

variable x_1 enters the basis.

Second Iteration:

		C_j	(-2	-1	0	0	0)
C_B	Y_B	X_B	x_1	x_2	s_1	s_2	s_3
-2	x_1	3/5	1	0	-3/5	1/5	0
-1	x_2	6/5	0	1	4/5	-3/5	0
0	s_3	0	0	0	1	-1	1
$Z_j^* - C_j$		-12 / 5	0	0	2/5	1/5	0

Since all $Z_j^* - C_j \geq 0$ and all $X_{Bi} \geq 0$, the current solution is optimal.

Therefore the optimum solution is Max Z* = -12 / 5, x_1 = 3/5 , x_2 = 6 / 5

But Min Z = -Max Z* = - (- 12 /5) = 12 / 5.

Problem : 5

Using Simplex method solve the LPP

$$\begin{aligned} &Maximize \quad Z = 3x_1 + 2x_2 + 5x_3 \\ &Subject\ to \quad x_1 + 4x_2 \leq 420 \\ &3x_1 + 2x_3 \leq 460 \\ &x_1 + 2x_2 + x_3 \leq 430 \quad and \quad x_1, x_2, x_3 \geq 0 \end{aligned}$$

Soln.

By introducing the slack variables s_1, s_2, s_3, the problem in standard form becomes

Maximize $Z = 3x_1 + 2x_2 + 5x_3 + 0s_1 + 0s_2 + 0s_3$

Subject to $x_1 + 4x_2 + s_1 = 420$

$3x_1 + 2x_3 + s_2 = 460$

$x_1 + 2x_2 + x_3 + 0s_3 = 430 \quad x_1, x_2, x_3 \geq 0$

Initial Iteration:

		C_j	(3	2	5	0	0	0)	
C_B	Y_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3	Ratio
0	s_1	420	1	4	0	1	0	0	
0	s_2	460	3	0	(2)	0	1	0	460 / 2 = 230
0	s_3	430	1	2	1	0	0	1	430 / 1 = 430
$Z_j - C_j$			-3	-2	-5	0	0	0	

Since all $(Z_j - C_j) < 0$ the current basic feasible solution is not optimal.

The leaving variable is the basic variable s_2 and the corresponding basic variable x_3 enters the basis.

First Iteration:

		C_j	(3	2	5	0	0	0)	
C_B	Y_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3	Ratio
0	s_1	420	1	4	0	1	0	0	420 / 4 = 105
5	x_3	230	3/2	0	1	0	½	0	-
0	s_3	200	-1/2	(2)	0	0	-1/2	1	200 / 2 = 100
$Z_j - C_j$			-3	-2	-5	0	0	0	

Since all $(Z_j - C_j) < 0$ the current basic feasible solution is not optimal. The leaving variable is variable s_3 and enter the variable is x_2.

Second Iteration:

		C_j	(3	2	5	0	0	0)	
C_B	Y_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3	Ratio
0	s_1	20	2	0	0	1	1	-2	
5	x_3	230	3/2	0	1	0	½	0	
2	x_2	100	-1/4	1	0	0	-1/4	1/2	
$Z_j - C_j$		1350	4	0	0	0	2	1	

Since all $(Z_j - C_j) \geq 0$ the current basic feasible solution is optimal. The optimal solution is Max. Z = 1350, $x_1 = 0$, $x_2 = 100$ and $x_3 = 230$.