HISTOGRAM PROCESSING:

The histogram of a digital image with gray levels in the range [0, L-1] is a discrete function of the form

$$H(r_k)=n_k$$

where r_k is the k^{th} gray level and n_k is the number of pixels in the image having the level r_k.. A normalized histogram is given by the equation

$$p(r_k)=n_k/n$$

for

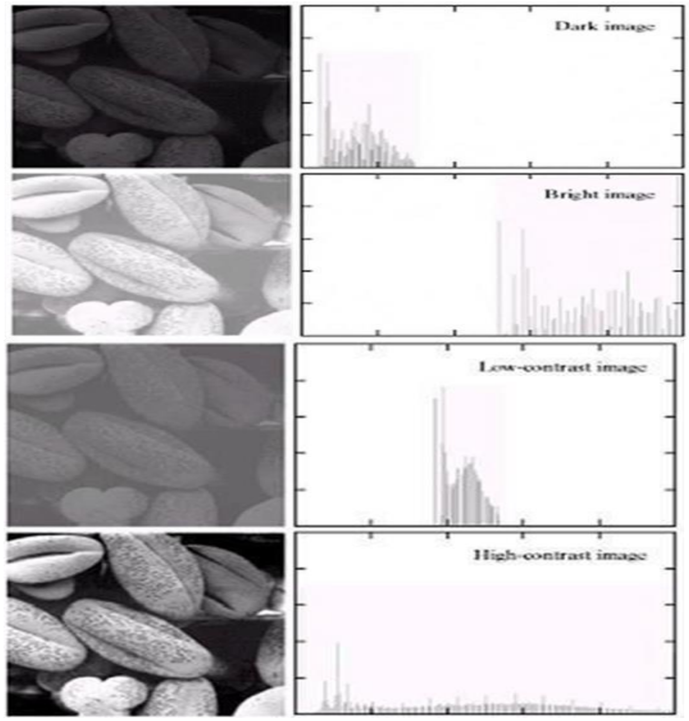
k=0,1,2,…..,L-1

$P(r_k)$ gives the estimate of the probability of occurrence of gray level r_k. The sum of all components of a normalized histogram is equal to 1.

The histogram plots are simple plots of $p(r_k)=n_k$ versus r_k.

In the dark image the components of the histogram are concentrated on the low (dark) side of the gray scale. In case of bright image the histogram components are baised towards the high side of the gray scale. The histogram of a low contrast image will be narrow and will be centered towards the middle of the grayscale

The components of the histogram in the high contrast image cover a broad range of the gray scale. The net effect of this will be an image that shows a great deal of gray levels details and has high dynamic range.



HISTOGRAM EQUALIZATION:

Histogram equalization is a common technique for enhancing the appearance of images. Suppose we have an image which is predominantly dark. Then its histogram would be skewed towards the lower end of the grey scale and all the image detail are compressed into the dark end of the histogram. If we could 'stretch out' the grey levels at the dark end to produce a more uniformly

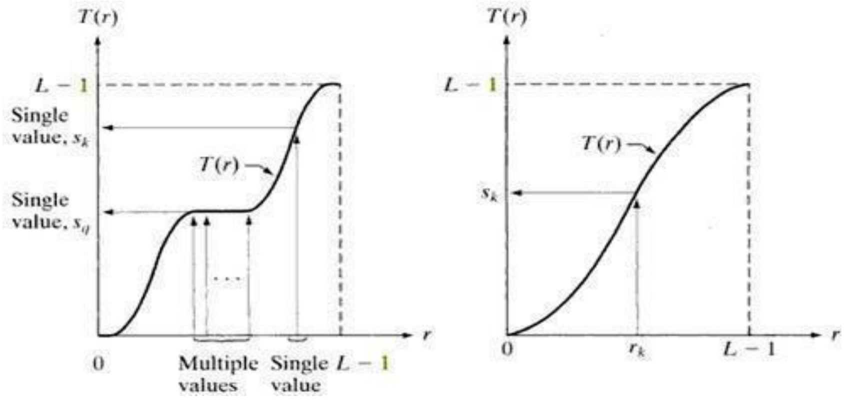
distributed histogram then the image would become much clearer.

Let there be a continuous function with r being gray levels of the image to be enhanced. The range of r is [0, 1] with r=0 representing black and r=1 representing white. The transformation function is of the form

$$S=T(r) \text{ where } 0<r<1$$

It produces a level s for every pixel value r in the original image.

a b
FIGURE
(a) Monotonically increasing function, showing how multiple values can map to a single value.
(b) Strictly monotonically increasing function. This is a one-to-one mapping, both ways.



The transformation function is assumed to fulfill two condition T(r) is single valued and monotonically increasing in the internal 0<T(r)<1 for 0<r<1.The transformation function should be single valued so that the inverse transformations should exist. Monotonically increasing condition preserves the increasing order from black to white in the output image. The second conditions guarantee that the output gray levels will be in the same range as the input levels. The gray levels of the image may be viewed as random variables in the interval [0.1]. The most fundamental descriptor of a random variable is its probability density function (PDF) Pr(r) and Ps(s) denote the probability density functions of random variables r and s respectively. Basic results from an elementary probability theory states that if Pr(r) and Tr are known and T-1(s) satisfies conditions (a), then the probability density function Ps(s) of the transformed variable is given by the formula

$$p_s(s) = p_r(r)\left|\frac{dr}{ds}\right|$$

Thus the PDF of the transformed variable s is the determined by the gray levels PDF of the input image and by the chosen transformations function.
A transformation function of a particular importance in image processing

$$s = T(r) = (L - 1)\int_0^r p_r(w)\,dw$$

This is the cumulative distribution function of r.

L is the total number of possible gray levels in the image.

HISTOGRAM-BASED THRESHOLDING

We will denote the histogram of pixel values by h0, h1,...,hN , where hk specifies the number of pixels in an image with grey scale value k and N is the maximum pixel value (typically 255). Ridler and Calvard (1978) and Trussell (1979) proposed a simple algorithm for choosing a single threshold. We shall refer to it as the inter means algorithm. First we will describe the algorithm in words, and then mathematically.

Initially, a guess has to be made at a possible value for the threshold. From this, the mean values of pixels in the two categories produced using this threshold are calculated. The threshold is repositioned to lie exactly half way between the two means. Mean values are calculated againand a new threshold is obtained, and so on until the threshold stops changing value. Mathematically, the algorithm can be specified as follows.

- Make an initial guess at t: for example, set it equal to the median pixel value, that is, the value for which

$$\sum_{k=0}^{t} h_k \geq \frac{n^2}{2} > \sum_{k=0}^{t-1} h_k,$$

where n^2 is the number of pixels in the $n \times n$ image.

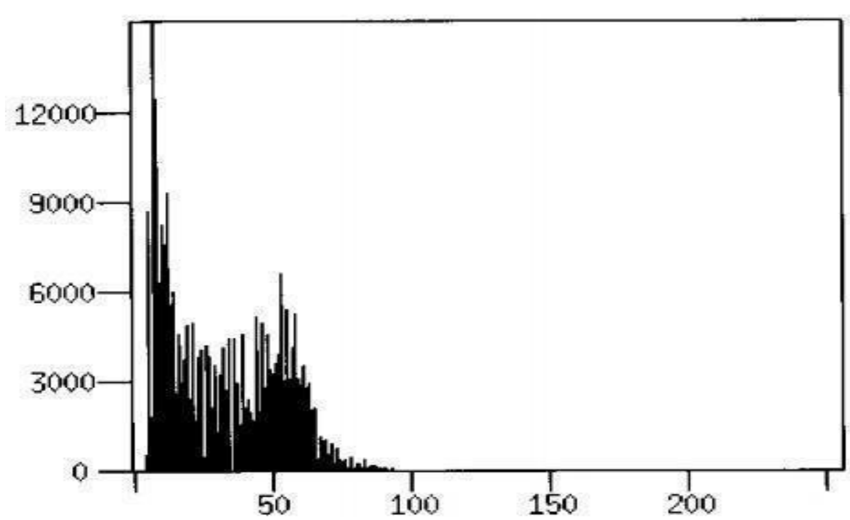


Figure 4.3: Histogram of soil image.

- Calculate the mean pixel value in each category. For values less than or equal to t, this is givenby:

$$\mu_1 = \sum_{k=0}^{t} kh_k \Bigg/ \sum_{k=0}^{t} h_k$$

Whereas, for values greater than t, it is given by:

$$\mu_2 = \sum_{k=t+1}^{N} kh_k \Bigg/ \sum_{k=t+1}^{N} h_k \, .$$

- Re-estimate t as half-way between the two means ,i.e.

$$t = \left[\frac{\mu_1 + \mu_2}{2}\right]$$

where [ ] denotes 'the integer part of' the expression between the brackets.

- Repeat steps (2) and (3) until 't' stops changing value between consecutive evaluations.

Fig 4.3 shows the histogram of the soil image. From an initial value of t = 28 (the median pixel value), the algorithm changed t to 31, 32, and 33 on the first three iterations, and then t remained unchanged. The pixel means in the two categories are 15.4 and 52.3. Fig 4.4(a) shows the result of using this threshold. Note that this value of t is considerably higher than the threshold value of 20 which we favored in the manual approach.

The inter means algorithm has a tendency to find a threshold which divides the histogram in two, so that there are approximately equal numbers of pixels in the two categories. In many applications, such as the soil image, this is not appropriate. One way to overcome this drawback is to modify the algorithm as follows.

Consider a distribution which is a mixture of two Gaussian distributions. Therefore, in the absence of sampling variability, the histogram is given by:

$$h_k = n^2\{p_1\phi_1(k) + p_2\phi_2(k)\}, \qquad \text{for } k = 0, 1, \ldots, N.$$

Here, p1 and p2 are proportions (such that p1 + p2 = 1) and φl(k) denotes the probability density of a Gaussian distribution, that is

$$\phi_l(k) = \frac{1}{\sqrt{2\pi\sigma_l^2}} \exp\left\{\frac{-(k-\mu_l)^2}{2\sigma_l^2}\right\} \qquad \text{for } l = 1, 2,$$

where μl and σ^2l are the mean and variance of pixel values in category l. The best classification criterion, i.e. the one which misclassifies the least number of pixels, allocates pixels with value k to category 1 if p

$$p_1\phi_1(k) \geq p_2\phi_2(k),$$

and otherwise classifies them as 2. After substituting for φ and taking logs, the inequality becomes

$$k^2\left\{\frac{1}{\sigma_1^2} - \frac{1}{\sigma_2^2}\right\} - 2k\left\{\frac{\mu_1}{\sigma_1^2} - \frac{\mu_2}{\sigma_2^2}\right\} + \left\{\frac{\mu_1^2}{\sigma_1^2} - \frac{\mu_2^2}{\sigma_2^2} + \log\frac{\sigma_1^2 p_2^2}{\sigma_2^2 p_1^2}\right\} \leq 0.$$

The left side of the inequality is a quadratic function in k. Let A, B and C denote the three terms in curly brackets, respectively. Then the criterion for allocating pixels with value k to category 1 is:

$$k^2A - 2kB + C \leq 0.$$

There are three cases to consider:

- If A = 0 (i.e. $\sigma 1^2 = \sigma 2^2$), the criterion simplifies to one of allocating pixels with value k to category 1 if

$$2kB \geq C.$$

- (If, in addition, p1 = p2 and μ1 < μ2, the criterion becomes k ≤ 1/ 2 {μ1 + μ2}. Note that this is the intermeans criterion, which implicitly assumes that the two categories are of equal size.)

- If B < AC, then the quadratic function has no real roots, and allpixels are classified as 1 if A
- < 0 (i.e. $\sigma 1^2 > \sigma 2^2$), or as 2 if A > 0
- Otherwise, denote the roots t1 and t2 , where t1 ≤ t2and

$$t_1, t_2 = \frac{B \pm \sqrt{B^2 - AC}}{A}.$$

The criteria for category 1 are

$$t_1 < k \leq t_2 \qquad \text{if } A > 0,$$

$$k \leq t_1 \text{ or } k > t_2 \qquad \text{if } A < 0.$$

In practice, cases (a) and (b) occur infrequently, and if μ1 < μ2 the rule simplifies to thethreshold:

$$\text{category 1 if a pixel value} \quad k \leq \frac{B + \sqrt{B^2 - AC}}{A}.$$

Kittler and Illingworth (1986) proposed an iterative minimum-error algorithm, which is based on this threshold and can be regarded as a generalization of the inter means algorithm. Again, we will describe the algorithm in words, and then mathematically.

From an initial guess at the threshold, the proportions, means and variances of pixel values in the two categories are calculated. The threshold is repositioned according to the above criterion, and proportions, means and variances are recalculated. These steps are repeated until there are no changes in values between iterations.

Mathematically:

- ❖ Make an initial guess at a value fort.

$$p_1 = \frac{1}{n^2}\sum_{k=0}^{t} h_k,$$

$$\mu_1 = \frac{1}{n^2 p_1}\sum_{k=0}^{t} k h_k,$$

$$\text{and } \sigma_1^2 = \frac{1}{n^2 p_1}\sum_{k=0}^{t} k^2 h_k \quad - \quad \mu_1^2.$$

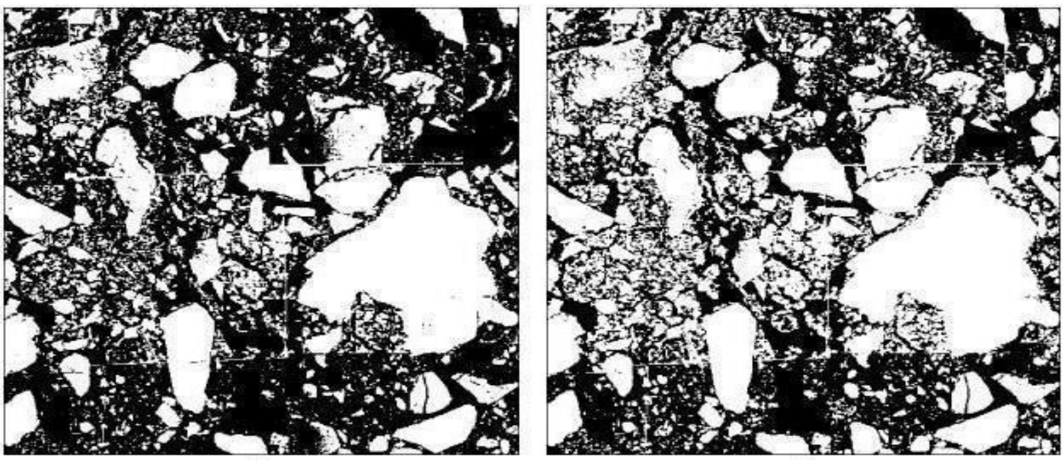
- ❖ Estimate p1, μ1 and σ1 ^{2} for pixels with values less than or equal to t,by

- ❖ Similarly, estimate p2, μ2 and σ2 ^{2} for pixels in the range t + 1 to N.

- ❖ Re-estimate tby

$$t = \left[\frac{B + \sqrt{B^2 - AC}}{A}\right],$$

- ❖ where A, B, C and [ ] have already been defined.
- ❖ Repeat steps (2) and (3) until *t* converges to a stable value.



(a) (b)

Figure 4.4: Segmentations of the soil image, obtained by thresholding at: **(a)** 33, selected by the intermeans algorithm, **(b)** 24, selected by the minimum-error algorithm.

When applied to the soil image, the algorithm converged in 4 iterations to t = 24. Fig 4.4(b) shows the result, which is more satisfactory than that produced by the inter means

Algorithm because it has allowed for a smaller proportion of air pixels ($p1 = 0.45$, as compared with $p2 = 0.55$). The algorithm has also taken account of the air pixels being less variable in value than those for the soil matrix ($\sigma1^2 = 30$, whereas $\sigma2^2 = 186$). This is in accord with the left- most peak in the histogram plot (Fig 4.3) being quite narrow.