

DESIGN OF EXPERIMENTS

ANOVA – Analysis of Variance

Working Rule (One – Way Classification)

Set the null hypothesis H_0 : There is no significance difference between the treatments.

Set the alternative hypothesis H_1 : There is a significance difference between the treatments.

Step: 1 Find N = number of observations

Step: 2 Find T = The total value of observations

Step: 3 Find the Correction Factor $C.F = \frac{T^2}{N}$

Step: 4 Calculate the total sum of squares and find the total sum of squares

$$TSS = (\sum X_1^2 + \sum X_2^2 + \sum X_3^2 + \dots) - C.F$$

Step: 5 Column sum of squares $SSC \left(\frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_2} + \frac{(\sum X_3)^2}{N_3} + \dots \right) - C.F$

Where N_i = Total number of observation in each column ($i = 1, 2, 3, \dots$)

Step: 6 Prepare the ANOVA to calculate F – ratio

Source of variation	Sum of Degrees	Degrees of Freedom	Mean Square	F - Ratio
Between Samples	SSC	$K - 1$	$MSC = \frac{SSC}{K-1}$	$F_c = \frac{MSC}{MSE}$ if $MSC > MSE$
Within Samples	SSE	$N - K$	$MSE = \frac{SSE}{N-K}$	$F_c = \frac{MSE}{MSC}$ if $MSE > MSC$

Step: 7 Find the table value (use chi square table)

Step: 8 Conclusion:

Calculated value $<$ Table value, then we accept null hypothesis.

Calculated value $>$ Table value, then we reject null hypothesis.

PROBLEMS ON ONE WAY ANOVA

1.A completely randomised design experiment with 10 plots and 3 treatments gave the following results.

Plot No	1	2	3	4	5	6	7	8	9	10
Treatment	A	B	C	A	C	C	A	B	A	B
Yield	5	4	3	7	5	1	3	4	1	7

Analyse the result for treatment effects.

Solution:

Set the null hypothesis H_0 : There is no significance difference between the treatments.

Set the alternative hypothesis H_1 : There is a significance difference between the treatments.

Treatments	Yields from plots			
A	5	7	3	1
B	4	4	7	-
C	3	5	1	-

TABLE:

Treatment A		Treatment B		Treatment C	
X_1	X_1^2	X_2	X_2^2	X_3	X_3^2
5	25	4	16	3	9
7	49	4	16	5	25
3	9	7	49	7	7
1	1	-	-	-	-
$\sum X_1 = 16$	$\sum X_1^2 = 84$	$\sum X_2 = 5$	$\sum X_2^2 = 81$	$\sum X_3 = 9$	$\sum X_3^2 = 35$

Step: 1 $N = 10$

Step: 2 Sum of all the items (T) = $\sum X_1 + \sum X_2 + \sum X_3 = 16 + 15 + 9 = 40$

Step: 3 Find the Correction Factor C . $F = \frac{T^2}{N} = \frac{(40)^2}{10} = 160$

Step: 4 TSS = Total sum of squares

= sum of squares of all the items – C. F

$$\begin{aligned} \text{TSS} &= (\sum X_1^2 + \sum X_2^2 + \sum X_3^2 + \dots) - C.F \\ &= (84 + 81 + 35) - 160 = 40 \end{aligned}$$

Step: 5 SSC = Sum of squares between samples

$$\begin{aligned} \text{SSC} &= \left(\frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_2} + \frac{(\sum X_3)^2}{N_3} + \dots \right) - C.F \\ \text{SSC} &= \left(\frac{(16)^2}{4} + \frac{(15)^2}{3} + \frac{(9)^2}{3} + \dots \right) - 160 \\ &= 64 + 75 + 27 - 160 = 6 \end{aligned}$$

Step: 6 MSC = Mean squares between samples

$$\begin{aligned} &= \frac{\text{Sum of squares between samples}}{d.f} \\ &= \frac{6}{2} = 3 \end{aligned}$$

SSE = Sum of squares within samples

$$\begin{aligned} &= \text{Total sum of squares} - \text{Sum of squares between samples} \\ &= 40 - 6 = 34 \end{aligned}$$

Step: 7 MSE = Mean squares within samples

$$\begin{aligned} &= \frac{\text{Sum of squares within samples}}{d.f} \\ &= \frac{34}{7} = 4.86 \end{aligned}$$

ANOVA TABLE

Source of variation	Sum of Degrees	Degrees of Freedom	Mean Square	F - Ratio
Between Samples	SSC = 6	$K - 1 = 3 - 1 = 2$	$\text{MSC} = \frac{\text{SSC}}{K-1} = 3$	
Within Samples	SSE = 34	$N - K = 10 - 3 = 7$	$\text{MSE} = \frac{\text{SSE}}{N-K} = 4.86$	$F_c = \frac{\text{MSE}}{\text{MSC}} = 1.62$

d.f for (7, 2) at 5% level of significance is 19.35

Step: 8 Conclusion:

Calculated value < Table value, then we accept null hypothesis.

2. Three different machines are used for a production. On the basis of the outputs, set up one – way ANOVA table and test whether the machines are equally effective.

Outputs		
Machine I	Machine II	Machine III
10	9	20
15	7	16
11	5	10
10	6	14

Given that the value of F at 5% level of significance for (2, 9) d. f is 4.26

Solution:

Set the null hypothesis H_0 : The machines are equally effective.

TABLE:

Treatment A		Treatment B		Treatment C	
X_1	X_1^2	X_2	X_2^2	X_3	X_3^2
10	100	9	81	20	400
15	225	7	49	16	256
11	121	5	25	10	100
20	400	6	36	14	196
$\sum X_1 = 56$	$\sum X_1^2 = 846$	$\sum X_2 = 27$	$\sum X_2^2 = 191$	$\sum X_3 = 60$	$\sum X_3^2 = 952$

Step: 1 $N = 12$

Step: 2 Sum of all the items (T) = $\sum X_1 + \sum X_2 + \sum X_3 = 56 + 27 + 60 = 143$

Step: 3 Find the Correction Factor C . $F = \frac{T^2}{N} = \frac{(143)^2}{12} = 1704.08$

Step: 4 TSS = Total sum of squares

= sum of squares of all the items – C. F

$$\begin{aligned} \text{TSS} &= (\sum X_1^2 + \sum X_2^2 + \sum X_3^2 + \dots) - C.F \\ &= (846 + 191 + 952) - 1704.08 = 284.92 \end{aligned}$$

Step: 5 SSC = Sum of squares between samples

$$\begin{aligned} \text{SSC} &= \left(\frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_2} + \frac{(\sum X_3)^2}{N_3} + \dots \right) - C.F \\ \text{SSC} &= \left(\frac{(56)^2}{4} + \frac{(27)^2}{4} + \frac{(60)^2}{4} + \dots \right) - 1704.08 \\ &= 784 + 182.25 + 900 - 1704.08 = 162.17 \end{aligned}$$

Step: 6 MSC = Mean squares between samples

$$\begin{aligned} &= \frac{\text{Sum of squares between samples}}{d.f} \\ &= \frac{162.17}{2} = 81.085 \end{aligned}$$

SSE = Sum of squares within samples

$$\begin{aligned} &= \text{Total sum of squares} - \text{Sum of squares between samples} \\ &= 284.92 - 162.17 = 122.75 \end{aligned}$$

Step: 7 MSE = Mean squares within samples

$$\begin{aligned} &= \frac{\text{Sum of squares within samples}}{d.f} \\ &= \frac{122.75}{9} = 13.63 \end{aligned}$$

ANOVA TABLE

Source of variation	Sum of Degrees	Degrees of Freedom	Mean Square	F - Ratio
Between Samples	SSC = 162.17	$K - 1 = 3 - 1 = 2$	$\text{MSC} = \frac{\text{SSC}}{K-1} = 81.085$	
Within Samples	SSE = 122.75	$N - K = 12 - 3 = 9$	$\text{MSE} = \frac{\text{SSE}}{N-K} = 13.63$	$F_c = \frac{\text{MSE}}{\text{MSC}} = 5.95$

d.f for (2, 9) at 5% level of significance is 4.26.

Step: 8 Conclusion:

Calculated value $>$ Table value, then we reject the null hypothesis.

i.e., the three machines are not equally effective.

