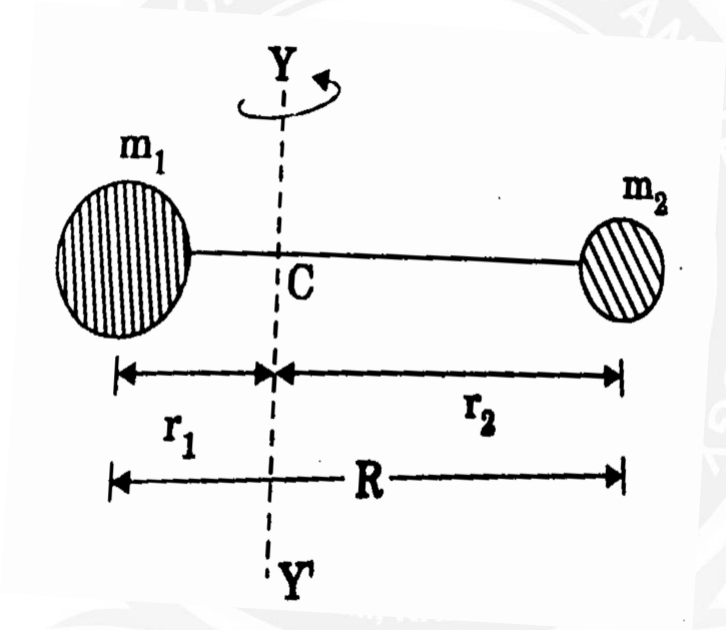
UNIT I

Mechanics

1.4 Moment of Inertia of a diatomic molecules

A diatomic molecule, in its stable equilibrium position consists two atoms that are at a distance 'R' apart. The distance 'R' is called the band length between the two atoms.

Presently we can consider that it consists of two tiny spheres at either end of a thin weightless rigid rod, as shown in fig. This kind of arrangement can be called as rigid rotor.



Let 'C' be the center of mass of the molecule and r_1 and r_2 respective distances of the two atoms from it.

Then

$$r_1 + r_2 = R \qquad \text{-------- (1)}$$

and

$$m_1 r_1 = m_2 r_2 \qquad \text{-------- (2)}$$

where m_1 and m_2 are the masses of two atoms respectively.

From eqn. (1),

$$r_1 = R - r_2 \qquad \text{--------} (3)$$

and from eqn. (2),

$$r_2 = \frac{m_1 r_1}{m_2} \qquad \text{--------} (4)$$

so,

$$r_1 = R - \frac{m_1 r_1}{m_2}$$

$$R = r_1 + \frac{m_1 r_1}{m_2} = r_1[\,1 + \frac{m_1}{m_2}]\text{--------} (5)$$

$$R = \frac{R}{[\,1 + \frac{m_1}{m_2}]} \qquad \text{--------} (6)$$

Now, the moment of inertia of the molecule (i.e., of the two atoms) about an axis passing through the centre of mass 'C' and perpendicular to the bond is given as

$$I = m_1 r_1^2 + m_2 r_2^2 \quad \text{--------} (7)$$

$$I = m_1 r_1 r_1 + m_1 r_1 r_2 \quad \text{--------} (6) \quad [\therefore \text{ from eqn. } (2)]$$

$$I = m_1 r_1 (r_1 + r_2)$$

(or) by using eqn. (1),

$$I = m_1 r_1 R \quad \text{--------} (8)$$

Substituting eqn. (6) in (8) gives

$$I = m_1 R \,[\frac{R}{[\,1 + \frac{m_1}{m_2}]}]$$

$$I = \frac{m_1 R^2}{[\,1 + \frac{m_1}{m_2}]}$$

$$= \frac{m_1 R^2}{[\frac{m_2 + m_1}{m_2}]}$$

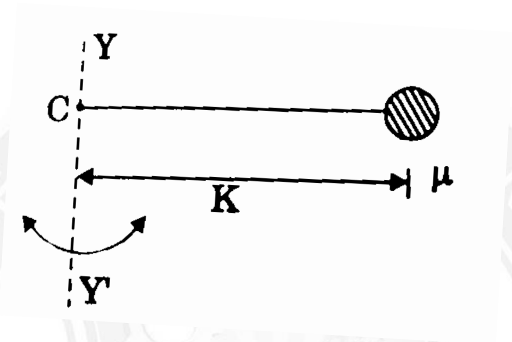
$$= \frac{m_1 m_2 R^2}{m_2 + m_1}$$

(or)

$$I = \left[\frac{m_1 m_2}{m_2 + m_1} \right] R^2$$

$$I = \mu R^2 \qquad \text{-------- (9)}$$

Where $\mu = \frac{m_1 m_2}{m_2 + m_1}$ is called as reduced mass of the molecule. Thus the figure can also be redrawn as



In figure, K=R, which is called radius of gyration, so moment of inertia

$$I = \mu K^2 \qquad \text{-------- (10)}$$