4.6. Quantum Channels

Introduction

Quantum information must be transmitted or processed through physical systems. A quantum channel is the mathematical description of this evolution. It can represent transmission through optical fibre, free space, a quantum memory, or an operation inside a quantum processor. A channel may preserve information or introduce noise and loss.

Definition

A quantum channel is a completely positive, trace-preserving (CPTP) map that transforms an input density operator ρ into an output state E(ρ).

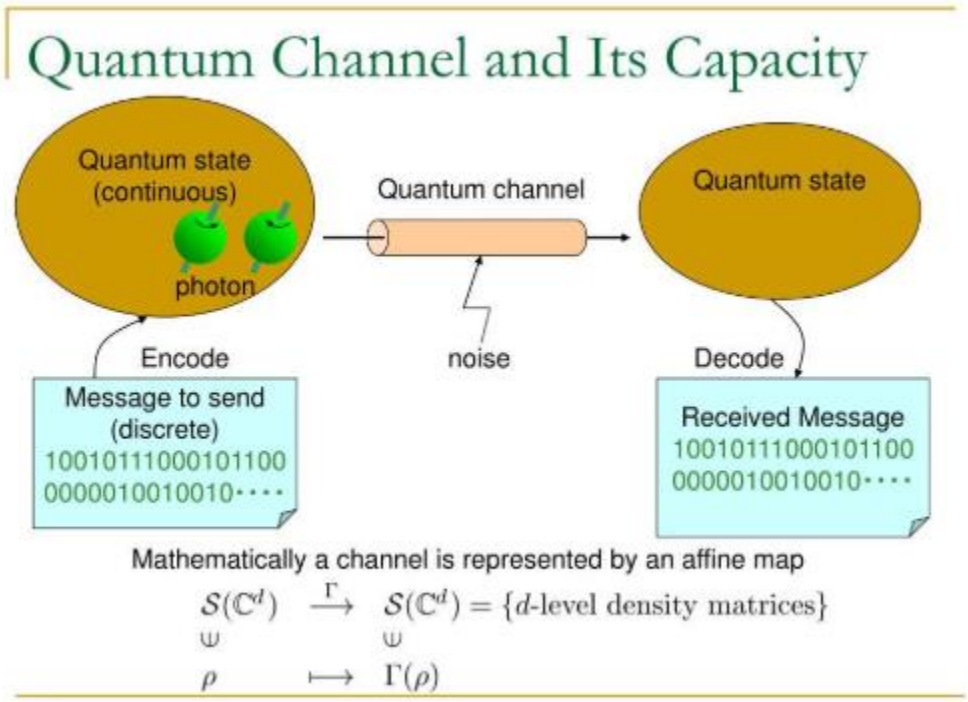
Main purposes of quantum channels:

1. To transfer quantum information between two locations.
2. To describe noise and errors during transmission.
3. To support quantum communication and quantum networks.
4. To transmit quantum states between quantum computers.
5. To maintain the accuracy and reliability of quantum information.

Basic Diagram of a Quantum Channel

1. **Sender:** The sender generates the original quantum information.
2. **Quantum State:** Information is represented using qubits.
3. **Quantum Channel:** The qubits travel through a communication medium.
4. **Noise:** During transmission, external disturbances may affect the quantum state.
5. **Receiver:** The receiver obtains the transmitted quantum information.

6. **Output State:** The receiver gets the transmitted state, which may be affected by noise.



Working Principle of Quantum Channels

The working of a quantum channel involves the following steps:

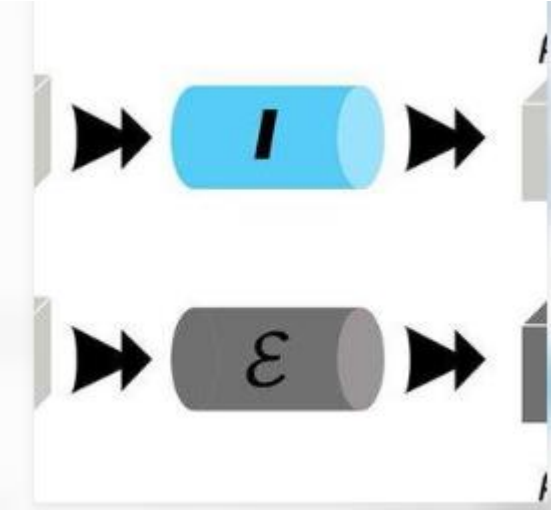
1. The sender prepares a quantum state using one or more qubits.
2. The quantum state is given as input to the channel.
3. The channel transfers the quantum state through a communication medium.
4. During transmission, noise or environmental disturbances may occur.
5. The channel transforms the original quantum state into an output state.
6. The receiver receives the output quantum state.
7. The receiver performs measurements or further quantum operations to obtain useful information.

Types of Quantum Channels

There are several types of quantum channels. Each channel represents a different way in which quantum information can be transmitted or affected by noise.

4.1 Identity Quantum Channel

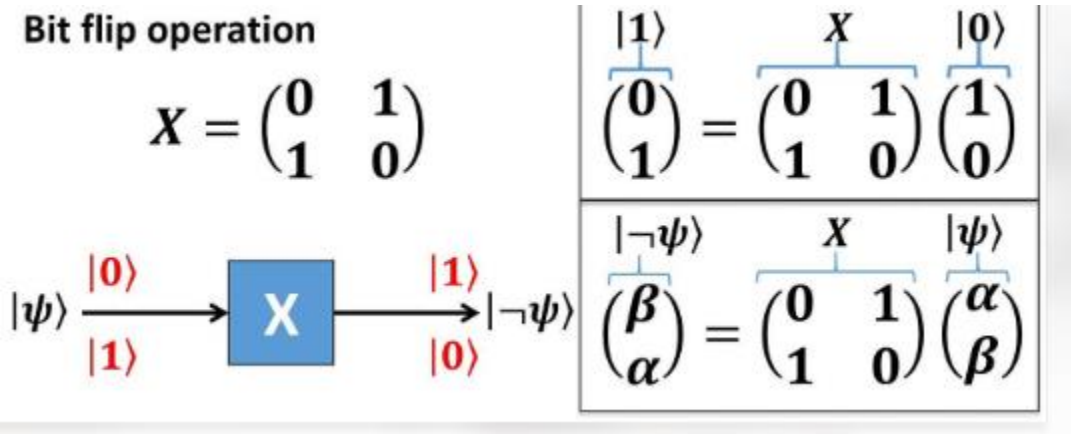
An identity quantum channel transfers quantum information without changing the original quantum state.



- The original quantum state enters the channel.
- No noise or disturbance occurs.
- The output state remains the same as the input state.
- It is an ideal model of quantum communication.

4.2 Bit-Flip Quantum Channel

A bit-flip channel changes the value of a qubit from 0 to 1 or from 1 to 0 with some probability.



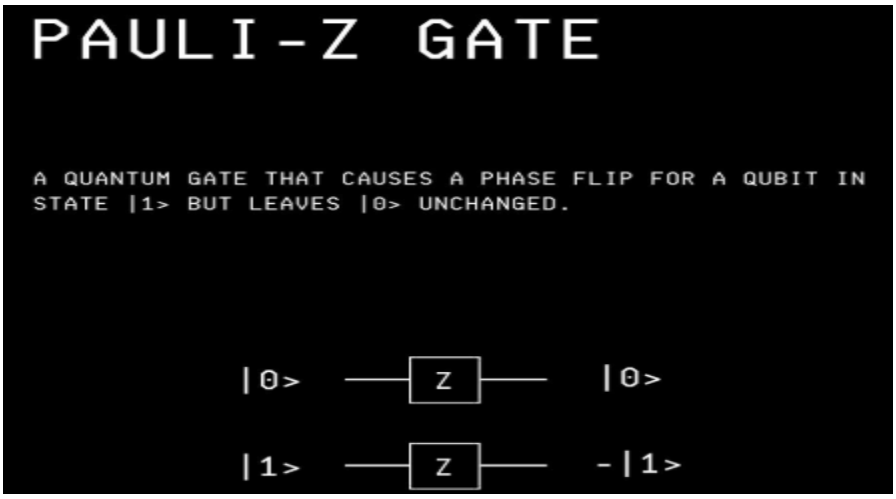
- The sender transmits a qubit.
- Noise may change the qubit value.
- The Pauli-X operation represents the bit-flip error.
- The receiver may receive an incorrect bit value.

Example:

Application: Quantum error correction and reliable quantum communication.

4.3 Phase-Flip Quantum Channel

A phase-flip channel changes the phase of a quantum state without changing its computational basis bit value.



- The quantum state passes through the channel.
- Noise changes the relative phase of the quantum state.
- The Pauli-Z operation represents the phase-flip error.
- The output state has a changed phase.

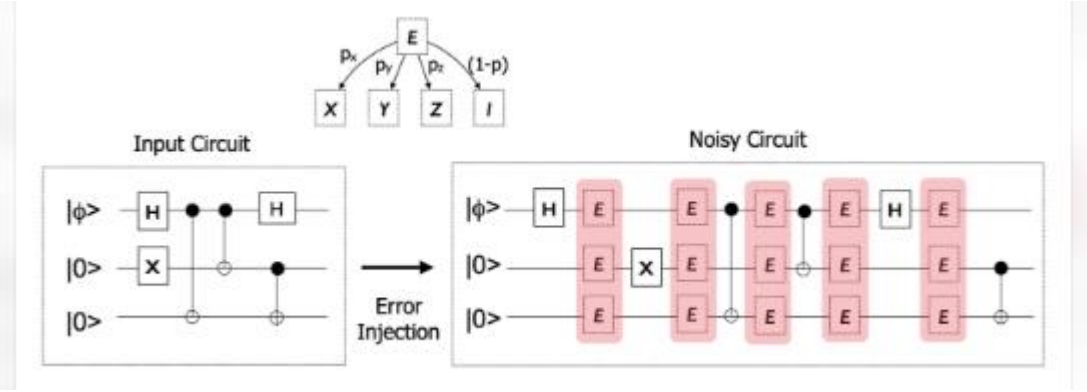
Example:

$$|0\rangle \rightarrow |0\rangle$$

$$|1\rangle \rightarrow -|1\rangle$$

4.4 Depolarizing Quantum Channel

A depolarizing channel introduces random errors into a quantum state. It can cause bit-flip, phase-flip, or combined bit-and-phase errors.



- The original quantum state enters the channel.
- Random quantum errors may occur.
- The quantum state becomes mixed or less pure.
- The receiver receives a noisy quantum state.

Application: Modelling noise in quantum computers and quantum communication systems.

4.5 Amplitude Damping Channel

An amplitude damping channel describes the loss of energy from a quantum system.

Noise Channels

Channel	Kraus set $\{E_m\}$	Short description
Bit flip	$E_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$	Flips the qubit state of $\|0\rangle$ and $\|1\rangle$
Phase flip	$E_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$	Imprints a phase flip on $\|0\rangle$ and $\|1\rangle$
Depo-larizing	$E_0 = \sqrt{1-p}\, I_2$ $E_1 = \sqrt{\frac{p}{}}\, X$ $E_2 = \sqrt{\frac{p}{}}\, Y$ $E_3 = \sqrt{\frac{p}{}}\, Z$	Random Pauli process on the full Bloch sphere
Ampli-tude damping	$E_0 = \|0\rangle\langle 0\| + \sqrt{1-p}\|1\rangle$ $E_1 = \sqrt{p}\,\|0\rangle\langle 1\|$	Energy loss to the environment
Ampli-tude	$FK\|0\rangle = \nabla 0\|/1$	Energy loss to the environment

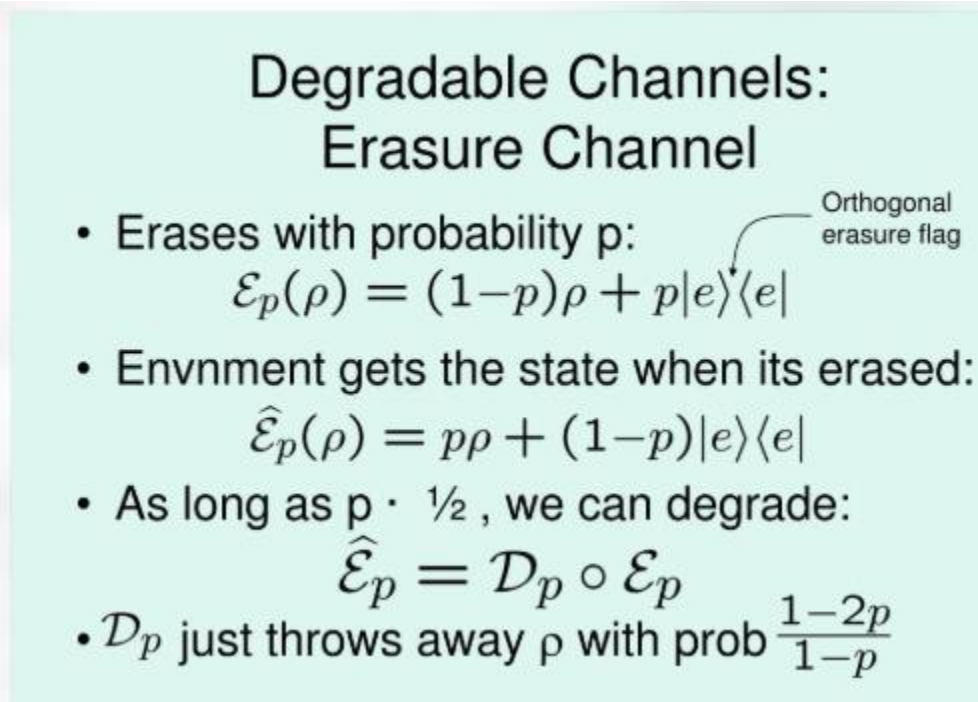
Explanation:

- The qubit may initially be in an excited state.
- Due to environmental interaction, it can lose energy.
- The qubit may change from state \(|1\rangle\) to state \(|0\rangle\).
- This process represents energy loss or relaxation.

Application: Modelling energy loss in physical quantum devices.

4.6 Erasure Quantum Channel

An erasure channel represents a situation in which quantum information is lost during transmission, but the receiver knows that the information has been erased.



- The sender transmits quantum information.
- The channel may lose the transmitted information.
- The receiver identifies that an erasure has occurred.
- The lost information cannot be directly recovered from that transmission.

Application: Quantum communication networks and error correction.

4.6.1 Mathematical representation

A general quantum channel can be written in Kraus form: $E(\rho) = \Sigma_k E_k \rho E_k^\dagger$, where the Kraus operators satisfy $\Sigma_k E_k^\dagger E_k = I$ for a trace-preserving channel. Complete positivity ensures the map remains physically valid even when the system is entangled with another system.

4.6.2 Ideal channel

An ideal identity channel leaves the state unchanged: $E(\rho) = \rho$. A unitary channel applies a reversible quantum operation U, giving $E(\rho) = U\rho U^\dagger$. These models are useful for describing perfect gates and ideal transmission.

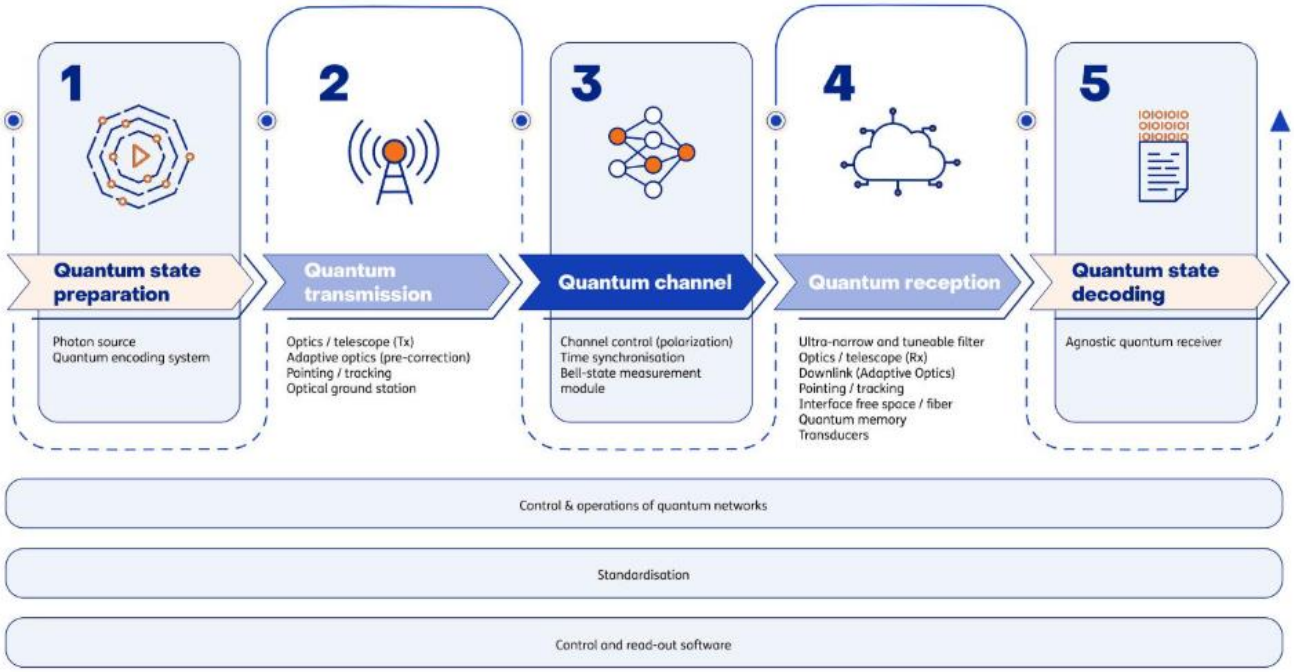
4.6.3 Common noisy channels

A bit-flip channel applies the Pauli-X operation with some probability. A phase-flip channel applies Pauli-Z with some probability. A depolarizing channel replaces part of the state with a maximally mixed state. Amplitude damping models energy loss, while dephasing reduces phase coherence without necessarily changing the energy population.

4.6.4 Channel parameters and performance

Channel quality can be described using error probability, loss probability, fidelity, capacity and other measures. The relevant metric depends on the task. A channel suitable for transmitting classical measurement outcomes may not be adequate for preserving entanglement or transmitting unknown quantum states.

Overall Quantum Channel Block Diagram



1. Quantum state preparation generates the required qubits.
2. Quantum encoding prepares the information for transmission.
3. The quantum channel carries the encoded quantum information.
4. Noise may affect the information during transmission.
5. Quantum decoding processes the received quantum information.
6. The receiver obtains the final quantum state.

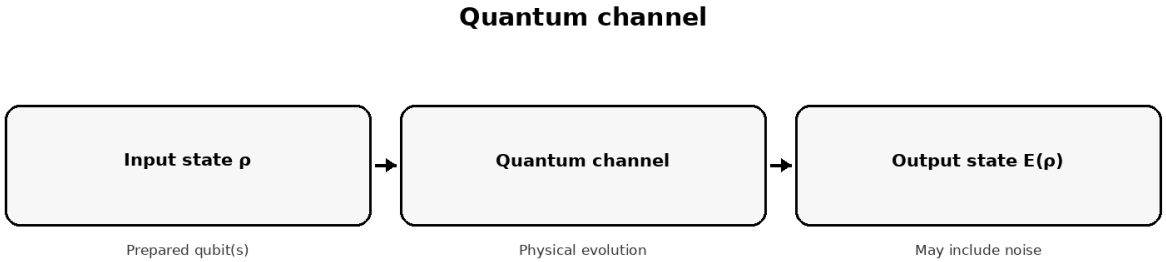
4.6.5 Classical versus quantum channels

A classical channel transmits classical symbols or bits. A quantum channel transmits quantum states and must preserve their coherence sufficiently for the intended task. Quantum channels can also carry classical information, but their defining feature is that they act on quantum states and may interact with entangled reference systems.

4.6.6 Channel uses and limitations

Quantum channels are used to model communication links, gates, memories and interactions with environments. Their practical performance is limited by photon loss, thermal noise, imperfect controls, detector errors and decoherence. Channel characterisation is important before building a reliable quantum network.

Diagram



The input density matrix ρ enters a quantum channel E and emerges as E(ρ). The channel may be ideal or noisy. The mathematical map describes the complete physical effect on the quantum state.

Advantages

- Enables the transmission of quantum information.
- Supports secure quantum communication protocols.
- Helps develop quantum networks.
- Allows researchers to study quantum noise.
- Supports quantum error detection and correction.
- Enables communication between quantum computers.

Disadvantages / limitations

- Quantum information is sensitive to environmental noise.
- Maintaining quantum states over long distances is difficult.

- Quantum communication equipment can be expensive.
- Signal loss can affect transmission quality.
- Quantum states cannot be copied perfectly because of the no-cloning theorem.
- Quantum repeaters and error correction require advanced technology.

Applications

Quantum channels are used in several important areas:

1. **Quantum Communication:** Used to transfer quantum information between distant locations.
2. **Quantum Cryptography:** Helps in secure communication and quantum key distribution.
3. **Quantum Computing:** Describes noise and errors in quantum processors.
4. **Quantum Networks:** Connects different quantum computers and quantum devices.
5. **Quantum Teleportation:** Supports the transfer of quantum states using shared entanglement and classical communication.
6. **Quantum Error Correction:** Helps study and correct errors occurring during quantum information transmission.
7. **Quantum Internet:** Provides the communication foundation for connecting quantum devices.

Thus quantum channel is the basic model for quantum information transfer and processing. CPTP maps provide a rigorous way to describe ideal operations, noise, loss and realistic communication links.